- **Formulas**
- Conditional Probability: $P(e \wedge h) = P(e \mid h) * P(h)$
- Bayes' Rule: $P(e \mid h) * P(h) = P(h \mid e) * P(e)$
- Chain Rule: $P(a_1 \wedge a_2 \wedge a_3) = P(a_1 \mid a_2 \wedge a_3) * P(a_2 \mid a_3) * P(a_3)$
- Filtering Formula: $P(s_i \mid o_{0\ldots i}) = \frac{P(o_i|s_i)*\sum_{s_{i-1}\in S_{i-1}} P(s_i|s_{i-1})*P(s_{i-1}|o_{0\ldots 1})*P(o_{0\ldots i-1})}{P(o_{0\ldots i})}$ where $P(o_{0\ldots i})$ means $P(o_0 \wedge o_1 \wedge \cdots \wedge o_i)$
- **Axioms of probability**
- $P(a) \geq 0$
- $P(true) = 1$
- $P(a \vee b) = P(a) + P(b)$ if a and b are mutually exclusive
- **Propositions**
- $P(\neg a) = 1 - P(a)$
- $P(a) = \sum_{x \in domain(X)} P(a \wedge X = x)$
- $P(a \vee b) = P(a) + P(b) - P(a \wedge b)$
- If $a \leftrightarrow b$ then $P(a) = P(b)$
- $P(a) = P(a \wedge b) + P(a \wedge \neg b) = \sum_{b \in B}(a \wedge b)$
- **Concepts**
- Random Variables: $P(X)$ is a function from the domain of X to $[0 \ldots 1]$, e.g. $P_X(32) = P(X = 32)$
- Expected Value: Let $f(w)$ be a function on worlds. Given a probability measure P, the expected value of $f \in P$ is $E_P(f) = \sum_{\omega \in \Omega} f(w) * P(w)$
- Expected Value Special Case:

$$f(\omega) = \begin{cases} 1 & if\ a\ is\ true\ in\ \omega \\ 0 & if\ a\ is\ false\ in\ \omega \end{cases}$$

$E_P(f) = P(a) = \sum_{\omega:\ a\ true\ in\ \omega} P(\omega)$

- Conditional Expected Value: $E_P(f \mid e) = \sum_{\omega \in \Omega} f(\omega) * P(\omega \mid e)$
- Independence: X is *independent* of Y if $P(X \mid YZ) = P(X \mid Z)$
- Discounted Reward: $V_1 = r_1 + \gamma r_2 + \gamma^2 r_3 + \cdots + \gamma^{n-1} r_n$, which implies $V_i = r_i + \gamma V_{i+1}$. Note that n may be ∞. V converges in most cases where $\gamma < 1$
- Value of a Policy: $V^{\Pi}(s)$ (which means value of Π at s) $\implies$ what reward you can expect if you start at s and follow policy Π
- **Belief Networks and Markov Chains**
- *Stationary Markov Chain:* dynamics are the same for every arrow
- *Ergodic Markov Chain:* any state has a non-zero probability of being reached from any state
- *Periodic Markov Chain:* states can only be reached during regularly occurring times
- All Markov Chains have an equivalent stationary chain
- Belief Networks only have states
- Markov Chains and Hidden Markov Models are special types of belief networks
- **Utility Theory**
- *completeness:* every outcome can be compared to all others
- *lottery:* probability distribution over outcomes
- *decomposability:* no preference between lotteries with equivalent outcomes and probability distributions
- *substitutability:* no preference between decomposable lotteries where an outcome in one lottery is different from its corresponding outcome in the other lottery if both outcomes have equivalent utilities
- *money pump:* $a \succ a$, $b \succ c$, $c \succ a$ (cycle that breaks transitivity)
- Utility $\sim$ Expected Value, math-wise