

# Laboratory 7 - Roots and Extrema

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Lab Section 1B, Tuesday 8:30-11:20 AM

October 29th, 2017 I understand and have adhered to all the tenets of the Duke Community Standard in completing every part of this assignment. I understand that a violation of any part of the Standard on any part of this assignment can result in failure of this assignment, failure of this course, and/or suspension from Duke University.

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## 1 Basic Root-Finding Problems

Function	Real Roots	Roots
$f(x) = 20e^{-4x} - 36e^{-2x} + 18e^{-x} - 1$	3	4.5651e-02, 6.3358e-01, 2.7546e+00
$f(x) = x^5 + 100\cos(2x)$	3	-7.8392e-01, 7.8691e-01, 2.1295e+00
$f(x) = \frac{10}{x-2} - 90e^{-(x/2)}, x \neq 2$	2	2.3619e+00, 7.9669e+00

## 2 Basic Min/Max-Finding Problems

Function	Counts	Extrema
$f(x) = 20e^{-4x} - 36e^{-2x} + 18e^{-x} - 1$	1 min, 1 max	min: $f(2.4510e - 01) = -1.4596e + 00$ max: $f(1.3002e + 00) = 1.3421e + 00$
$f(x) = x^5 + 100\cos(2x)$	2 min, 2 max	min 1: $f(-1.6682e + 00) = -1.1103e + 02$ min 2: $f(1.5063e + 00) = -9.1415e + 01$ max 1: $f(-2.4916e + 00) = -6.9275e + 01$ max 2: $f(8.7055e - 06) = 1.0000e + 02$

## 3 Chapra 6.16

$V, m^3$	10	20	30	40	50	60
$h, m$	8.6492e-01	1.4210e+00	1.9292e+00	2.4326e+00	2.9685e+00	3.6373e+00

## 4 Chapra 6.20

The value of d when h is 0.43 is 1.6672e-01. Numerical evidence that this graph is that by plugging two values of h into the formula ( $h=0.25, h=0.5$ ). Plugging these values of h into the formula  $\frac{2*40d^{5/2}}{5} + \frac{1}{2}40,000*d^2 - 95*9.81*d - 95*9.81*h$  the output for  $h = 0.25$  is  $d = 0.133693$  and the output for  $h = 0.5$  is  $d = 0.177671$ . These appear to be points on the graph, giving numerical evidence that the graph is correct.

## 5 Chapra 6.21

The two angles that achieve the desired velocity at the desired distance are  $3.7959e + 01^\circ$  and  $5.1532e + 01^\circ$ .

## 6 Chapra 7.25, 7.26, and 7.27(b/c)

The minimum value for the equation in Chapra 7.25 was -6.5165e-01 at the coordinates (5.5233e-01, 7.4472e-01).  
The maximum value for the equation in Chapra 7.26 was 4.3440e+00 at the coordinates (9.6756e-01, 6.5585e-01).  
The minimum value for the equation in Chapra 7.27 was -1.7333e+01 at the coordinates (3.3333e+00, -6.6665e-01).

