10.1 Infinite Sequences

Main Ideas

- **Sequences**

  are lists of numbers where each term is defined by its index n

  $$\{\ a_n\ \} = \{\ a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ ...\ \}$$

  the n-th term of a sequence is defined by some formula

  $$a_n = f(n)$$

  $$\textbf{example:}\quad a_n = 2^n \implies \{\ a_n\ \} = \{2, 4, 8, 16, ...\}$$

  sometimes the n-th term can be defined recursively, meaning it is a function of the previous terms

  $$\textbf{example:}\quad a_n = a_{n-1} + a_{n-2}\ \text{ for } n \geq 3, \quad a_1 = 1, \quad a_2 = 1 \implies \quad \{\ a_n\ \} = \{1, 1, 2, 3, 5, 8, ...\}$$

- **Continuous Function Theorem**

  If $\{\ a_n\ \}$ is a sequence, and f is a continuous function where $f(n) = a_n$ for all n, then

  $$\lim_{n\to\infty} a_n \quad = \quad \lim_{x\to\infty} f(x)$$

  if $f(x)$ converges to a finite number L when $x \to \infty$, then a_n converges to L

- **Sandwich Theorem**

  If $\{\ a_n\ \}$, $\{\ b_n\ \}$, $\{\ c_n\ \}$ are all sequences with $a_n \leq b_n \leq c_n$ for all n beyond some N, and if

  $$\lim_{n\to\infty} a_n \quad = \quad \lim_{n\to\infty} c_n \quad = \quad L$$

  meaning that $\{\ a_n\ \}$ and $\{\ c_n\ \}$ both converge to L, then

  $$\lim_{n\to\infty} b_n \quad = \quad L$$

- **Nonincreasing** means that the terms of a sequence $\{\ a_n\ \}$ satisfy $a_{n+1} \leq a_n$ for all $n \geq N$ for some N
- **Nondecreasing** means that the terms of a sequence $\{\ a_n\ \}$ satisfy $a_{n+1} \geq a_n$ for all $n \geq N$ for some N
- **Monotonic** means that a sequence is either nondecreasing or nonincreasing
- **Bounded above** means that $a_n < M$ for all n for some M
- **Bounded bellow** means that $a_n > M$ for all n for some M
- **Monotonic Sequence Theorem**

  If a sequence is both bounded bellow and above, and monotonic, then it converges