Homework1

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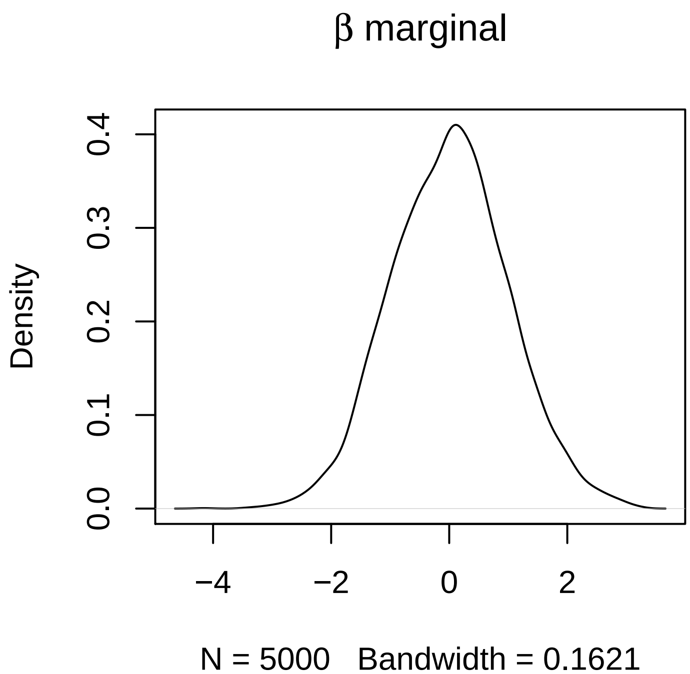
Bayesian Adaptive Lasso

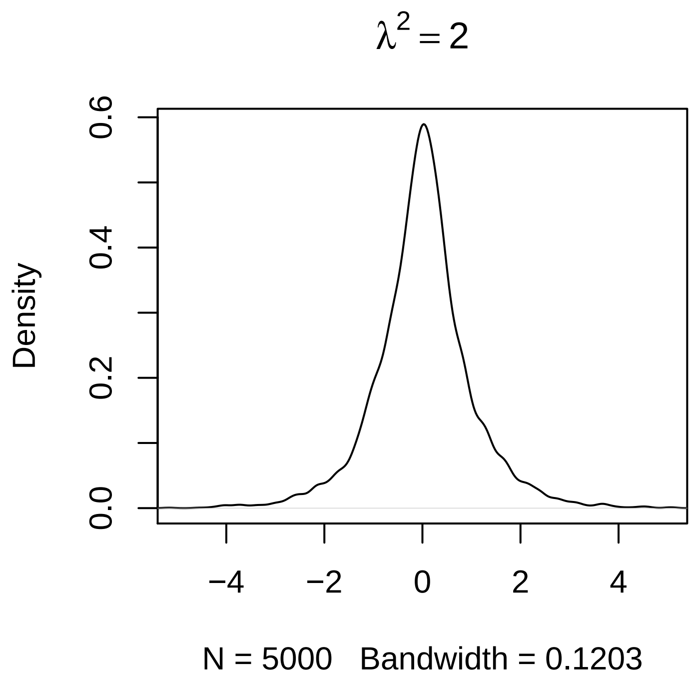
Part a)

```
n <- 5000
plot(density(rnorm(n,0,1)), main=TeX(paste("$\\beta$", "marginal")))
```



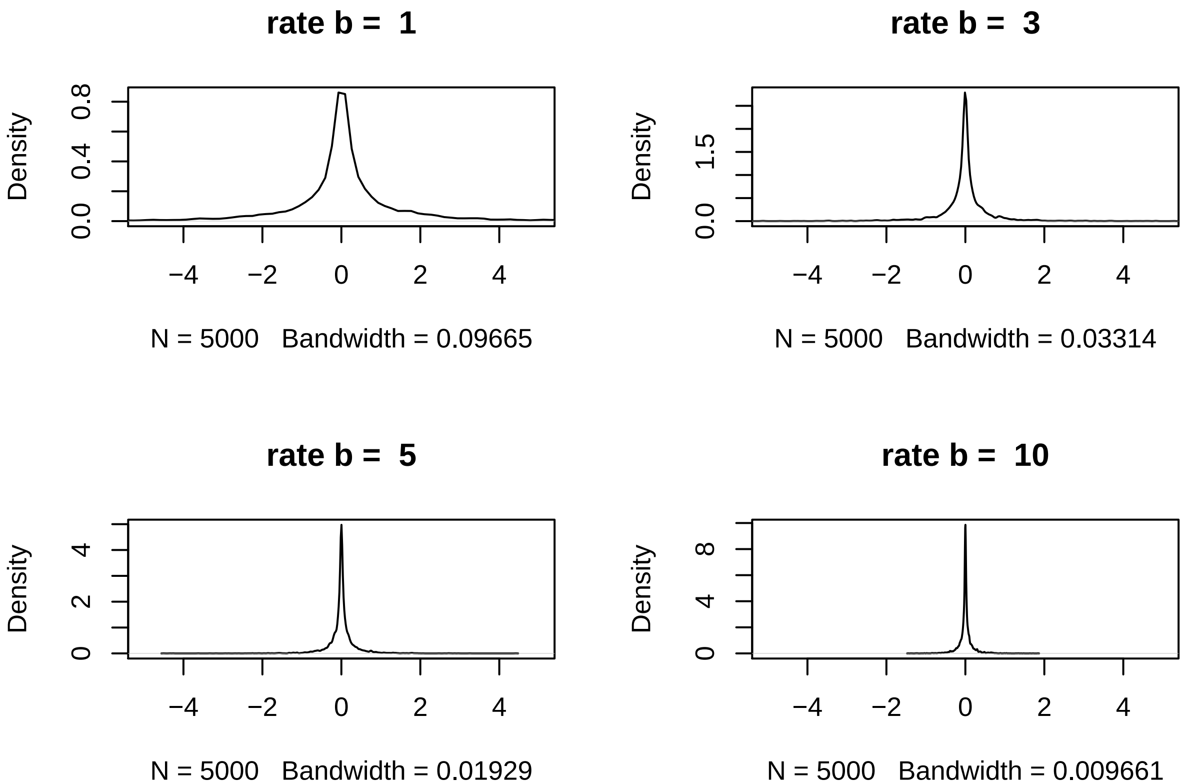
Part b)

```
lambda <- sqrt(2)
tau.sq <- rgamma(n,shape=1,rate = lambda^2/2)
beta.marginal <- rnorm(n,0,sqrt(tau.sq))
plot(density(beta.marginal), main=TeX(paste("$\\lambda^2 = 2$")), xlim=c(-5,5))
```



