# ASEN 6080 Homework 6

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## I. Problem 1

### I.A. UKF Perforamnce Using HW3 Data

Plots of residuals have been emmitted because they are essentially identical in each case; they all very closely resemble the measurement noise.

#### *I.A.1. Case $\alpha = 1$, $\beta = 2$, no process noise*

Figure 1 shows the state errors and $3\sigma$ bounds.

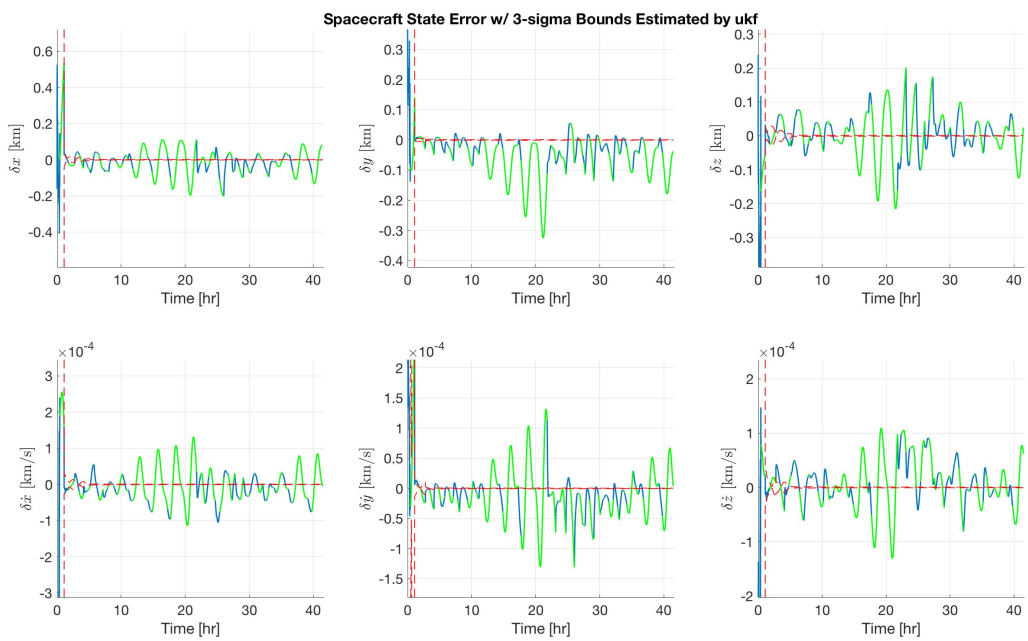


**Figure 1. UKF State Errors and $3\sigma$ Error Bounds, $\alpha = 1$, $\beta = 2$, No Process Noise**

The 3D RMS values are shown in Table 1

| | |
|---|---|
| 3D RMS, Position [m] | 135.0388 |
| 3D RMS, Velocity [m/s] | 0.01032 |

**Table 1. 3D RMS Values for UKF, $\alpha = 1$, No Process Noise**

#### *I.A.2. Case Case $\alpha = 1$, $\beta = 2$, with process noise*

Figure 2 shows the state errors and $3\sigma$ bounds.

