Semester Project

Magnetic Detumbling control of a spacecraft using magnetorquers

ASEN 5010

Mid-Project (1-3) Report Due Date: April 12^{th} 2018
Final Project Report Due Date: April 26^{th} 2018

The high-level goal of this ASEN 5010 project is to detumble a spacecraft in Low Earth Orbit (LEO) using torque rods as attitude actuators. In order to satisfy the mission requirements, the spacecraft's angular rates must be driven down to below a desired threshold. A Magnetorquer or torque rod uses earth's local magnetic field to generate a control torque, by magnetizing its coil using electric current. You will implement a torque control law $\boldsymbol{u}$, which will generate this desired 'command magnetic dipole' $\boldsymbol{m}$ that provides the torque to drive the satellite's tumbling angular velocity $\boldsymbol{\omega}_{B/N}$ below the threshold. The scope of this project encompasses modeling of the geomagnetic field, rigid body attitude kinematics and dynamics formulation and nonlinear feedback control. By the end of this project, you will have gained valuable practical experience in these areas thanks to analytic derivations and software implementation of the different project milestones.

1 Mission Profile

A spacecraft has just been inserted into a circular, inclined, Low Earth Orbit (LEO) and it is tumbling with high angular velocities given by $\boldsymbol{\omega}_{B/N}$. For simplicity, it is assumed that the spacecraft is in a simple two-body circular motion. The spacecraft's orbit can be characterized by an altitude of $h =$ 450 km (i.e., the orbit radius is $r = R_{Earth} + h$ where $R_{Earth} = 6378$ km), an inclination i, and a right ascension Ω. These two angles with be defined later in the project. Since the LEO orbit is circular, its angular rate $\dot{\theta} = \sqrt{\mu/r^3}$ is constant, where $\theta(t_0)$ will be defined later and $\mu = 398600$ km^{2}s^{-2}.

This LEO spacecraft is equipped with multiple magnetorquers to provide control torques. These actuators are located along on the three orthogonal principle body axes of the spacecraft. In the following sections, you will derive the expressions for the attitude of the satellite, as well as the associated angular rate error. Once these quantities are known and related to the spacecraft's current state, you will eventually implement a B-dot control law that drives the spacecraft angular rates below a threshold using its local magnetic field to generate a control torque. The two variations of the B-dot control used, will be a modulating control law and a bang-bang control law. You will analyze the performance of each control law using a Monte-Carlo analysis, and draw conclusions on their performance bounds such as steady state error and settling time.

2 Mission Parameters

The following frames of reference will be employed in this mission.

- Inertial reference frame $\mathcal{N} = \{O, \hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}_3\}$
- Geomagnetic Earth fixed frame $\mathcal{M} = \{O, \hat{\boldsymbol{m}}_1, \hat{\boldsymbol{m}}_2, \hat{\boldsymbol{m}}_3\}$
- LVLH orbit reference frame $\mathcal{H} = \{O, \hat{\boldsymbol{i}}_r, \hat{\boldsymbol{i}}_\theta, \hat{\boldsymbol{i}}_h\}$
- Satellite body fixed frame $\mathcal{B} = \{H, \hat{\boldsymbol{b}}_1, \hat{\boldsymbol{b}}_2, \hat{\boldsymbol{b}}_3\}$

Each of these frames are defined in detail below.

2.1 Earth Magnetic Field Modeling

In order to use torque rods as attitude actuators, we need to simulate earth's magnetic field as observed by a low earth satellite. This can be done using the tilted geomagnetic dipole model depicted in Fig. 2., and is setup for this project as shown in Fig. 3.

First an inertial reference frame $\mathcal{N} = \{\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}_3\}$ is setup as follows. The origin is at the center of the earth with the $\hat{\boldsymbol{n}}_1 - \hat{\boldsymbol{n}}_2$ plane being the Earth's equatorial plane and $\hat{\boldsymbol{n}}_1$ axis along the autumnal equinox. The $\hat{\boldsymbol{n}}_3$ axis is along the polar axis towards the geographical north pole.

