

# Executive Summary

This project implements an end-to-end portfolio construction and risk analysis framework that explicitly separates **allocation decisions** from **risk evaluation**. Portfolios are constructed using standard, well-established rolling out-of-sample optimizers (Sharpe maximization, target volatility, mean–variance utility, and CVaR-constrained optimization), with transaction costs applied to evaluate implementable performance.

Risk is then analyzed *conditional on the realized portfolio* using a factor-based simulation layer. Time-varying macro factor exposures are estimated via rolling regressions, and portfolio outcomes are stress-tested under correlated shocks drawn from Gaussian and fat-tailed (multivariate Student- $t$ ) distributions. The simulation can additionally condition the factor covariance matrix on market regimes (e.g., low- vs. high-volatility environments), allowing dependence structure and tail behavior to differ across states of the world.

The objective is not return forecasting. Instead, the framework is designed to answer a risk question: **what drives portfolio losses in extreme market scenarios?** To that end, the project produces tail metrics (VaR/CVaR) and a tail decomposition that attributes losses in the worst simulated outcomes to individual macro risk drivers (equity risk, rates, credit spreads, inflation surprises, and the U.S. dollar factor). The result is a transparent and extensible research framework for diagnosing portfolio risk beyond variance-based summaries.

## 1 Framework Overview

The framework is organized into two clearly separated layers: a **portfolio construction layer** and a **risk analysis layer**.

Figure ?? provides a schematic overview of the workflow.

### 1.1 Portfolio Construction Layer

In the first layer, portfolios are constructed in a rolling out-of-sample setting using monthly asset returns. At each rebalancing date, the portfolio is optimized based solely on information available at that time, ensuring a clean separation between estimation and evaluation.

The framework supports multiple, well-established allocation objectives, including:

- maximization of the Sharpe ratio,
- target-volatility allocation,
- mean–variance utility maximization,
- maximization of expected return subject to a CVaR constraint.

All strategies operate under explicit asset-level constraints and allow for transaction costs, making the resulting portfolios implementable rather than purely theoretical. The output of this layer is a time series of realized portfolio weights and out-of-sample returns.

## 1.2 Factor-Based Risk Representation

In the second step, realized portfolio returns are mapped into a macro factor space. Portfolio-level factor exposures are estimated using rolling regressions of realized returns on a set of macroeconomic factors. This yields time-varying estimates of factor betas that summarize the portfolio's systematic risk profile at each point in time.

Importantly, factor modeling is used exclusively for risk representation and diagnostics. It does not feed back into the portfolio optimization stage and is not used for return forecasting.

## 1.3 Scenario Simulation

Conditional on the estimated factor exposures, portfolio risk is analyzed through simulation. Factor return scenarios are generated using multivariate distributions calibrated to historical data, allowing for both Gaussian and fat-tailed dynamics. Correlations across factors are explicitly preserved through the covariance structure.

The framework further allows the covariance matrix to depend on market regimes, defined by the volatility state of a selected factor. This enables the analysis of portfolio behavior under regime-dependent dependence structures, such as periods of elevated market stress.

Simulated factor realizations are mapped to portfolio returns via the estimated factor loadings, producing a distribution of hypothetical portfolio outcomes for a given point in time.

## 1.4 Tail Risk Diagnostics

From the simulated return distribution, tail-focused risk measures are computed, including Value-at-Risk and Conditional Value-at-Risk. Beyond scalar risk metrics, the framework decomposes extreme portfolio losses into factor-level contributions.

This decomposition identifies which macroeconomic risk drivers dominate portfolio losses in adverse scenarios, providing insight into the sources of tail risk rather than merely its magnitude.

## 1.5 Outputs

The framework produces a consistent set of outputs for each strategy, including realized performance, tail risk measures, regime-conditioned risk summaries, and factor-level loss decompositions. These outputs are

designed to support interpretation and diagnostic analysis rather than to optimize portfolio decisions directly.

## 2 Data and Asset Universe

The analysis is conducted using **monthly total return data** for a diversified set of liquid exchange-traded funds (ETFs), selected to serve as **investable proxies** for major asset classes. The use of ETFs ensures that portfolio allocations are fully implementable, transparent, and representative of instruments commonly used in practice.

### 2.1 Asset Universe

The portfolio universe includes the following asset classes and corresponding ETF proxies:

Asset Class	ETF Proxy
U.S. Equities (Broad Market)	SPY
U.S. Treasury Bonds (7–10Y)	IEF
U.S. Treasury Bills (Cash Proxy)	SHY
Chinese Equities	FXI
U.S. Real Estate (REITs)	VNQ
Gold	GLD

These assets are selected to provide meaningful diversification across equity risk, interest-rate risk, real-asset exposure, and defensive characteristics, while remaining sufficiently liquid over the full sample period.

### 2.2 Sample Period and Frequency

- **Frequency:** Monthly
- **Sample start:** January 2005
- **Sample end:** Latest available observation at the time of analysis

Monthly frequency is chosen to balance economic interpretability, robustness of estimation, and relevance for strategic asset allocation decisions.

### 2.3 Return Construction

Returns are computed as **monthly total returns**, incorporating price changes and distributions where applicable. All series are aligned to a common month-end calendar. Missing observations are handled conservatively, and assets are retained only when sufficient historical coverage is available for rolling estimation windows.

## 2.4 Rolling Out-of-Sample Framework

Portfolio optimization is performed in a **rolling out-of-sample setting** using a fixed-length estimation window:

- **Estimation window:** 60 months
- **Rebalancing frequency:** Monthly

At each rebalancing date, portfolio weights are determined using only information available up to that point in time. This ensures that all reported results reflect realistic, implementable decision rules and avoid look-ahead bias.