Part c)

```
set.seed(1)
par(mfrow=c(2,2))
rates <- c(1,3,5,10)
for(b in rates){
  lambda <- 1/rgamma(n,1,b)
  tau.sq <- rgamma(n,shape=1,rate = lambda^2/2)
  beta.marginal <- rnorm(n,0,sqrt(tau.sq))
  plot(density(beta.marginal), main=paste("rate b = ",b),xlim=c(-5,5))
}
```



Part d) and e)

I will implement a Gibbs Sampler. The model is

$$\begin{aligned}
\boldsymbol{Y}|\boldsymbol{\beta}, \sigma^2 &\sim N(\boldsymbol{X}\boldsymbol{\beta}, \sigma^2 \boldsymbol{I}) \\
\beta_j|\tau_j^2 &\sim N(0, \tau_j^2) \\
\tau_j^2 &\sim \text{Gamma}(1, \frac{\lambda^2}{2}) \\
\lambda &\sim \text{Inverse-Gamma}(a, 1/b) \\
\sigma^2 &\sim \text{Inverse-Gamma}(0.1, 0.1).
\end{aligned}$$

I need the full conditionals

$$\{\beta_j|\boldsymbol{Y}, \tau_j^2, \sigma^2, \lambda\} \text{ and } \{\sigma^2|\boldsymbol{Y}, \tau_j^2, \beta_j^2, \lambda\}.$$

I'll start with the posterior

$$\begin{aligned}
p(\beta_1, \ldots, \beta_p, \tau_1^2, \ldots, \tau_p^2, \sigma^2, \lambda|\boldsymbol{Y}) \propto\ & p(\boldsymbol{Y}|\beta_1, \ldots, \beta_p, \tau_1^2, \ldots, \tau_p^2, \sigma^2, \lambda) \\
&\times p(\beta_1, \ldots, \beta_p|\tau_1^2, \ldots, \tau_p^2) \\
&\times p(\tau_1^2, \ldots, \tau_p^2|\lambda)p(\sigma^2).
\end{aligned}$$

As a function of $\boldsymbol{\beta}$ and σ^2, this is proportional to

$$\begin{aligned}
& p(\boldsymbol{Y}|\beta_1, \ldots, \beta_p, \tau_1^2, \ldots, \tau_p^2, \sigma^2, \lambda)p(\beta_1, \ldots, \beta_p|\tau_1^2, \ldots, \tau_p^2)p(\sigma^2) \\
= \quad & N(\boldsymbol{X}\boldsymbol{\beta}, \sigma^2\boldsymbol{I})N(0, \Sigma)IG(a, b) \\
=. &
\end{aligned}$$

Similarly, I can get the full conditional of σ^2 since it's proportional to

$$\begin{aligned}
& p(\boldsymbol{Y}|\beta_1, \ldots, \beta_p, \tau_1^2, \ldots, \tau_p^2, \sigma^2, \lambda)p(\sigma^2) \\
= \quad & N(\boldsymbol{X}\boldsymbol{\beta}, \sigma^2\boldsymbol{I})IG(a, b).
\end{aligned}$$

Now that I have the full conditionals, a Gibbs Sampling routine will follow

$i \leftarrow 10$
for s in *niter* **do**
  $\boldsymbol{\beta}^{(s+1)} \sim p(\boldsymbol{\beta}|\sigma^{2(s)},)$
  $\sigma^{2(s+1)} \sim p(\sigma^2|\boldsymbol{\beta}^{(s+1)},)$
end for