Improper Integrals

Calculus II §8.8

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Main Ideas

- Improper integrals are definite integrals with infinities or asymptotes within their bounds.
- This can be dealt with by replacing the infinity or asymptote with a limit.
- If the limit does not converge, the integral does not exist.

Examples

- This integral has a infinite bounds

$$\int_1^\infty \frac{1}{x^2} dx$$

we can fix this by adding a limit

$$= \lim_{b\to\infty} \int_1^b \frac{1}{x^2} dx$$

and the integral will exist if the limit exists

$$= \lim_{b\to\infty} \left[\frac{-1}{x} \right]_1^b = \lim_{b\to\infty} \left[\frac{-1}{b} - (-1) \right]$$

$$= \frac{-1}{\infty} + 1 = 0 + 1 = 1$$

- This function goes to infinity within the integral's bounds

$$\int_0^1 \frac{dx}{\sqrt{x}}$$

we can also add a limit here

$$= \lim_{a\to 0} \int_a^1 \frac{dx}{\sqrt{x}} = \lim_{a\to 0} \left[\, 2\sqrt{x} \, \right]_a^1$$
$$= \lim_{a\to 0} \left[\, 2\sqrt{1} - 2\sqrt{a} \, \right] = 2$$