## 2.5 Factor Data

In addition to asset returns, a set of **macro-economic factor return series** is constructed to capture key underlying economic risks, including equity market conditions, interest rate movements, credit spreads, inflation dynamics, and currency fluctuations. These factor returns are used exclusively for **ex post risk attribution and scenario-based simulation**, and are not employed for return forecasting or portfolio optimization.

All factors are constructed using liquid, exchange-traded funds (ETFs) to ensure transparency, investability, and consistent data availability. Where appropriate, factor returns are defined as spreads between ETFs in order to isolate the intended economic risk while minimizing exposure to unrelated market components.

**Equity Market Risk.** Equity market risk is proxied by the return spread between the MSCI All Country World Index ETF (ACWI) and short-term U.S. Treasury bills (SHY). This construction captures global equity risk relative to a low-risk cash proxy and serves as a broad measure of risk-on versus risk-off conditions in global markets.

**Interest Rate (Duration) Risk.** Interest rate risk is represented by the return spread between intermediate-term U.S. Treasury bonds (IEF) and short-term Treasury bills (SHY). This factor isolates duration risk by capturing changes in the slope of the Treasury yield curve while abstracting from equity and credit risk.

**Credit Risk.** Credit risk is measured using the return spread between investment-grade corporate bonds (LQD) and intermediate-term U.S. Treasuries (IEF). This construction captures variation in corporate credit spreads and credit conditions while controlling for interest rate movements.

**Inflation Risk.** Inflation risk is proxied by the return spread between U.S. Treasury Inflation-Protected Securities (TIP) and nominal Treasuries (IEF). This factor reflects changes in inflation expectations and real-rate dynamics, isolating inflation-related risk from duration effects.

**U.S. Dollar Risk.** U.S. dollar exposure is represented by the return on the euro–U.S. dollar exchange rate ETF (FXE). This factor captures movements in the U.S. dollar relative to a major global currency and serves as a proxy for dollar strength in international asset pricing.

All factor returns are computed at monthly frequency and aligned with portfolio returns for rolling regression-based exposure estimation and simulation analysis.

## 2.6 Scope and Extensibility

The choice of assets, factors, and sample period is intended to be illustrative rather than exhaustive. The framework is fully modular and can be extended to alternative asset universes, higher-frequency data, or different factor definitions without altering the core methodology.

# 3 Portfolio Construction Methods

Portfolio allocation is performed using a set of well-established optimization frameworks commonly employed in academic research and professional asset management. The objective of this stage is **not** to forecast returns or exploit predictive signals, but to generate economically meaningful portfolios whose risk characteristics can later be analyzed in a consistent and comparable manner.

All portfolio strategies are implemented in a rolling out-of-sample setting, using only information available at the time of each rebalancing date. Optimization is conducted at monthly frequency using a fixed-length estimation window of 60 months.

## 3.1 General Setup

Let  $\mathbf{r}_t$  denote the vector of asset returns at time  $t$ , and let  $\mathbf{w}_t$  denote the portfolio weights chosen at the end of period  $t - 1$ . Portfolio returns are given by:

$$R_{p,t} = \mathbf{w}_t^\top \mathbf{r}_t$$

At each rebalancing date, portfolio weights are determined by solving an optimization problem subject to:

- Full investment:  $\sum_i w_i = 1$
- Long-only constraints
- Asset-specific allocation bounds

The same asset universe, constraints, and estimation window are used across all portfolio construction methods to ensure comparability.

### 3.2 Maximum Sharpe Ratio Portfolio

The maximum Sharpe ratio portfolio serves as a benchmark representing classical mean–variance optimal allocation. Portfolio weights are chosen to maximize the expected excess return per unit of volatility:

$$\max_w \frac{\mathbb{E}[R_p - R_f]}{\sqrt{\text{Var}(R_p)}}$$

Expected returns and the covariance matrix are estimated using historical sample moments over the rolling estimation window. A short-term Treasury ETF is used as a proxy for the risk-free rate.

This strategy reflects standard practice in strategic asset allocation and provides a natural reference point for comparison with alternative risk-based approaches.

### 3.3 Target Volatility Portfolio

The target volatility strategy seeks to construct a portfolio whose ex ante volatility matches a predefined annualized target level. Formally, portfolio weights are chosen to satisfy:

$$\text{Vol}(R_p) \leq \sigma^*$$

while remaining fully invested and respecting allocation constraints.

This approach decouples the allocation decision from return forecasts and is used in risk-controlled portfolio construction. It provides a useful contrast to unconstrained mean–variance optimization by explicitly anchoring portfolio risk.

### 3.4 Mean–Variance Utility Maximization

The utility-based strategy maximizes a quadratic mean–variance utility function:

$$\max_w \mathbb{E}[R_p] - \frac{\lambda}{2} \text{Var}(R_p)$$

where  $\lambda > 0$  represents the investor’s risk aversion parameter. This formulation nests mean–variance optimization within a preference-based framework and allows for smooth trade-offs between expected return and risk.

The risk aversion parameter is held constant across the sample period to isolate the impact of changing return and covariance estimates.

### 3.5 CVaR-Constrained Return Maximization

To explicitly address downside risk, a Conditional Value-at-Risk (CVaR) constrained strategy is implemented. Portfolio weights are chosen to maximize expected return subject to a constraint on tail risk:

$$\max_{\mathbf{w}} \mathbb{E}[R_p] \quad \text{s.t.} \quad \text{CVaR}_{\alpha}(R_p) \leq c$$

where  $\text{CVaR}_{\alpha}$  denotes expected losses beyond the  $\alpha$ -quantile, and  $c$  is a predefined monthly risk limit.

This formulation directly controls extreme downside outcomes and contrasts with variance-based approaches that treat positive and negative deviations symmetrically.

### 3.6 Transaction Costs and Turnover

To assess implementation realism, all strategies are evaluated both before and after transaction costs. Transaction costs are modeled as proportional to portfolio turnover:

$$\text{Cost}_t = \kappa \sum_i |w_{i,t} - w_{i,t-1}|$$

where  $\kappa$  represents the cost per unit of traded portfolio weight.