The geomagnetic reference frame $\mathcal{M} = \{O, \hat{\boldsymbol{m}}_1, \hat{\boldsymbol{m}}_2, \hat{\boldsymbol{m}}_3\}$ is defined as an Earth fixed coordinate frame, with its origin same as the inertial frame,

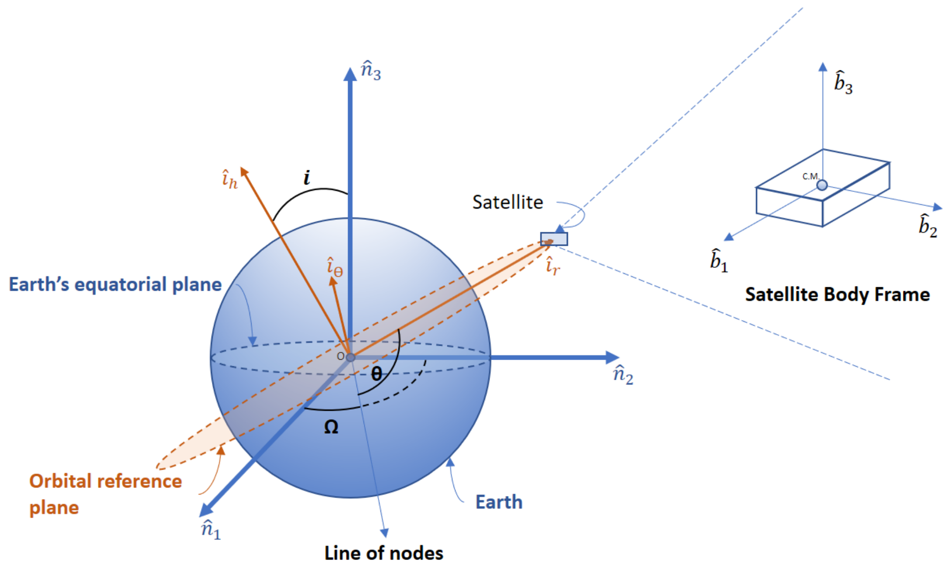

Figure 1: Inertial frame $\mathcal{N} = \{O, \hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}_3\}$, Hill frame $\mathcal{H} = \{O, \hat{\boldsymbol{i}}_r, \hat{\boldsymbol{i}}_\theta, \hat{\boldsymbol{i}}_h\}$ and body frame $\mathcal{B} = \{H, \hat{\boldsymbol{b}}_1, \hat{\boldsymbol{b}}_2, \hat{\boldsymbol{b}}_3\}$.

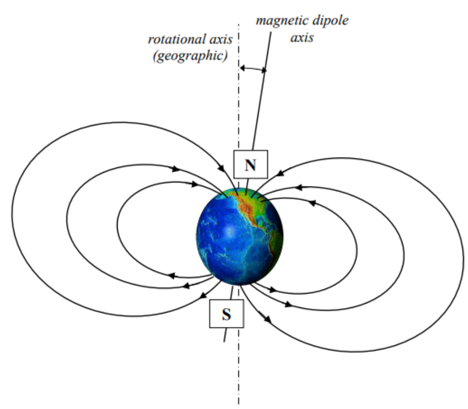

Figure 2: Tilted magnetic dipole model of geomagnetic field.[1]

at the Earths center with the $\hat{\boldsymbol{m}}_3$ axis directed along Earth's tilted magnetic dipole. This dipole is approximated to have a dipole moment of $\mathcal{M} = 7.838 \times 10^6$ Tkm^3 with a tilt angle $\gamma_m = 17°$ as shown in Fig. 3. The $\hat{\boldsymbol{m}}_1$- $\hat{\boldsymbol{m}}_2$ plane is the geomagnetic equatorial plane. The $\hat{\boldsymbol{m}}_1$ axis is along the intersection of the geomagnetic equatorial plane and the Earth's equatorial