## A Codes

### A.1 Problem741.m

```
1 % I have adhered to all the tenets of the
2 % Duke Community Standard in creating this code.
3 % Signed: [ih52]
4 %% Initialize workspace
5 clear; format short e
6 %% Establish functions
7 f1 = @(x) 20.*exp(-4*x)-36.*exp(-2*x)+18.*exp(-x)-1;
8 f2 = @(x) x.^5 + 100.*cos(2*x);
9 f3 = @(x) 10./(x-2)-90.*exp(-(x/2));
10 %% Function one plot
11 figure(1); clf
12 x = linspace(-10,10,1000);
13 subplot(2,1,1)
14 plot(x, f1(x), 'k-')
15 xlabel('x')
16 ylabel('f(x)')
17 title('Plot of f(x)= 20e^{-4x}-36e^{-2x}+18e^{-x}-1 (ih52)')
18 subplot(2,1,2)
19 plot(x, sign(f1(x)), 'k-')
20 xlabel('x')
21 ylabel('sign(f(x))')
22 title('Plot of sign(f(x))')
23 axis([-10 10 -1.2 1.2])
24 print -depsc F1Plot
25 %% Function one roots
26 [Root11,ValueRoot11] = fzero(@(xdummy) f1(xdummy), [-1 0.2])
27 [Root12,ValueRoot12] = fzero(@(xdummy) f1(xdummy), [0.2 1])
28 [Root13,ValueRoot13] = fzero(@(xdummy) f1(xdummy), [2.5 3])
29 %% Function two plot
30 figure(2);clf
31 x = linspace(-10,10,10000);
32 subplot(2,1,1)
33 plot(x, f2(x), 'k-')
34 xlabel('x')
35 ylabel('f(x)')
36 title('Plot of f(x)=x^5+100cos(2x) (ih52)')
37 subplot(2,1,2)
38 plot(x, sign(f2(x)), 'k-')
39 xlabel('x')
40 ylabel('sign(f(x))')
41 title('Plot of sign(f(x))')
42 axis([-10 10 -1.2 1.2])
43 print -depsc F2Plot
44 %% Function two roots
45 [Root21,ValueRoot21] = fzero(@(xdummy) f2(xdummy), [-2 0])
46 [Root22,ValueRoot22] = fzero(@(xdummy) f2(xdummy), [0 1.5])
47 [Root23,ValueRoot23] = fzero(@(xdummy) f2(xdummy), [1.5 3])
48 %% Function three plot
49 figure(3);clf
50 x = linspace(-10,10,10000);
51 subplot(2,1,1)
52 plot(x, f3(x), 'k-')
53 xlabel('x')
```

```

54 ylabel('f(x)')
55 title('Plot of f(x)=10/(x-2)-90e^{-(x/2)}, x~2 (ih52)')
56 subplot(2,1,2)
57 plot(x, sign(f3(x)), 'k-')
58 xlabel('x')
59 ylabel('sign(f(x))')
60 title('Plot of sign(f(x))')
61 axis([-10 10 -1.2 1.2])
62 print -depsc F3Plot
63 %% Function Three Roots
64 [Root31,ValueRoot31] = fzero(@(xdummy) f3(xdummy), [2.1 3])
65 [Root32,ValueRoot32] = fzero(@(xdummy) f3(xdummy), [7 9])

```

## A.2 Problem742.m

```

1 % I have adhered to all the tenets of the
2 % Duke Community Standard in creating this code.
3 % Signed: [ih52]
4 %% Initialize workspace
5 clear; format short e
6 %% Establish functions
7 f1 = @(x) 20.*exp(-4*x)-36.*exp(-2*x)+18.*exp(-x)-1;
8 f2 = @(x) x.^5 + 100.*cos(2*x);
9 %% Find local minima
10 [F1MinX,F1MinY] = fminbnd(@(dummyx) f1(dummyx),-1,1)
11 [F2Min1X,F2Min1Y] = fminbnd(@(dummyx) f2(dummyx),-2,-1)
12 [F2Min2X,F2Min2Y] = fminbnd(@(dummyx) f2(dummyx),1,2)
13 %% Find local maxima
14 [x1,y1] = fminbnd(@(dummyx) -f1(dummyx),1,3);
15 [x21,y21] = fminbnd(@(dummyx) -f2(dummyx),-3,-2);
16 [x22,y22] = fminbnd(@(dummyx) -f2(dummyx),-1,1);
17 F1MaxX = x1
18 F1MaxY = -1*y1
19 F2Max1X = x21
20 F2Max1Y = -1*y21
21 F2Max2X = x22
22 F2Max2Y = -1*y22

```

## A.3 Chapra616.m

```

1 % I have adhered to all the tenets of the
2 % Duke Community Standard in creating this code.
3 % Signed: [ih52]
4 %% Initialize workspace
5 clear; format short e
6 %% Establish functions
7 V = @(r,h,L) (r.^2.*acos((r-h)/r)-(r-h).*sqrt(2.*r.*h-h.^2)).*L;
8 Height = zeros(1,6);
9 x = linspace(10,60,6)
10 for k = 1:numel(x)
11     [Height(k)] = fzero(@(dummyv) V(2,dummyv,5)-x(k),[0 4]);
12 end
13 Height = Height

