1 Compressible benchmark

The set of non dimentional equations that have to be solved,

$$\nabla \cdot (\rho_r \mathbf{u}) = \mathbf{0} - \delta \mathbf{P} + \nabla \cdot \tau = \mathbf{TRae_z} \tag{1}$$

The model setup to reproduce the compressible benchmark of Tan and Gurnis is as follows

1.1 Model domain

The model domain is a unit box with 16, 32 or 64 elements in each direction (equal spacing) free slip boundary conditions are applied.

1.2 viscosity

the viscosity variations are restricted to the vertical direction (1D) to be able to do a FFT decomposition of the equation allowing for a semi analytical solution. The 1D viscosity profile is given as,

$$\eta = e^{az} \tag{2}$$

with a a constant either 0 or 2

1.3 Right hand side

The model is driven by a lateral temperature perturbarton of the form

$$T(x,z) = sin(\pi z) cos(\pi k x) \tag{3}$$

with k the wavenumber

1.4 Density

the reference density is expressed as

$$\rho_r(z) = e^{\beta(1-z)} \tag{4}$$

with $\beta = Di/\gamma$
for the ssible incompreBousinesq approximation (BA) $Di = 0$ and $\gamma = \inf$ for the truncated anelastic approximation (TALA) $Di = 0.5$ and $\gamma = 1$. The density anomaly is given b,

$$\Delta\rho(x,z) = \rho_r(z) T(x,z) T(x,z) = sin(\pi z) cos(\pi k x) \tag{5}$$

The analytical solution to the flow is given as

$$\partial \begin{bmatrix} U_z \\ U_x \\ \sum_{zz}/2\eta_0 k \\ \sum_{xz}/2\eta_0 k \end{bmatrix} = \begin{bmatrix} \beta & -k & 0 & 0 \\ k & 0 & 0 & 2k/\eta^* \\ 0 & 0 & 0 & -\kappa \\ -\beta\eta^* & 2\kappa\eta^* & k & 0 \end{bmatrix} \cdot \begin{bmatrix} U_z \\ U_x \\ \sum_{zz}/2\eta_0 k \\ \sum_{xz}/2\eta_0 k \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \Omega Ra/2\eta_0 k \\ 0 \end{bmatrix} \tag{6}$$