10.6 Alternating Series, Conditional Convergence

Main Ideas

- **Alternating Series Test**

$$\text{the series} \quad \sum_{n=1}^{\infty}(-1)^{n+1}\, u_n = u_1 - u_2 + u_3 - u_4 + ... \quad \text{converges if and only if}$$

1. The u_n terms are all positive
2. The u_n terms are eventually non-increasing ($u_{n+1} \leq u_n$ for all $n \geq N$ for some N)
3. The u_n terms approach 0 ($\lim u_n = 0$ as $n \to \infty$)

- **Conditional Convergence**

If a series is convergent, but not absolutely convergent, then it is conditionally convergent

$$\text{if} \quad \sum_{n=1}^{\infty}|a_n| \quad \text{converges, then} \quad \sum_{n=1}^{\infty}a_n \quad \text{is absolutely convergent}$$

Summary of Convergence Tests

1. **N-th term test**
2. **Geometric series**
3. **P-series**
4. **Integral test**
5. **Direct comparison**
6. **Limit comparison**
7. **Root test**
8. **Ratio test**
9. **Absolute convergence**
10. **Alternating series test**