```

## A.4 Chapra620.m

```
1 % I have adhered to all the tenets of the
2 % Duke Community Standard in creating this code.
3 % Signed: [ih52]
4 %% Initialize workspace
5 clear; format short e
6 %% Establish function & solve Chapra equation
7 MyFun = @(d,h) (2.*40.*d.^(5/2))./5+1/2.*40000.*d.^2-95.*9.81.*d-95.*9.81.*h;
8 ChapraFunZeros = fzero(@(ddummy) MyFun(ddummy,0.43), [0 1])
9 %% Graph of d for 50 values of h
10 h = linspace(0.0001,1,50);
11 dvals = zeros(1,50);
12 for k = 1:numel(h)
13     dvals(k) = fzero(@(ddummy) MyFun(ddummy,h(k)), [0.00001 10]);
14 end
15 figure(1); clf
16 plot(h,dvals,'k-')
17 grid on
18 title('Plot of Chapra Problem 6.20')
19 xlabel('h (m)')
20 ylabel('d (m)')
21 print -depsc Chapra620Fig
22
23
```

## A.5 Chapra621Bball.m

```
1 % I have adhered to all the tenets of the
2 % Duke Community Standard in creating this code.
3 % Signed: [ih52]
4 %% Initialize workspace
5 clear; format short e
6 %% Establish functions
7 MyFun = @(x,v,theta) tan(theta).*x-9.81/(2.*v.^2.*(cos(theta).^2)).*x.^2+1.8;
8 %% Get angle in radians
9 FunZero1 = fzero(@(dummytheta) MyFun(90,30,dummytheta)-1,[0.1]);
10 FunZero2 = fzero(@(dummytheta) MyFun(90,30,dummytheta)-1,[1]);
11 %% Plot trajectories
12 x = linspace(0,90,1000);
13 y1 = MyFun(x,30,FunZero1);
14 y2 = MyFun(x,30,FunZero2);
15 figure(1); clf
16 plot(x,y1,'k-',x,y2,'c-')
17 legend('37.959 Degrees','51.532 Degrees')
18 axis([0 90 0 50])
19 xlabel('x (m)')
20 ylabel('y (m)')
21 print -depsc Chapra621BballPlot
22 %% Convert angle to degrees
23 Angle1 = radtodeg(FunZero1)
24 Angle2 = radtodeg(FunZero2)
25
26
```

## A.6 Chapra725.m

```
1 % I have adhered to all the tenets of the
2 % Duke Community Standard in creating this code.
3 % Signed: [ih52]
4 %% Initialize workspace
5 clear; format short e
6 %% Chapra 7.25 minimum
7 MyFun1 = @(x,y) 2.*y.^2-2.225.*x.*y-1.75.*y+1.5.*x.^2;
8 [MinX1, MinY1] = fminsearch(@(dummyv) MyFun1(dummyv(1),dummyv(2)),[0 0])
9 %% Chapra 7.25 surface plot
10 figure(1);clf
11 x = linspace(-4,4,20);
12 [X,Y] = meshgrid(x);
13 surfc(X,Y,MyFun1(X,Y))
14 colormap jet
15 title('Surface Plot w/ Contours of Chapra 7.25')
16 xlabel('x')
17 ylabel('y')
18 zlabel('f(x,y)')
19 print -depsc Chapra725Plot
20 %% Chapra 7.26 maximum
21 MyFun2 = @(x,y) 4.*x+2.*y+x.^2-2.*x.^4+2.*x.*y-3.*y.^2;
22 [MaxX2,NegMaxY2] = fminsearch(@(dummyv) -MyFun2(dummyv(1),dummyv(2)),[0 0]);
23 MaxX2 = MaxX2
24 MaxY2 = -1.*NegMaxY2
25 %% Chapra 7.26 surface plot
26 figure(2);clf
27 surfc(X,Y,MyFun2(X,Y))
28 colormap jet
29 title('Surface Plot w/ Contours of Chapra 7.26')
30 xlabel('x')
31 ylabel('y')
32 zlabel('f(x,y)')
33 print -depsc Chapra726Plot
34 %% Chapra 7.27 minimum
35 MyFun3 = @(x,y) -8.*x+x.^2+12.*y+4.*y.^2-2.*x.*y;
36 [MinX3,MinY3] = fminsearch(@(dummyv) MyFun3(dummyv(1),dummyv(2)),[0,0])
37 %% Chapra 7.27 surface plot
38 figure(3);clf
39 surfc(X,Y,MyFun3(X,Y))
40 colormap jet
41 title('Surface Plot w/ Contours of Chapra 7.27')
42 xlabel('x')
43 ylabel('y')
44 zlabel('f(x,y)')
45 print -depsc Chapra727Plot
```