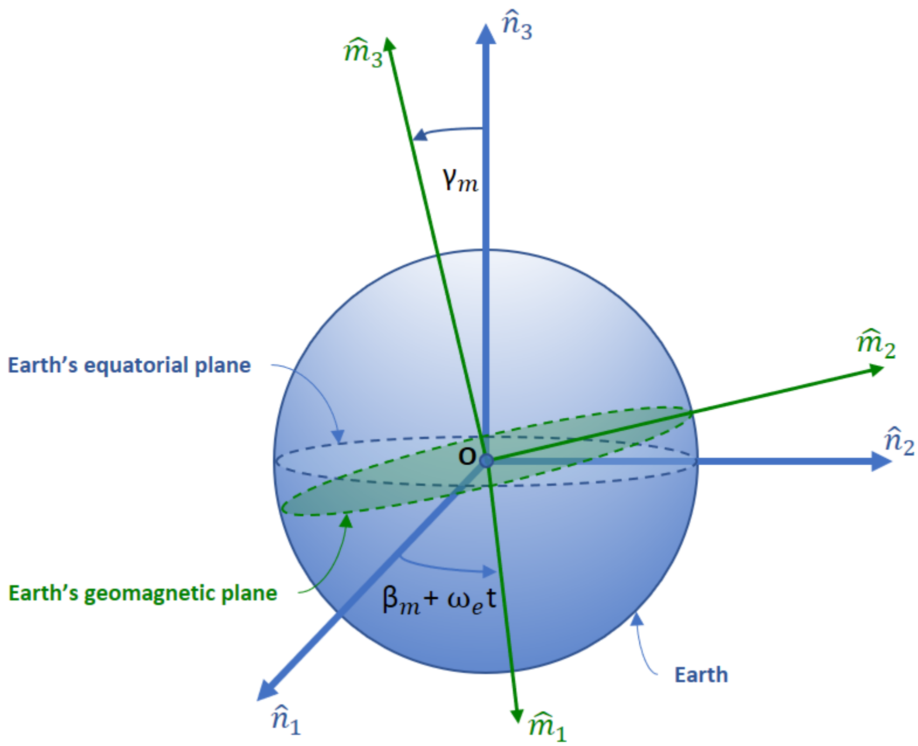

Figure 3: Inertial frame $\mathcal{N} = \{O, \hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}_3\}$, and Geomagnetic frame $\mathcal{M} = \{O, \hat{\boldsymbol{m}}_1, \hat{\boldsymbol{m}}_2, \hat{\boldsymbol{m}}_3\}$

plane. The angle between $\hat{\boldsymbol{n}}_1$ and $\hat{\boldsymbol{m}}_1$ axes is given by $\beta_m = \beta_{m0} + \omega_e t$, where β_{m0} is the angle at $t = 0$ and $\omega_e = 7.2921159 \times 10^{-5}$ rad/s is the earth's rotation rate about its spin axis $\hat{\boldsymbol{n}}_3$.

The LVLH or Hill reference frame $\mathcal{H} = \{O, \hat{\boldsymbol{i}}_r, \hat{\boldsymbol{i}}_\theta, \hat{\boldsymbol{i}}_h\}$ is a rotating coordinate frame. The circular orbit can be easily described in this so-called Hill frame. Figure 1 shows the orientation of this frame with respect to the inertial frame (denoted by $\mathcal{N}$). Its $\hat{\boldsymbol{i}}_r$ axis always tracking the satellite's position along its position vector $\boldsymbol{r}$. The $\hat{\boldsymbol{i}}_h$ axis is along the orbit normal, tracking the orbit angular momentum vector. Finally, $\hat{\boldsymbol{i}}_\theta = \hat{\boldsymbol{i}}_h \times \hat{\boldsymbol{i}}_r$ vector completes the right-handed coordinate frame. It is important to note that the Hill frame is given through the (3-1-3) Euler angle set $\{\Omega, i, \theta\}$ with $\theta = \omega + \nu$. However, since the satellite orbit is circular, arg. of periapsis ω is undefined and θ is simply the true anomaly. The local magnetic vector as seen by this orbit frame is formulated as given below[3].

$$
{}^{H}\boldsymbol{b} = \frac{M}{r^3}\begin{bmatrix} cos(\dot{\theta}t - \eta_m)sin(\xi_m) \\ cos(\xi_m) \\ -2sin(\dot{\theta}t - \eta_m)sin(\xi_m) \end{bmatrix} \tag{1}
$$

where,

$$
\xi_m = cos^{-1}(cos(i)cos(\gamma_m) + sin(i)sin(\gamma_m)cos(\Omega - \beta_m)) \tag{2}
$$

and

$$
\eta_m = sin^{-1}(sin(\gamma_m)sin(\Omega - \beta_m)/sin(\xi_m)) \tag{3}
$$

2.2 Mission Overview

First, this section describes spacecraft initial tumbling states. The 30 kg spacecraft's initial MRP, initial angular velocity, and inertia tensor at orbit insertion are

$$
\boldsymbol{\sigma}_{B/N} = \begin{bmatrix} 0.3 \\ 0.2 \\ 0.4 \end{bmatrix} \tag{4}
$$

$$
\boldsymbol{\omega}_{B/N} = \begin{bmatrix} 15 \\ 8 \\ 12 \end{bmatrix}_{\mathcal{B}} \text{deg/s} \tag{5}
$$

$$
{}^{\mathcal{B}}[I] = \begin{bmatrix} 3.5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 8 \end{bmatrix} \text{kg m}^2 \tag{6}
$$

The initial conditions for the 450 km circular orbit are as follows:

LEO initial angles:

$$
\{\Omega = 0, i = 45^\circ, \theta_{t0} = 0^\circ\} \tag{7}
$$

The spacecraft detumble phase should be achieved within three orbits. The detumble threshold is $3^\circ/s$ for each body axes.