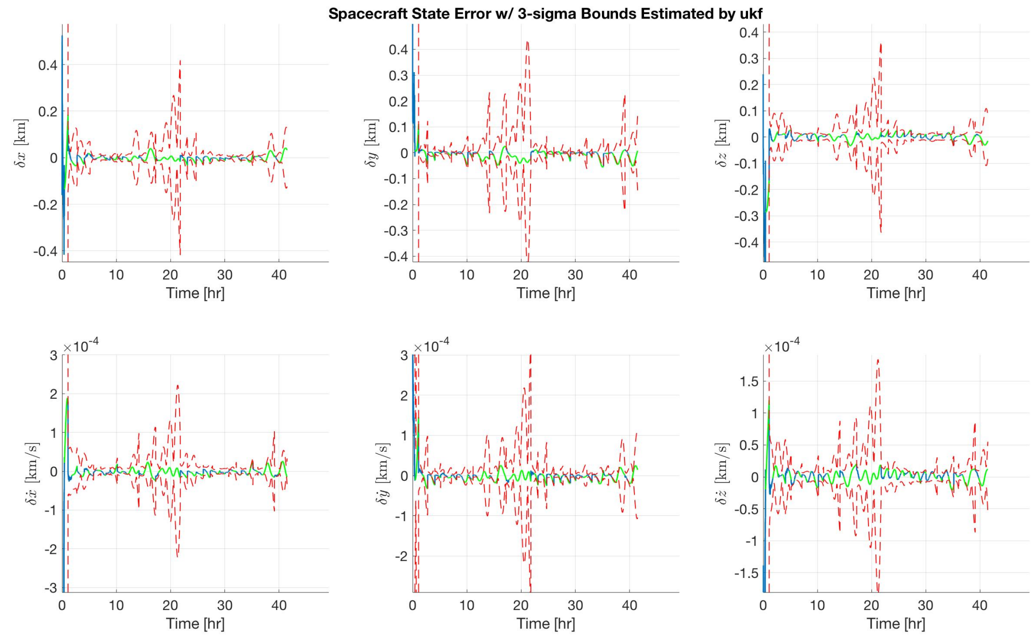


**Figure 2. UKF State Errors and $3\sigma$ Error Bounds, $\alpha = 1$, $\beta = 2$, With Process Noise**

The 3D RMS values are shown in Table 2

| | |
|---|---|
| 3D RMS, Position [m] | 63.3550 |
| 3D RMS, Velocity [m/s] | 0.0714 |

**Table 2. 3D RMS Values for UKF, $\alpha = 1$, With Process Noise**

### *I.A.3. Case Case $\alpha = 1 \times 10^{-4}$, $\beta = 2$, with process noise*

Figure 3 shows the state errors and $3\sigma$ bounds.

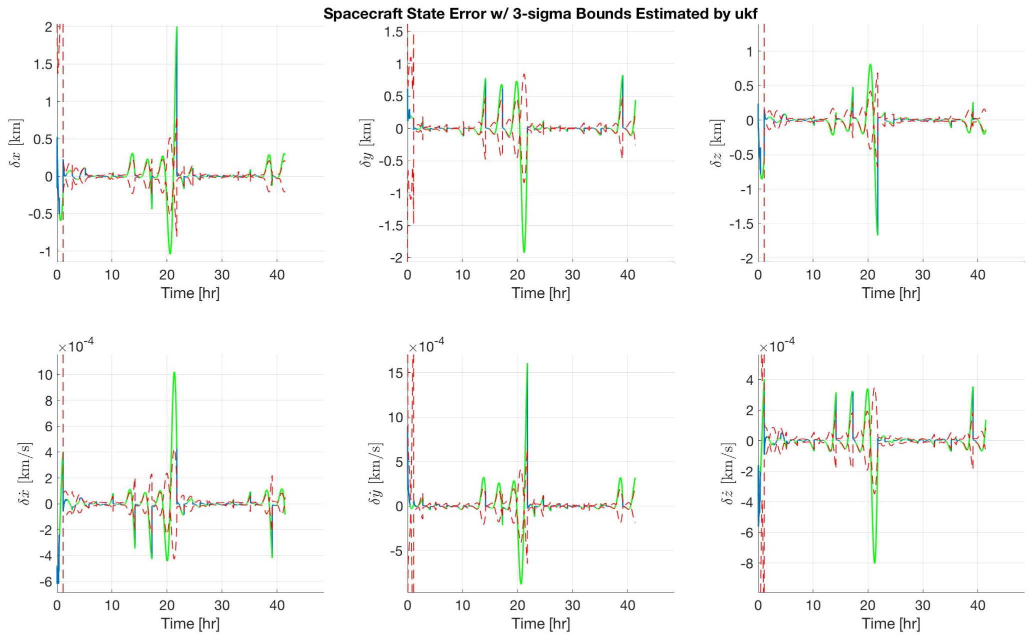


**Figure 3. UKF State Errors and $3\sigma$ Error Bounds, $\alpha = 1 \times 10^{-4}$, $\beta = 2$, With Process Noise**

The 3D RMS values are shown in Table 3

| | |
|---|---|
| 3D RMS, Position [m] | 438.3139 |
| 3D RMS, Velocity [m/s] | 0.2781 |

**Table 3. 3D RMS Values for UKF, $\alpha = 1 \times 10^{-4}$, With Process Noise**

#### *I.A.4. Comparison of Cases*

Comparing the three cases, the lowest residuals and the best-bounded states with the smallest bounds come from case 2, where $\alpha = 1$ and there is process noise. These residuals are 1/2 the RMS values of when there is no process noise and about 1/7 the RMS vaues of when there is process noise but $\alpha = 1 \times 10^{-4}$, which has the highest residuals and the most poorly bounded states.