Net-of-cost returns are used for risk analysis whenever transaction costs materially affect portfolio behavior.

### 3.7 Role Within the Overall Framework

Importantly, portfolio construction represents only the first stage of the framework. No factor information, regime classification, or simulation-based assumptions are incorporated into the optimization process itself.

All risk analysis—factor exposure estimation, tail risk simulation, and regime-conditional covariance modeling—is performed **after** portfolio weights are determined.

## 4 Factor Exposure Estimation and Rolling Betas

To analyze portfolio risk beyond asset-level volatility, portfolio returns are decomposed into exposures to a small set of macroeconomic and market-wide risk factors. This step provides a transparent link between portfolio performance and underlying sources of systematic risk.

Factor exposures are estimated *after* portfolio construction, ensuring that allocation decisions are not influenced by factor modeling assumptions.

## 4.1 Factor Model Specification

Let  $R_{p,t}$  denote the realized portfolio return at time  $t$ . Portfolio returns are modeled using a linear factor structure:

$$R_{p,t} = \alpha + \boldsymbol{\beta}_t^\top \mathbf{F}_t + \varepsilon_t$$

where:

- $\mathbf{F}_t$  is a vector of factor returns at time  $t$ ,
- $\boldsymbol{\beta}_t$  represents time-varying factor exposures,
- $\alpha$  captures average unexplained return,
- $\varepsilon_t$  is an idiosyncratic residual.

The factor set is designed to capture broad macro-financial risks rather than asset-specific characteristics.

## 4.2 Rolling Estimation of Factor Exposures

Factor loadings are estimated using rolling ordinary least squares regressions over a fixed-length window of 60 months. At each time  $t$ , the regression uses information available up to  $t - 1$ :

$$\hat{\boldsymbol{\beta}}_t = \arg \min_{\boldsymbol{\beta}} \sum_{s=t-60}^{t-1} (R_{p,s} - \alpha - \boldsymbol{\beta}^\top \mathbf{F}_s)^2$$

This rolling estimation approach allows factor exposures to evolve gradually over time in response to changes in portfolio composition and market conditions.

## 4.3 Motivation for Time-Varying Betas

Assuming constant factor exposures is often unrealistic for dynamically rebalanced portfolios. Even in the absence of explicit market timing, changes in asset weights and cross-asset correlations can materially alter a portfolio's effective exposure to macroeconomic risks.

Rolling beta estimation captures:

- Shifts in risk concentration driven by rebalancing
- Changes in diversification effectiveness across market regimes
- The evolving relationship between portfolio returns and macro factors

This time variation is central to understanding portfolio behavior during periods of market stress, when correlations and risk transmission mechanisms tend to change.

## 4.4 Use of Factor Exposures in Risk Analysis

Estimated rolling betas serve two purposes within the broader framework:

1. They provide an interpretable decomposition of realized portfolio risk into macroeconomic drivers.
2. They form the mapping from simulated factor returns to simulated portfolio returns in the scenario and tail-risk analysis.

Importantly, factor exposures are treated as *state variables* summarizing the portfolio's sensitivity to systematic risks at a given point in time, rather than as predictive signals.

## 4.5 Alignment with Out-of-Sample Evaluation

To avoid look-ahead bias, only beta estimates available at the end of the estimation window are used for subsequent risk simulations. Factor realizations and beta estimates are aligned strictly by date, and all scenario analyses are conducted conditional on the most recently estimated exposures. This ensures that the risk analysis reflects the information set that would have been available to an investor at the time of evaluation.

# 5 Scenario Simulation and Regime-Conditional Risk Analysis

This section evaluates portfolio risk conditional on realized factor exposures. The objective is not return forecasting, but to analyze how portfolio risk behaves under tail events and changing market environments, given an already constructed portfolio.

Risk analysis is performed *after* portfolio construction, ensuring a clear separation between allocation decisions and risk diagnostics.

## 5.1 Factor Return Simulation

Portfolio risk is analyzed through simulations conducted in factor space. Let  $\mathbf{F}_t \in \mathbb{R}^K$  denote the vector of factor returns at time  $t$ . Future factor realizations are simulated from a multivariate distribution calibrated to historical factor data.

Two distributional assumptions are considered:

- A multivariate Gaussian distribution, serving as a benchmark.
- A multivariate Student- $t$  distribution, allowing for excess kurtosis and fat tails.

In both cases, the covariance structure of factor returns is preserved. Expected returns are set to zero, such that simulations focus exclusively on risk rather than return predictability.

## 5.2 Mapping Factor Shocks to Portfolio Returns

Simulated factor returns are mapped into portfolio returns using estimated factor exposures. Let  $\beta \in \mathbb{R}^K$  denote the vector of rolling factor loadings associated with the realized portfolio. Simulated portfolio returns are then given by:

$$R_p = \beta^\top \mathbf{F}. \quad (1)$$

This approach avoids resampling realized portfolio returns or asset-level returns. All risk dynamics originate from factor shocks and their interaction with the portfolio's exposure profile.

## 5.3 Tail Risk Metrics

Tail risk is evaluated using Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR), computed on the distribution of simulated portfolio returns. Losses are defined as negative returns, and tail metrics are calculated over the worst  $\alpha\%$  of outcomes.

In addition to standard tail statistics, the probability of exceeding fixed loss thresholds is reported to provide intuitive benchmarks for downside risk.

## 5.4 Tail Risk Decomposition

To understand the drivers of extreme portfolio losses, tail outcomes are decomposed into factor-level contributions. For a given simulated draw, the contribution of factor  $k$  is defined as:

$$C_k = \beta_k F_k. \quad (2)$$

Conditioning on the worst  $\alpha\%$  of portfolio outcomes, summary statistics of factor contributions are computed. This decomposition reveals which factors dominate portfolio losses in extreme scenarios and how their importance varies across environments.