3 Project Milestones

1. **(15 points) Co-ordinate frame relations:**
   Determine analytic expressions for [HN], [MN] (using 3-1-3 Euler angles), as well as for [HM] direction cosine matrices.

2. **(20 points) Numerical Integrator:**
   In order to integrate the attitude, you obviously need a numerical integrator. Write an RK4 integrator in the language that you have chosen (i.e., do not use a built-in integrator such as Matlab's ode45). The propagated state $\boldsymbol{X}$ is

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{\sigma}_{B/N} \\ \boldsymbol{\omega}_{B/N} \end{bmatrix} \tag{8}$$

   Assume that the spacecraft's dynamics obey Equation (9)[4]:

$$[I]\dot{\boldsymbol{\omega}}_{B/N} = -\boldsymbol{\omega}_{B/N} \times [I]\boldsymbol{\omega}_{B/N} + \boldsymbol{u} \tag{9}$$

   Demonstrate that your integrator works properly by integrating $\boldsymbol{X}$ forward for 100 seconds with the control torque $\boldsymbol{u} = 0$ (no control). Use initial conditions given in mission overview. Plot the magnitude of the spacecraft's inertial angular momentum vector and rotational kinetic energy over time and show that they are indeed conserved.

3. **(15 points) Geomagnetic Modeling:**
   Using the co-ordinate frame descriptions above, model the tilted dipole geomagnetic field. With the initial angle $\beta_{m0} = 0°$, plot the local magnetic field ${}^{H}\boldsymbol{b}$ as seen by the orbit frame $\mathcal{H}$ (over one orbit) and as seen by the satellite body frame ${}^{B}\boldsymbol{b}$ (over 100 seconds). Use satellite and orbit initial conditions provided in mission overview.

4. **(30 points) Control Law Implementation:**
   Consider following two variations of B-dot control law.

   I) Modulating B-dot control law[2]:

   The command dipole, $\boldsymbol{m}$, required from the torque rods, can be calculated as,

$$\boldsymbol{m} = \frac{-k_{\omega}}{||\boldsymbol{b}||}\hat{\boldsymbol{b}} \times [(I_{3x3} - \hat{\boldsymbol{b}}\hat{\boldsymbol{b}}^{T})\boldsymbol{\omega}] \tag{10}$$

where $\hat{\boldsymbol{b}} = \boldsymbol{b}/||\boldsymbol{b}||$ is the unit vector parallel to the local geomagnetic vector expressed in terms of body-frame components. The maximum command dipole moment that each torque rod can generate is 3 Am^2. Make sure to include this dipole saturation constraint on your control law.The control gain is given by the following function.

$$k_w = 2\dot{\theta}(1 + sin(\xi_m))I_{\min} \tag{11}$$

The control torque $\boldsymbol{u}$ used in Eqn. (9) above is calculated as

$$\boldsymbol{u} = \boldsymbol{m} \times \boldsymbol{b} \tag{12}$$

Note that $\boldsymbol{b}$ is in body frame components and the control torque will be perpendicular to $\hat{\boldsymbol{b}}$.

II) Bang-Bang B-dot control law:

$$\boldsymbol{m} = -m_{\max}\text{sgn}(\boldsymbol{b}') \tag{13}$$

where $m_{\max}$ is the maximum torque rod dipole, and $\boldsymbol{b}'$ is the time derivative of the magnetic field as seen (measured) by the satellite frame. The control torque $\boldsymbol{u}$ is calculated as given in Eqn. (12).