Changing $\alpha$ to $1 \times 10^{-4}$ essentially tells the filter that the dynamics are extremely non-linear (i.e. cannot be linearized very well), and it seems as if this value does not correspond very well to the dynamics present in this homework, which makes sense, since although orbital equations of motion are nonlinear, they produce smooth trajectories.

### I.B. Comparison to EKF From HW3

Figure 4 shows the state errors and $3\sigma$ bounds of the EKF with process noise processing the same data as in the previous section. Both filters used the same process noise scalar, $\sigma_u = 1 \times 10^{-5}$.

The 3D RMS values are shown in Table 6

| | |
|---|---|
| 3D RMS, Position [m] | 64.9370 |
| 3D RMS, Velocity [m/s] | 0.0724 |

**Table 4. 3D RMS Values for EKF, With Process Noise**

The UKF ($\alpha = 1$, process noise) and EKF (process noise) cases are extremely comparable, with differences in position 3D RMS values of only 1 meter and in velocity of just 1 mm/s, which is equivalent to the measurement noise standard deviation. This analysis goes to show that although the UKF is technically

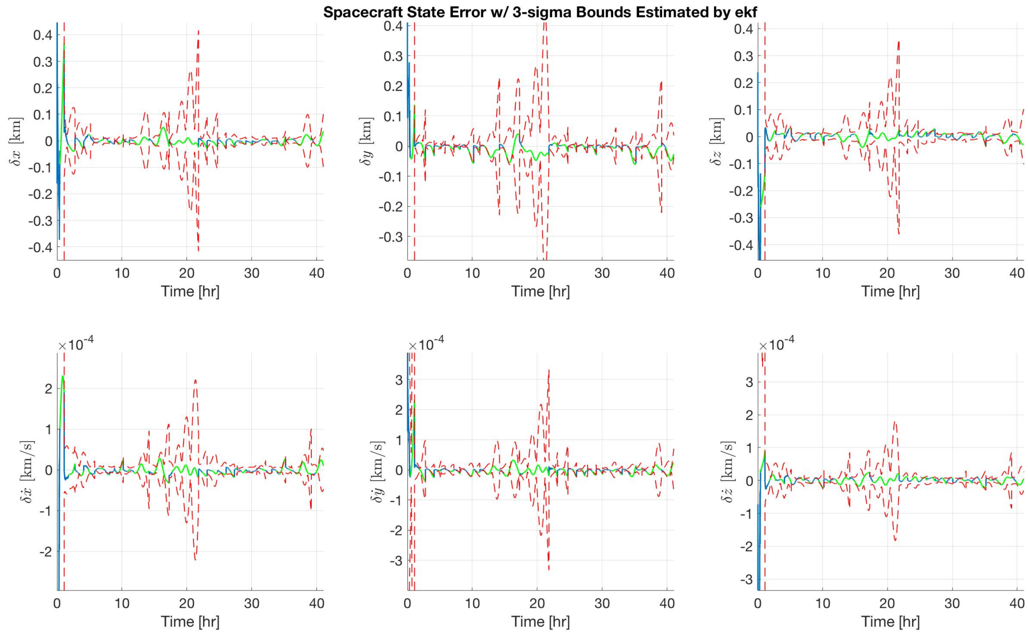


**Figure 4. EKF State Errors and $3\sigma$ Bounds, With Process Noise**

much better at estimating the state, out to third order, it is clear in this particular implementation that the EKF's linearization of the dynamics for the covariance time update is good enough.

## I.C. Robustness of UKF to Large Initial Perturbations Compared to EKF

In this case, the EKF (with process noise) was compared to the UKF (with process noise, $\alpha = 1$ for large initial perturbations. The initial perturbation used in the other cases was scaled by an integer value up to 15. Figure 5 shows the 3D RMS position and velocity values versus scale factor.

