10.7 Power Series

Main Ideas

- **Power Series**

  are series in the form

$$\sum_{n=0}^{\infty} c_n x^n \quad = \quad c_0 + c_1 x + c_2 x^2 + c_3 x^3 + ...$$

  a power series about (centered at) $x = a$ is in the form

$$\sum_{n=0}^{\infty} c_n (x-a)^n \quad = \quad c_0 + c_1 (x-a) + c_2 (x-a)^2 + c_3 (x-a)^3 + ...$$

- **Convergence Theorem for Power Series**

  1. If the power series

$$\sum_{n=0}^{\infty} c_n x^n \quad = \quad c_0 + c_1 x + c_2 x^2 + c_3 x^3 + ...$$

     converges at $x = c \neq 0$, then it converges for all $|x| < c$

  2. If it diverges for some $x = d$, then it diverges for all $|x| > |d|$

- **Testing for Convergence**

  1. Use the ratio test or the root test to find the interval of convergence

$$|x - a| < R$$

  2. If R is finite, test the endpoints for convergence
  3. If R is finite, the series diverges for all $|x - a| > R$