

Please describe all your work in clear terms, before implementing R code. Each question should include a description of your approach with clear indication of where I can download the associated source code. Your code should be attached to your assignment or uploaded online to a repository I can freely access.

### Generalized Linear Models.

In R load the package (`survival`) and consider the analysis of the data-set (`infert`). Ignoring dependence due to matching, consider a Bayesian analysis for a probit model relating case status to: age, parity, education, spontaneous and induced.

Assume case status  $y_i$  has density  $y_i \sim \text{Bern}(p_i)$ ,  $p_i = \Phi(X_i'\beta)$ , where  $\Phi(w)$  is the cdf of a standard Normal distribution evaluated at  $w$ . Consider a unit information prior  $\beta \sim N(0, n(X'X)^{-1})$ , where  $n$  is the sample size. We are interested in  $p(\beta | Y)$ .

- (1) Describe and implement a Metropolis-Hastings algorithm designed to obtain a MC with stationary distribution  $p(\beta | Y)$ .
- (2) Describe and implement a data augmented (DA-MCMC) strategy targeting  $p(\beta | Y)$ .
- (3) Describe and implement a parameter expanded - data augmentation (PX-DA MCMC) algorithm targeting  $p(\beta | Y)$ .
- (4) Assess mixing and convergence of the chains induced by the competing transition schemes implemented in 1,2 and 3. Comment on potential trade-offs involving: coding complexity, storage and cpu time.

Consider now a logit link, by assuming  $\log\left(\frac{p_i}{1-p_i}\right) = X_i'\beta$ , with  $\beta \sim N(0, n(X'X)^{-1})$ , as before.

- (5) Describe and implement a random walk Metropolis-Hastings algorithm designed to obtain a MC with stationary distribution  $p(\beta | Y)$ .
- (6) Describe and implement a Langevin-Hastings algorithm designed to obtain a MC with stationary distribution  $p(\beta | Y)$ .
- (7) Describe and implement an adaptive Metropolis-Hastings algorithm designed to obtain a MC with stationary distribution  $p(\beta | Y)$ .
- (8) Assess mixing and convergence of the chains induced by the competing transition schemes implemented in 5,6 and 7. Comment on potential trade-offs involving: coding complexity, storage and cpu time.