## B Figures

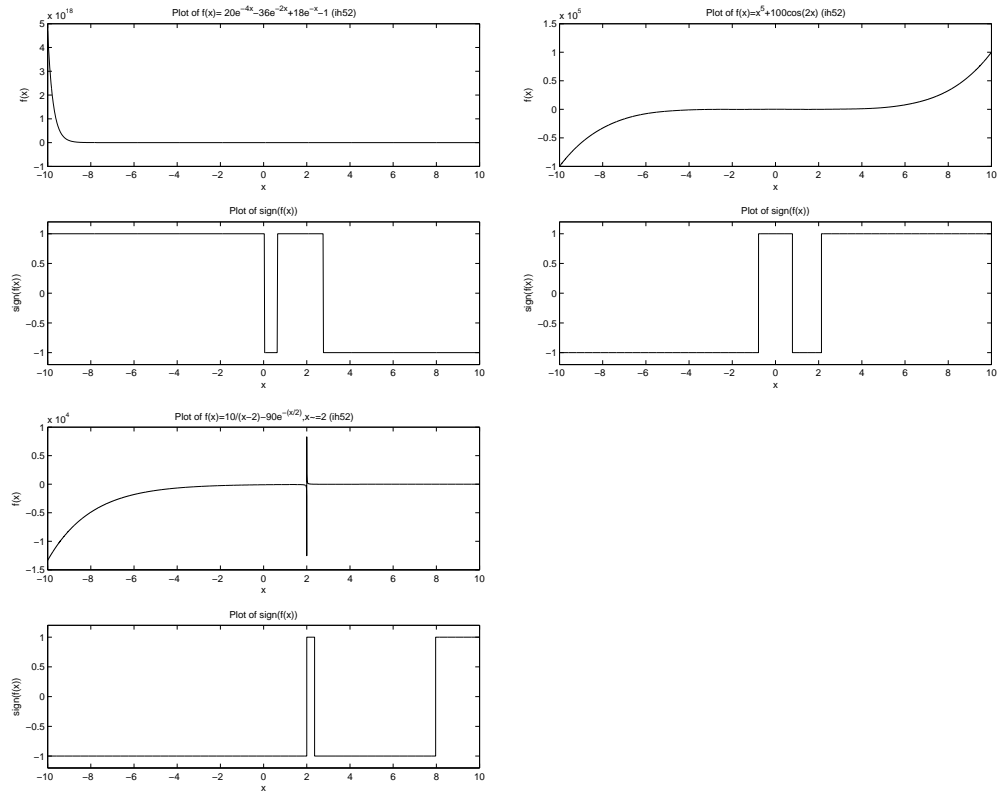


Figure 1: Basic Roots Problems

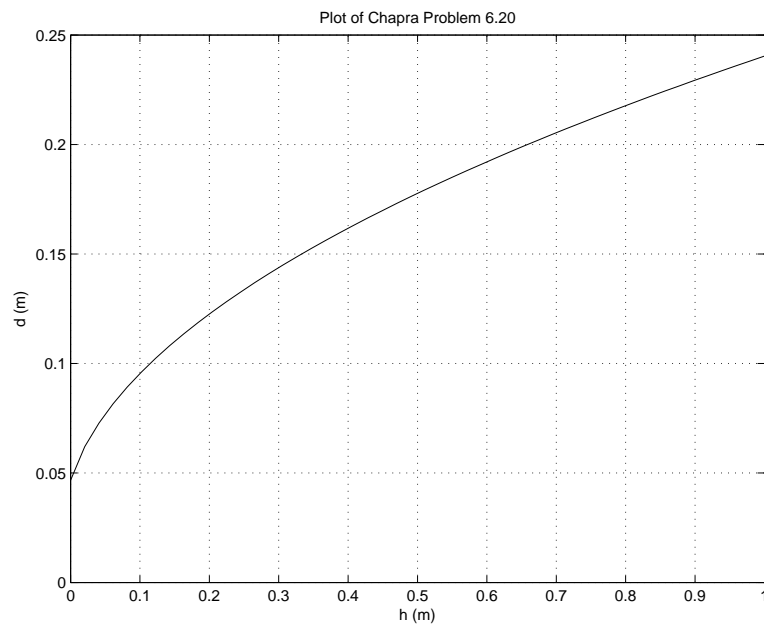


Figure 2: Plot of  $d$  vs.  $h$  from Chapra 6.20

