10.8 Taylor and Maclaurin Series

Main Ideas

• Taylor Series

If f is a function with derivatives of all orders throughout some interval containing a, then the <u>Taylor series</u> generated by f at x = a is

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!} (x-a)^k = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!} (x-a)^2 + \frac{f^{(3)}(a)}{3!} (x-a)^3 + \dots$$

where $f^{(k)}(x)$ is the k-th derivative of f(x)

• Maclaurin Series

The Maclaurin series of f is the Taylor series of f at a=0

\bullet A Taylor Polynomial of Order n

is the Taylor polynomial of f(x) up to the *n*-th order term

$$P_n(x) = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!} (x-a)^k = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!} (x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!} (x-a)^n$$