## 5.5 Regime-Conditional Covariance Estimation

Factor return covariances are allowed to vary across market regimes. Regimes are identified using rolling volatility of a selected factor, which serves as a proxy for overall market stress.

Observations are classified into low- and high-volatility regimes based on quantile thresholds. Covariance matrices are estimated separately within each regime, subject to a minimum observation requirement. If

insufficient observations are available, the procedure falls back to the unconditional covariance.

Importantly, regimes affect only second moments. Expected returns are not adjusted, and no regime forecasting is performed. Simulations are conditioned on the covariance structure corresponding to the prevailing volatility regime at the evaluation date.

## 5.6 Deterministic Stress Scenarios

In addition to probabilistic simulations, a small set of deterministic stress scenarios is evaluated. These scenarios represent economically interpretable factor shocks and serve as diagnostic tools rather than probabilistic forecasts.

Stress scenarios complement the simulation-based analysis by providing transparent, scenario-specific insights into portfolio vulnerabilities.

# 6 Results

This section presents the empirical results of the portfolio construction and risk analysis framework. We first analyze realized out-of-sample performance and then examine how portfolio composition and risk exposures differ across optimization objectives.

## 6.1 Out-of-Sample Performance

Figure 6.1 reports the cumulative out-of-sample performance of the rolling Max-Sharpe and CVaR-constrained portfolios based on realized monthly returns.

Both strategies exhibit sustained long-run growth and broadly similar performance over the full sample. Major drawdowns and recoveries are closely aligned, indicating that both portfolios are primarily driven by common macroeconomic forces. However, important differences emerge around major market stress episodes.

During the COVID-19 crash in 2020, the CVaR-constrained portfolio experiences a substantially larger drawdown than the Max-Sharpe portfolio. This outcome may appear counterintuitive given the explicit tail-risk constraint imposed on the CVaR strategy. The explanation lies in the nature of the constraint: the CVaR optimizer maximizes expected return subject to a cap on *ex-ante* estimated CVaR, rather than minimizing realized losses during an unprecedented shock. Given the rolling estimation window and the limited presence of comparable historical tail events prior to 2020, the CVaR constraint does not fully anticipate the magnitude of the COVID shock.

Following the crash, however, the CVaR portfolio recovers substantially faster and outperforms the Max-Sharpe strategy until early 2022. This stronger rebound coincides with a shift toward assets that performed well in the post-crisis environment, most notably equities and gold. From 2022 onward, both portfolios

experience similar losses during the inflation surge and rapid interest-rate tightening, and their performance converges again in the subsequent recovery phase.

To better understand these dynamics, we next examine the evolution of portfolio weights over time.

#### **Rolling out-of-sample cumulative performance based on realized monthly returns.**



## **6.2 Portfolio Composition: Max-Sharpe vs. CVaR**

Figures 6.2 and 6.2 show the time evolution of portfolio weights for the Max-Sharpe and CVaR-constrained strategies, respectively.

Both portfolios exhibit substantial time variation in asset allocations, reflecting changing return and risk estimates in the rolling optimization framework. However, the composition of the two portfolios differs markedly.

The Max-Sharpe portfolio displays a consistently stronger allocation to U.S. Treasury bonds, particularly between 2014 and 2022. During this period, the allocation to intermediate-term Treasuries reaches up to 60%, reflecting their historically favorable Sharpe ratio in a low-rate, low-volatility environment. In contrast, the CVaR-constrained portfolio maintains a much higher exposure to SP 500, frequently reaching the imposed upper bound of 70%.

Once the SP500 allocation constraint becomes binding, the CVaR optimizer reallocates capital to assets with return profiles similar to SP500, most notably REITs and the China equity market. This behavior follows mechanically from the optimization problem: after the SP500 cap is reached, the optimizer selects the remaining assets that offer the highest expected returns while satisfying the CVaR constraint.

From 2020 onward, the CVaR portfolio exhibits a pronounced increase in gold exposure, exceeding 30% shortly after the COVID crash and rising to nearly 60% by 2023. The Max-Sharpe portfolio, by contrast,

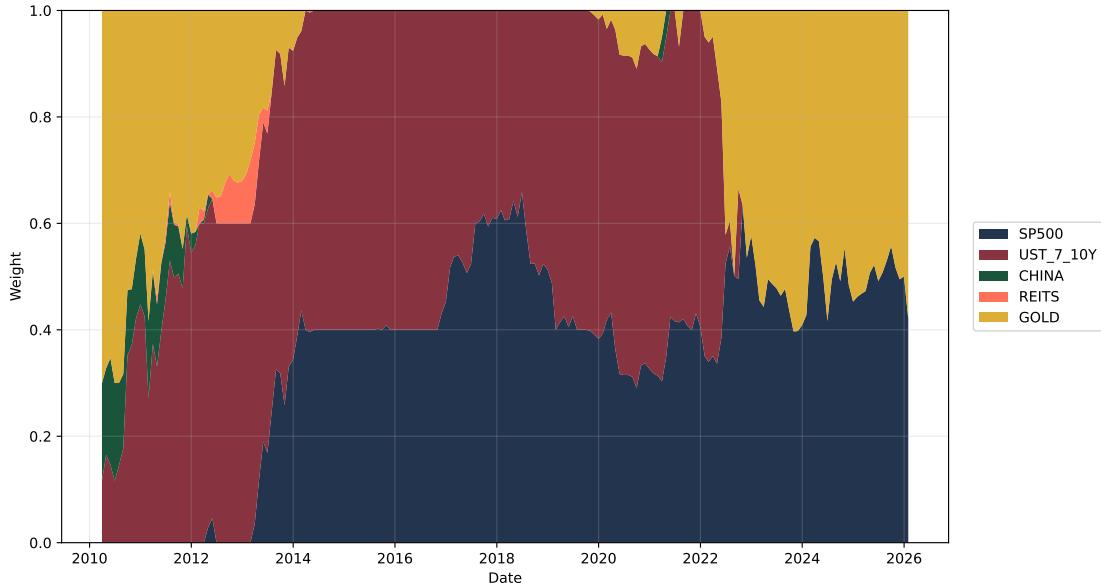
shifts from bonds to gold primarily during the 2022 rate-hike cycle, reaching allocations of around 50% only after bond performance deteriorates. The earlier and stronger allocation to gold in the CVaR strategy largely explains its superior post-COVID recovery.

