

10.8 Taylor and Maclaurin Series

Main Ideas

- **Taylor Series**

If f is a function with derivatives of all orders throughout some interval containing a , then the Taylor series generated by f at $x = a$ is

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!} (x-a)^k = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!} (x-a)^2 + \frac{f^{(3)}(a)}{3!} (x-a)^3 + \dots$$

where $f^{(n)}(x)$ is the n -th derivative of $f(x)$

- **Maclaurin Series**

The Maclaurin series of f is the Taylor series of f at $a = 0$

- **A Taylor Polynomial of Order n**

is the Taylor polynomial of $f(x)$ up to the n -th order term

$$P_n(x) = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!} (x-a)^k = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!} (x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!} (x-a)^n$$