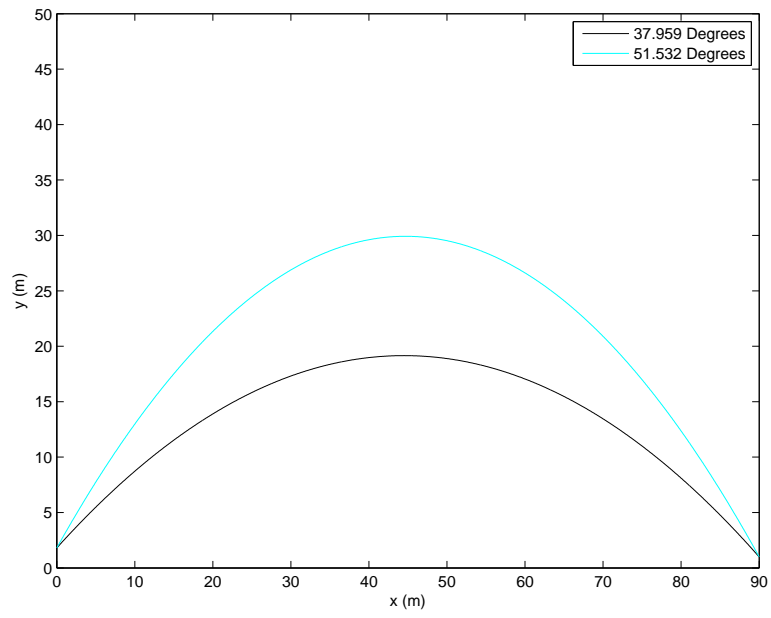


Figure 3: Plot of trajectories from Chapra 6.21

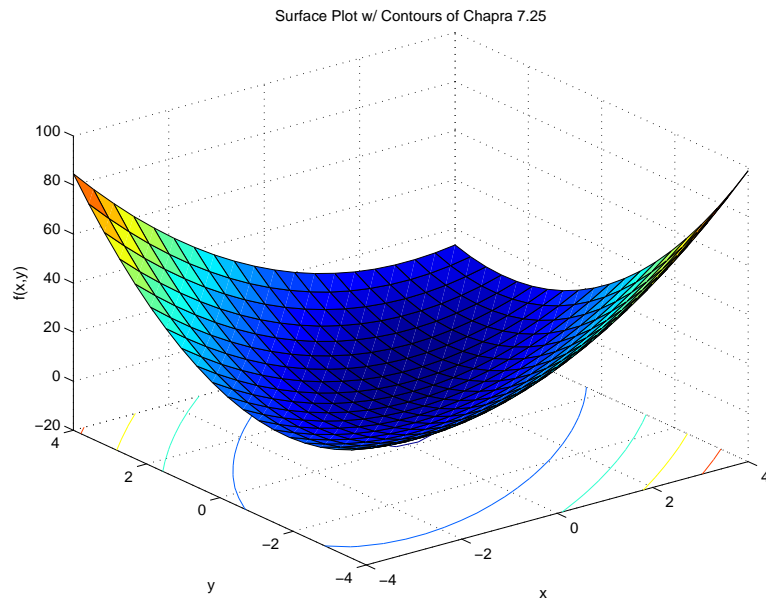


Figure 4: Surface plot with contours of Chapra 7.25