Both strategies display extremely conservative allocations in the aftermath of the 2008 financial crisis, with virtually no equity exposure until approximately 2013–2014. This result is driven by the five-year rolling estimation window, which continues to incorporate large crisis losses until they exit the sample. Once these observations roll off, both optimizers rapidly increase equity exposure, leading to a sharp reallocation around 2013. The Max-Sharpe portfolio begins this transition slightly earlier, which helps explain its stronger relative performance in the 2013–2014 period.

From an implementation perspective, the magnitude of allocation shifts is noteworthy. Both strategies exhibit large and occasionally abrupt changes in portfolio weights, reflecting the sensitivity of rolling optimizers to changes in estimated return and risk inputs. This behavior suggests that static or infrequently rebalanced allocations may be suboptimal in environments characterized by rapidly evolving risk–return conditions.

At the same time, the observed instability highlights a limitation of using equally weighted historical observations within a fixed rolling window. Alternative estimation approaches that place greater weight on more recent returns—such as exponentially weighted schemes or regime-aware estimators—could potentially reduce the severity of allocation jumps while preserving responsiveness to changing market conditions. Exploring such smoothing mechanisms represents a natural extension for improving the practical implementability of the framework.

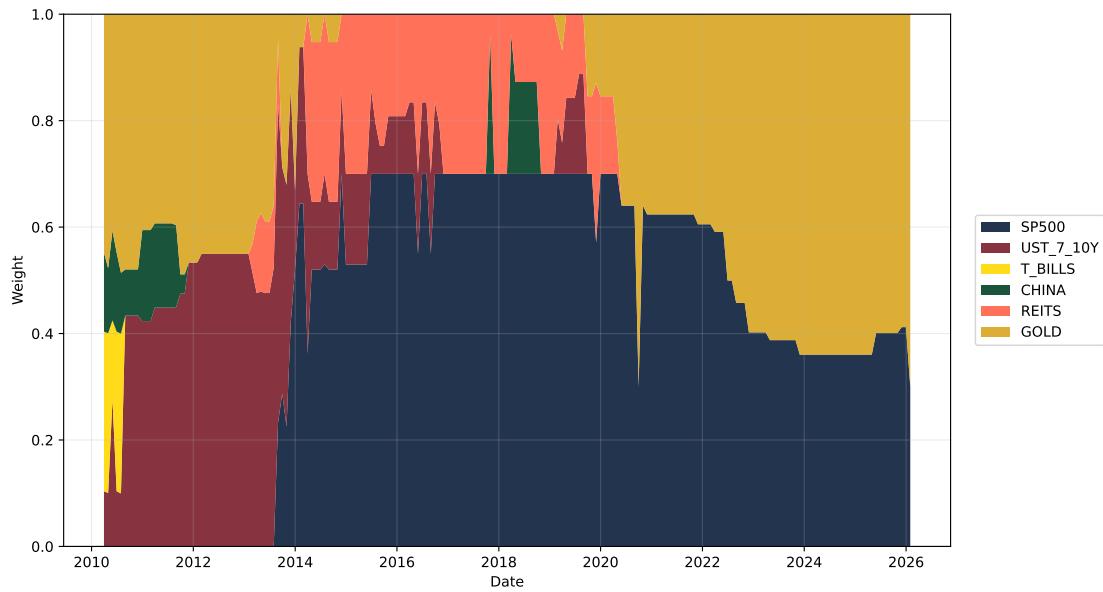
#### **Rolling portfolio weights - Max-Sharpe optimization.**



### **6.3 Risk Exposure**

We now examine the risk-factor exposures of both portfolios using the macro factor model.

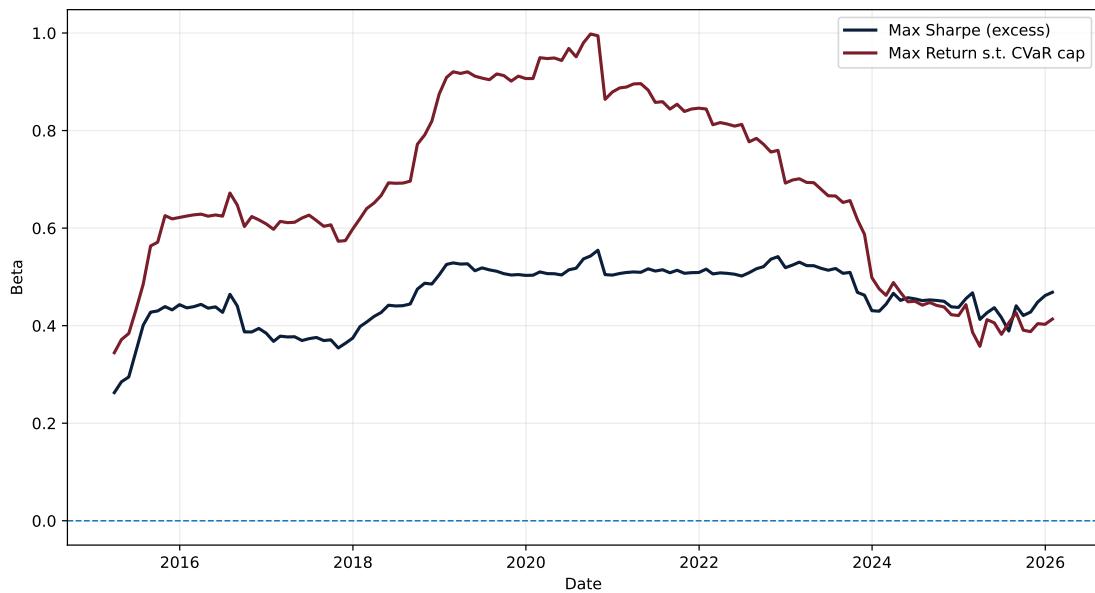
### Rolling portfolio weights - CVaR-constrained optimization.