The initial perturbation that was scaled is

$$\begin{bmatrix} 160.9497 \\ -615.1242 \\ -238.9927 \\ 0.6150 \\ -0.9014 \\ 0.3481 \end{bmatrix}$$

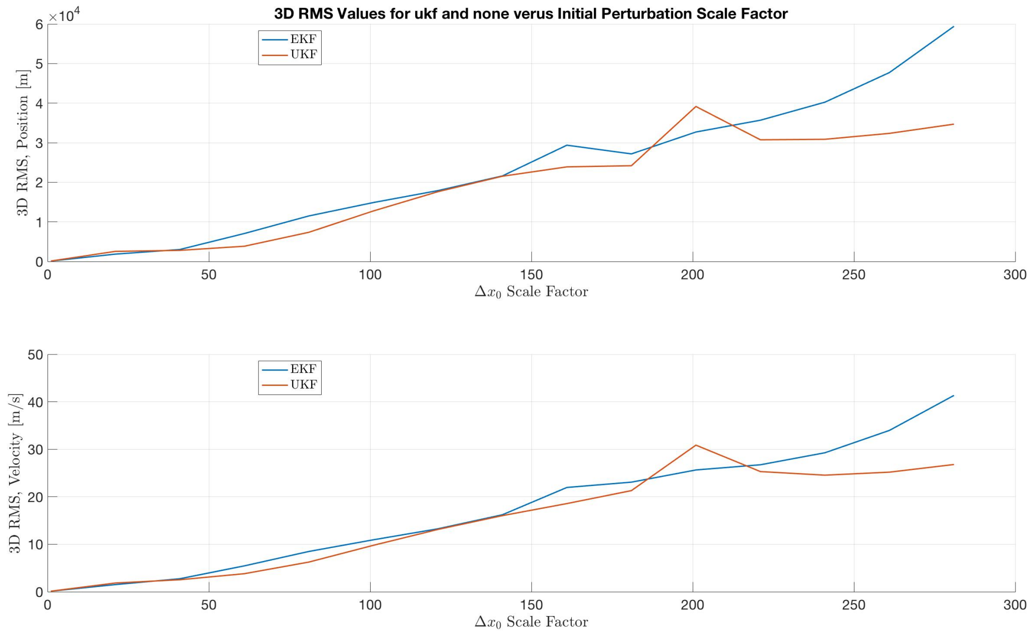


**Figure 5. EKF and UKF 3D RMS Values vs. Scale Factor**

The figure shows that either the UKF or EKF is a better option depending on the size of the initial perturbation, but as the scale factor moves past 215, the UKF has a clear advantage. Further simulation is needed to determine if this trend would contintue for even higher scale factors.

## I.D. J3 Addition to UKF Process Noise

J3 perturbations were added to the UKF dynamic model, and this was trivial because I had already coded support for J3 perturbations from a couple of homeworks ago.

Figure 6 shows the state errors and $3\sigma$ bounds for the UKF with J3 dynamics and process noise processing the same data as from HW3, and Figure 7 shows the same except without process noise.