Starting with the tumbling spacecraft, implement the control law given in Eqns. (10) and (12) to drive the angular rates below the desired threshold of $3°/s$ for each axis. Your implementation must compare the above two control laws for the following initial conditions (i.e., two different simulations):

1.
$$\boldsymbol{\sigma}_{B/N} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.4 \end{bmatrix} \qquad \boldsymbol{\omega}_{B/N} = \begin{bmatrix} 1 \\ 12 \\ 1 \end{bmatrix}_{\mathcal{B}} \text{deg/s} \tag{14}$$

2.
$$\boldsymbol{\sigma}_{B/N} = \begin{bmatrix} 0.35 \\ 0.2 \\ 0.15 \end{bmatrix} \qquad \boldsymbol{\omega}_{B/N} = \begin{bmatrix} 6 \\ 4 \\ 13 \end{bmatrix}_{\mathcal{B}} \text{deg/s} \tag{15}$$

Provide plots for each of the three initial condition scenarios showing the spacecraft's angular velocity and the components of its attitude MRP. Show a time history of the control error, as well as the control torque. For at least one simulation each of modulating and bang-bang control law, submit a plot of the command dipole $\boldsymbol{m}$ as well. Try different sets of gains to see how control performance varies with them. Be sure to list which gains you are using. Feel free to add any other plots you deem appropriate to demonstrate that the mission is being completed correctly.

5. **(10 points) Orbit Inclinations v/s Control Performance:** Implement the modulating and bang-bang control for two cases of orbit inclinations. One where the inclination is 15° and another where the inclination is 105°. Use the satellite initial conditions given in mission overview. Keep the gains same for each inclination case. Plot the resultant control errors over 3 orbits. What are your conclusions on performance of the control?

   What is the minimum size (in Am^2) of the torque rod dipoles needed to achieve a detumble threshold of less than $1°/s$ from these initial conditions within 3 orbits for the 15° inclination case?

6. **(10 points) Monte Carlo Analysis:** The objective of this section is to simulate the tumbling scenario under a random expected tumbling initial condition and visualize the performance of each iteration to draw broader conclusions on expected performance of the control. Here, you want to evaluate performance of bigger torque rods that can provide 4 Am^2 along each axis. For the angular velocity bounds given below, create a loop that will achieve the following:
   i. Generate a random angular velocity vector within the given bounds for each axis, i=1,2,3:

$$10°/s \leq |\boldsymbol{\omega}_{B/N(i)}| \leq 16°/s \tag{16}$$

   ii. Simulate detumbling scenario with this angular velocity over 3 orbits (use modulating control law).
   iii. Calculate the norm of the control error and plot on a logarithmic y-axis vs time.
   iv. Repeat the process for 25 random runs.

   All 25 control errors should be plotted in the same single plot. It is highly recommended to automate this loop as opposed to manually editing the initial condition and plotting the results, as this can get tedious over 25 runs. Change the control to bang-bang control law and repeat the Monte Carlo analysis (Steps i. to iv.). Analyze the results and comment on your expected steady state error and settling time for each control law, within the given time frame.

7. **(Stretch Goal) 3D Visualizer:** Write a routine that allows one to visuals the satellite's 3D attitude over time. To provide valuable insight into the satellite's dynamics, this visualizer routine should at least display the spacecraft's body fixed frame B, the inertial frame N. It is recommended to show the earth fixed geomagnetic frame and the satellite orbit frame. Feel free to add any other feature that you see fit. This stretch goal will not count as extra credit.

The final report must be written as an AIAA conference paper. Paper templates are available in MS Word and LATEX on the Desire2Learn website under the projects folder. Your final report must have an abstract, introduction, problem statement, sections explaining your development, numerical simulation, as well as a conclusion. Points will be deducted for poor presentation. The work you present must be your own. Be sure to properly reference other papers, figures or books that you use in support of this work. You need to submit your final report in electronic format (PDF preferred) to Desire2Learn in response to the project report assignment.

References

[1] Weigel, R. S., Bruntz, R., *CISM Knowledge Transfer Short Course*, Boston University, Boston, 2005.

[2] Avanzini, G. and Giulietti, F., *Magnetic Detumbling of a Rigid Spacecraft*, Journal of Guidance, Control, and Dynamics, Vol. 35, No. 4 (2012), pp. 1326-1334.

[3] McElvain, R. J., *Satellite Angular Momentum Removal Utilizing the Earth's Magnetic Field*, Torques and Attitude Sensing in Earth Satellites, edited by S. F. Singer, Academic, New York, 1964, pp. 137-158.

[4] Schaub, H., Junkins, J. L., *Analytical Mechanics of Space Systems, 3rd ed.*, AIAA Education Series, Virginia, 2014, Chs. 3, 4.