Overall, estimated factor exposures closely mirror the portfolio allocation patterns discussed in the previous section. Differences between the Max-Sharpe and CVaR portfolios arise primarily from their distinct responses to changing risk–return conditions under rolling estimation and binding constraints.

**Equity risk.** Equity exposure is consistently higher for the CVaR portfolio between 2014 and 2022, reflecting its systematically larger allocation to the S&P 500. Both portfolios exhibit positive equity betas throughout the sample, confirming that equity risk remains the dominant source of systematic risk for both strategies.

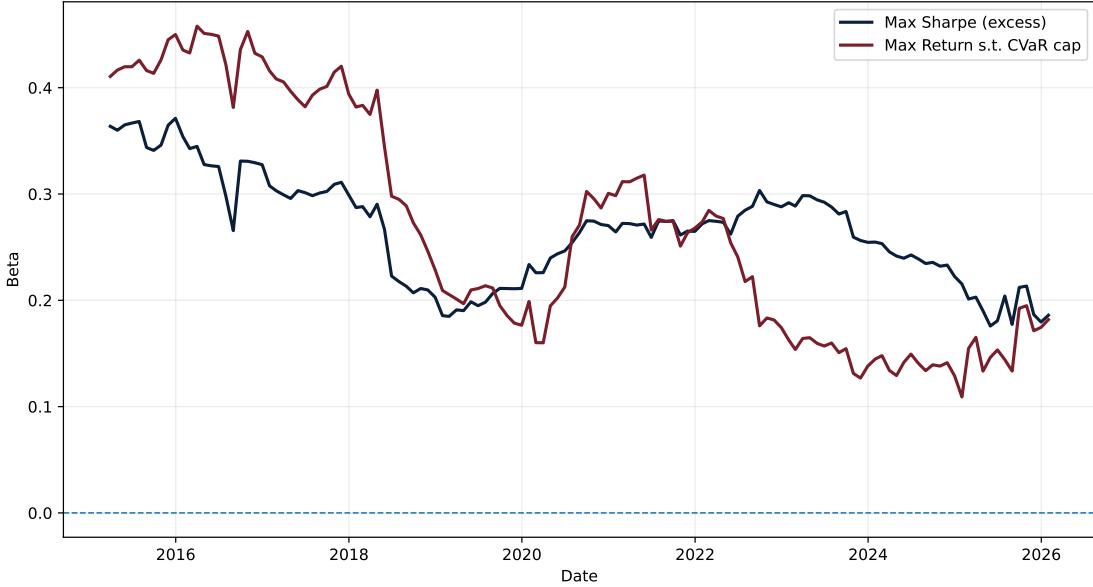
### Rolling betas - Equity Market Risk.



**Duration risk.** Duration exposure exhibits a clearer contrast across strategies, although both portfolios maintain positive duration exposure throughout the sample. Between 2015 and 2019, the CVaR portfolio displays higher duration exposure. Thereafter, its duration beta converges toward that of the Max-Sharpe portfolio and declines sharply from 2022 onward. By contrast, the Max-Sharpe portfolio maintains persistently higher duration exposure in the later part of the sample.

At first glance, the lower duration exposure of the CVaR portfolio in recent years may appear counterintuitive, given that both portfolios maintain allocations to U.S. equities and gold. However, factor regressions reported for 2026 are estimated over the 2021–2026 window. During this period, the Max-Sharpe strategy allocated more than 50% of capital to U.S. Treasuries in 2021 and 2022, whereas the CVaR portfolio held no Treasury exposure. Consequently, the estimated duration beta is mechanically higher for the Max-Sharpe portfolio.

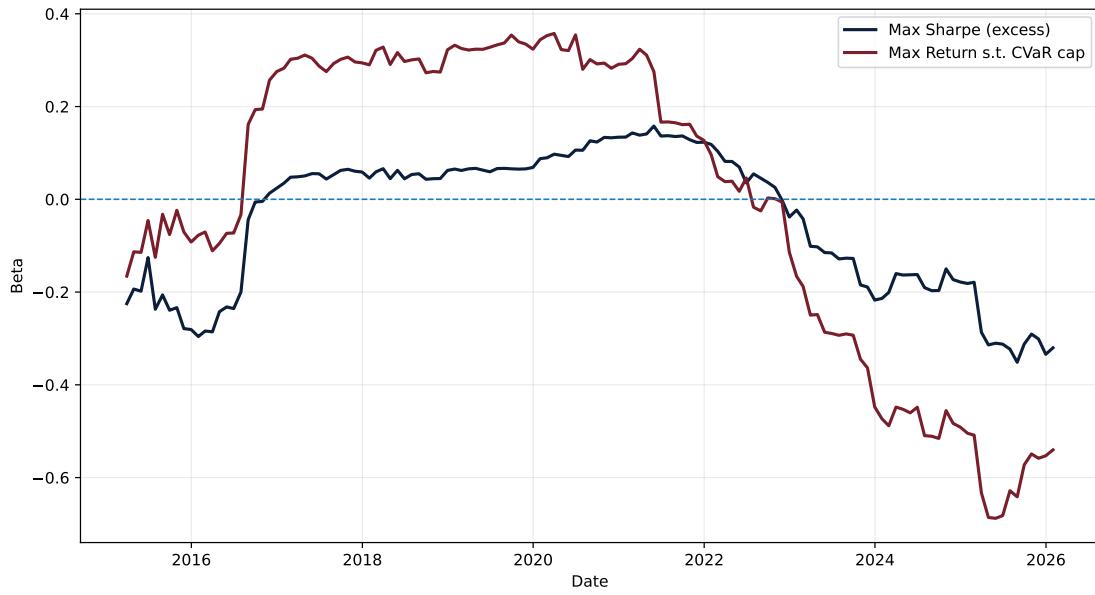
#### Rolling betas - Interest rate risk.