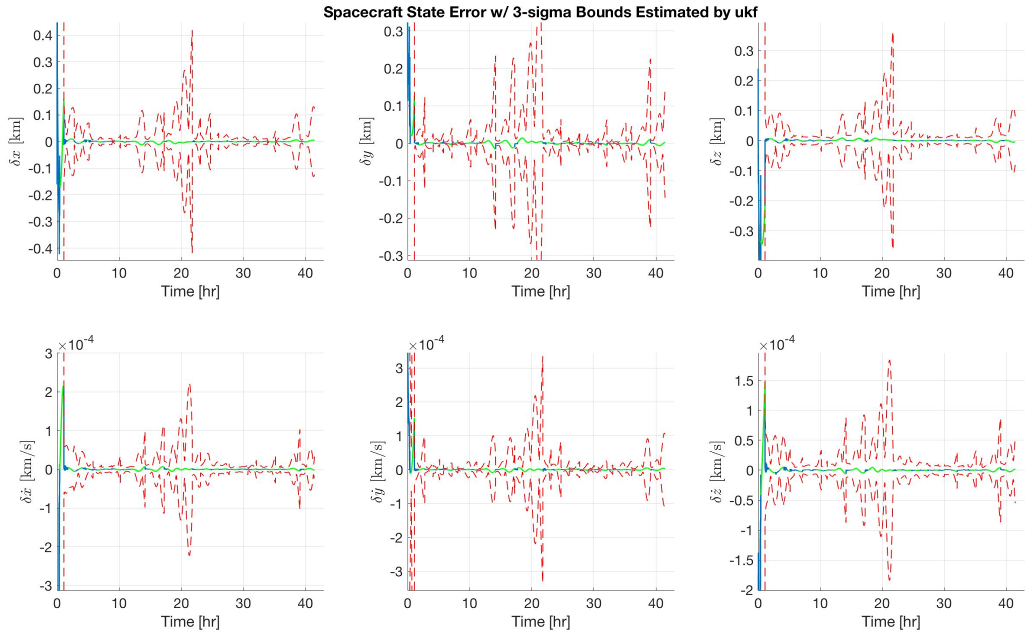


**Figure 6. State Errors and $3\sigma$ Bounds for UKF with J3 Dynamics and Process Noise**

The 3D RMS values are shown in Table 6

| | |
|---|---|
| 3D RMS, Position [m] | 64.0572 |
| 3D RMS, Velocity [m/s] | 0.0706 |

**Table 5. 3D RMS Values for EKF, With Process Noise**

The RMS values show that the UFK with J3 dynamics and process noise gives pretty much the same performance as the UFK with J2 dynamics and process noise, indicating that adding J3 did little to change the filter performance.

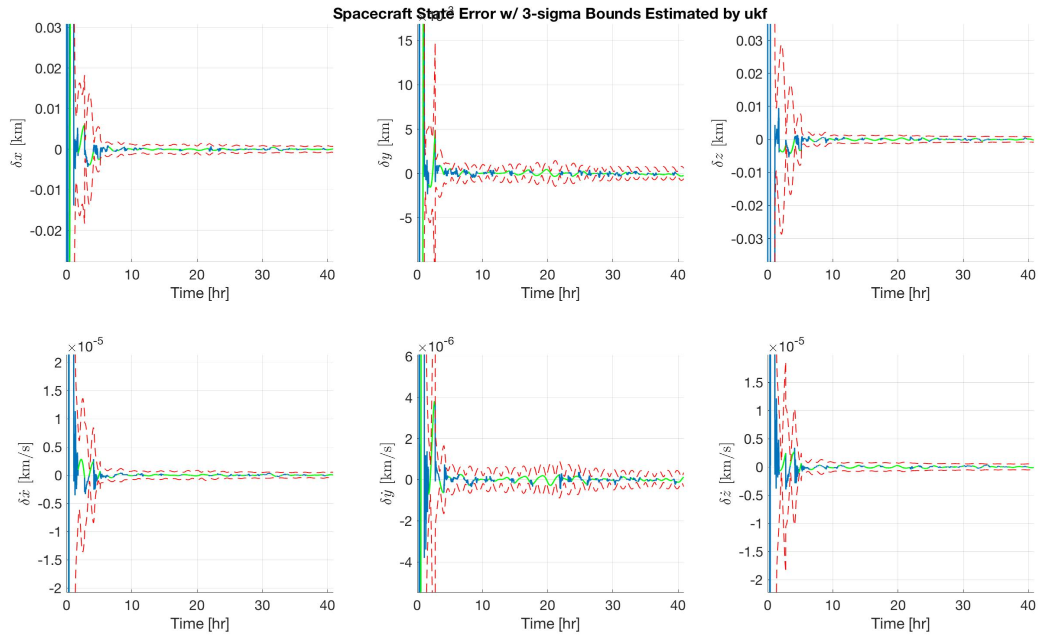


**Figure 7. State Errors and $3\sigma$ Bounds for UKF with J3 Dynamics and No Process Noise**

The 3D RMS values are shown in Table 6

| | |
|---|---|
| 3D RMS, Position [m] | 61.2377 |
| 3D RMS, Velocity [m/s] | 0.0740 |

**Table 6. 3D RMS Values for UKF, With J3 and No Process Noise**

The RMS values are only marginally smaller than those computed with the UKF with only J2 in the dynamics and process noise.