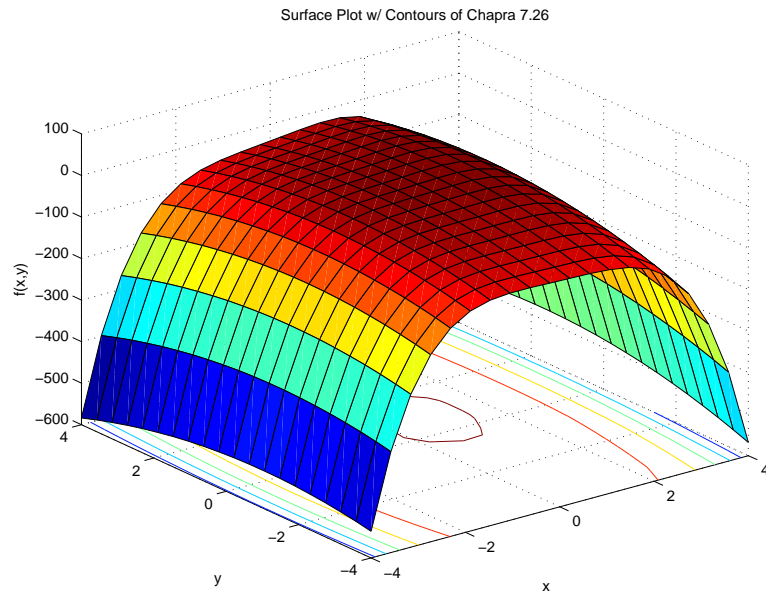


Figure 5: Surface plot with contours of Chapra 7.26

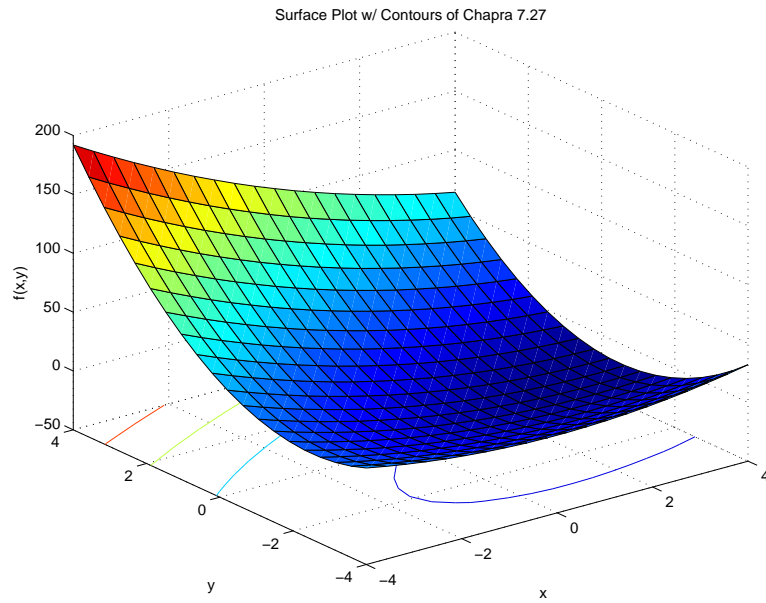


Figure 6: Surface plot with contours of Chapra 7.27