**U.S. dollar risk.** Exposure to the U.S. dollar factor is positive for both portfolios until approximately 2022 and is higher for the CVaR portfolio during this period. From 2022 onward, dollar exposure turns negative for both strategies, with the CVaR portfolio exhibiting a more pronounced decline.

This sign reversal reflects a regime shift in the relationship between the U.S. dollar and global risk assets. During the pre-2022 period, dollar appreciation coincided with favorable global growth conditions and accommodative financial markets, generating positive dollar exposure through equity and international asset holdings. Following the onset of aggressive monetary tightening and heightened global risk aversion, the dollar increasingly acted as a safe-haven asset. As a result, portfolios with greater exposure to risk assets—particularly equities and gold—exhibited negative dollar betas, while the sustained Treasury exposure in the Max-Sharpe portfolio partially offset this effect.

### Rolling betas - USD.



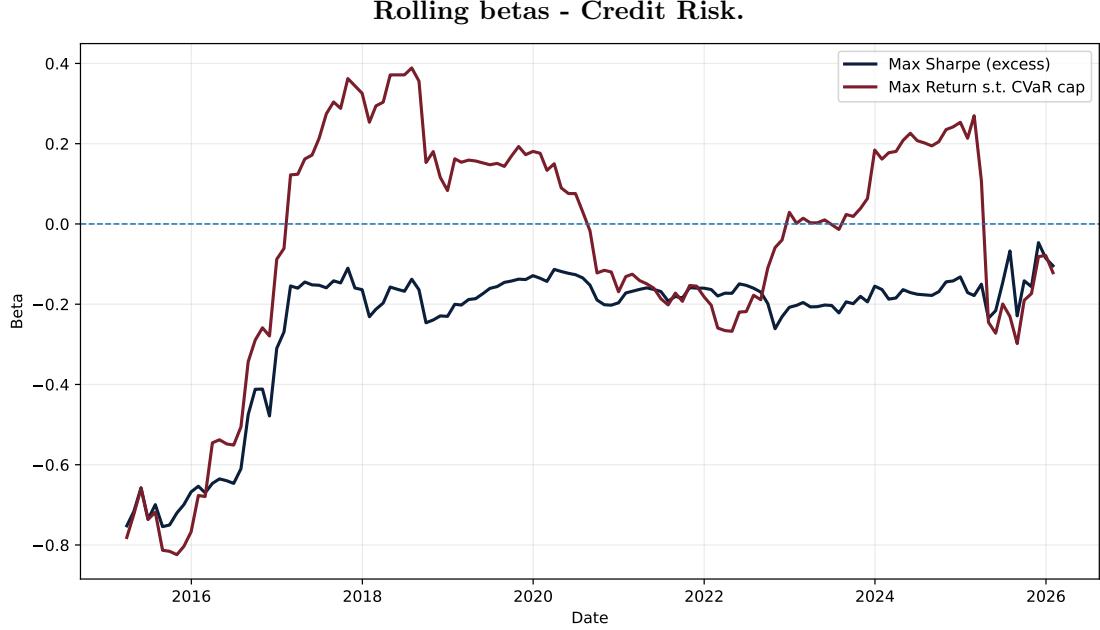
**Inflation risk.** Between 2018 and 2021, both portfolios exhibit negative inflation exposure, consistent with a prolonged low-inflation environment. From 2021 onward, inflation exposure becomes strongly positive for the CVaR portfolio, reflecting its substantial allocation to gold, which serves as an effective inflation hedge in the sample. The Max-Sharpe portfolio exhibits a weaker and more delayed increase in inflation exposure.

### Rolling betas - Inflation Risk.



**Credit risk.** Credit-factor exposure differs markedly across strategies. The Max-Sharpe portfolio maintains a consistently negative exposure to credit risk throughout the sample. In contrast, the CVaR portfolio exhibits positive credit exposure in several periods and substantially higher volatility in its credit beta. This

instability reflects the greater variability in portfolio composition induced by the binding CVaR constraint and its tendency to reallocate capital toward higher-return assets when tail-risk limits permit.



**Summary** Across all factors, risk exposures are markedly more time-varying for the CVaR strategy, whereas the Max-Sharpe portfolio exhibits comparatively smoother and more stable risk profiles. This contrast highlights a fundamental trade-off between adaptability and stability: tail-risk constraints induce dynamic reallocations and shifting factor exposures, while variance-based optimization produces more persistent risk characteristics that are slower to adjust across regimes.

## 6.4 Simulation-Based Risk Analysis

Figure 6.4 presents the full distribution of simulated portfolio returns under Student- $t$  distributional assumptions. The simulation is based on a conditional covariance matrix, where regimes are defined by the volatility of the equity factor. The most recent regime is classified as low volatility, and the corresponding covariance matrix is used for the baseline simulation.

Simulation results for the two portfolios are very similar, reflecting the convergence of portfolio allocations in recent years. Repeating the simulation under a high-volatility regime yields comparable results, suggesting that recent covariance structures are relatively stable across regimes. This stability likely reflects the strong and broadly correlated performance of most asset classes in the most recent sample period.

The most pronounced differences in simulated risk measures arise from the assumed return distribution. When returns are modeled using a Gaussian distribution instead of a Student- $t$  distribution, both portfolios exhibit substantially lower tail-risk measures. This result is expected, as the Student- $t$  distribution explicitly captures fat tails and extreme outcomes.

### Simulated distributions of portfolio returns.

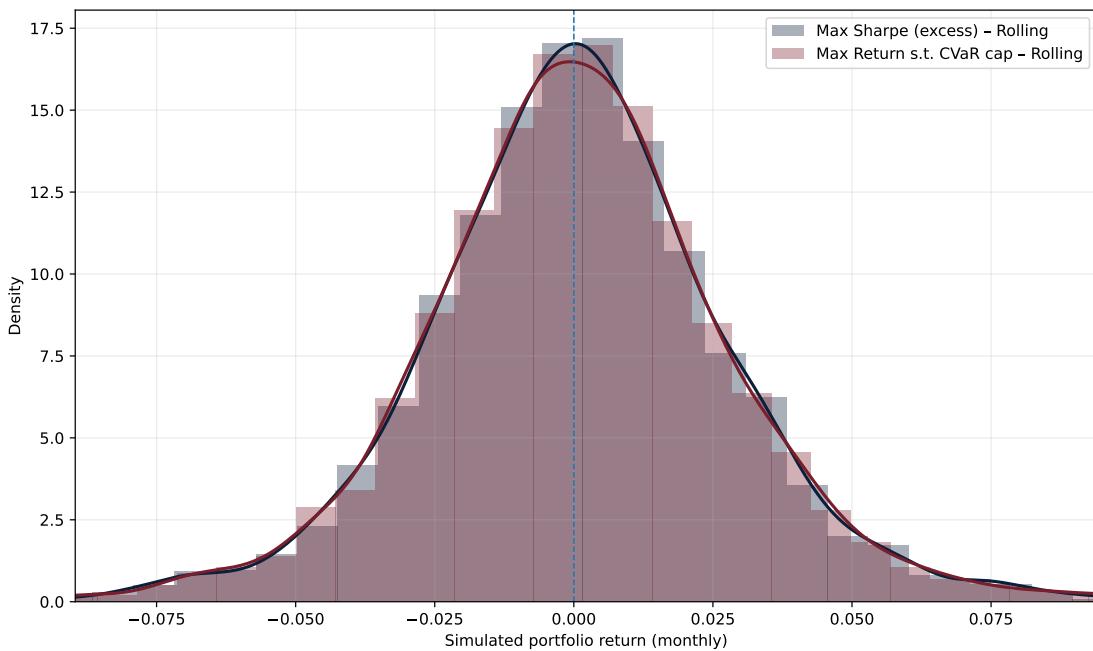
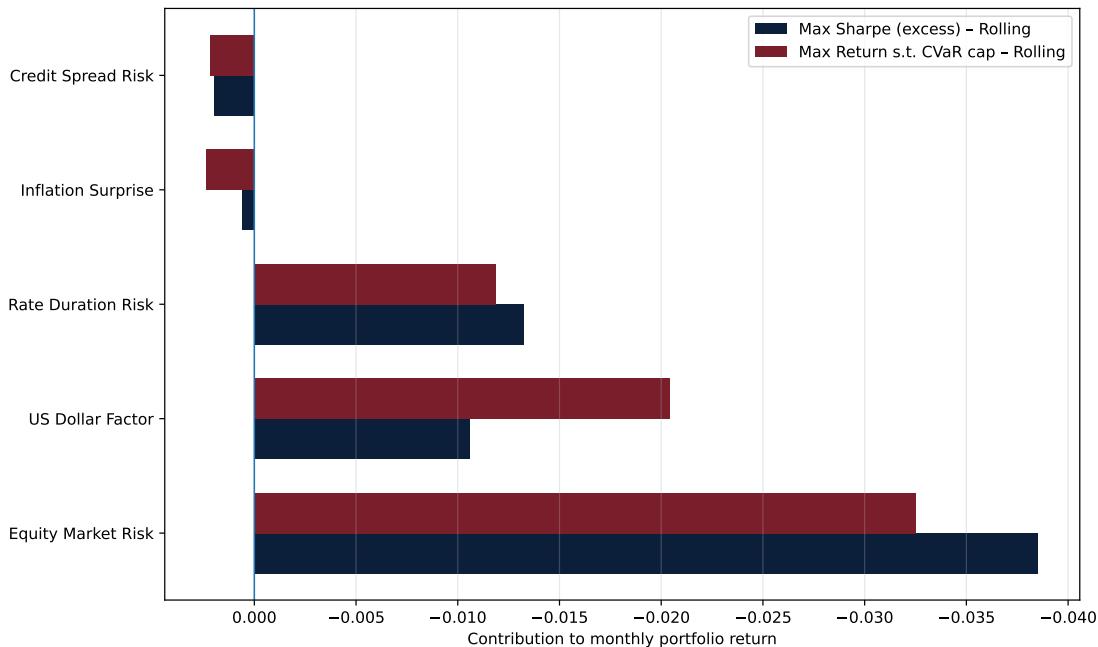


Figure 6.4 decomposes extreme portfolio losses into factor contributions. In both portfolios, equity risk is the dominant driver of tail losses, consistent with equities being the riskiest asset class in the investment universe. For the CVaR portfolio, the dollar factor represents the second-largest contributor, whereas rate risk plays a larger role in the Max-Sharpe portfolio. This difference again reflects the historically higher Treasury allocations in the Max-Sharpe strategy.

### Tail decomposition of portfolio losses.



Credit-spread and inflation-surprise factors contribute negatively to tail losses in both portfolios. This outcome is consistent with the portfolios' substantial exposure to gold, which tends to perform well during inflationary periods and episodes of economic stress associated with widening credit spreads.

## 7 Conclusion

This project demonstrates that meaningful insights into portfolio risk can be obtained by separating allocation decisions from risk evaluation. By combining standard portfolio construction techniques with factor-based simulation and regime-conditional risk analysis, the framework provides a flexible and interpretable approach to understanding portfolio vulnerabilities under extreme conditions.