10.1 Infinite Sequences

Main Ideas

- **Sequences**

  are lists of numbers where each term is defined by its index n

  $$\{\ a_n\ \} = \{\ a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ ...\ \}$$

- **Continuous Function Theorem**

  If $\{\ a_n\ \}$ is a sequence, and f is a continuous function where $f(n) = a_n$ for all n, then

  $$\lim_{n\to\infty} a_n \quad = \quad \lim_{x\to\infty} f(x)$$

  if $f(x)$ converges to a finite number L when $x \to \infty$, then a_n converges to L

- **Sandwich Theorem**

  If $\{\ a_n\ \}$, $\{\ b_n\ \}$, $\{\ c_n\ \}$ are all sequences with $a_n \leq b_n \leq c_n$ for all n beyond some N, and if

  $$\lim_{n\to\infty} a_n \quad = \quad \lim_{n\to\infty} c_n \quad = \quad L$$

  meaning that $\{\ a_n\ \}$ and $\{\ c_n\ \}$ both converge to L, then

  $$\lim_{n\to\infty} b_n \quad = \quad L$$

- **Nonincreasing**

  means that the terms of a sequence $\{\ a_n\ \}$ satisfy $a_{n+1} \leq a_n$ for all $n \geq N$ for some N

- **Nondecreasing**

  means that the terms of a sequence $\{\ a_n\ \}$ satisfy $a_{n+1} \geq a_n$ for all $n \geq N$ for some N

- **Monotonic**

  means that a sequence is either nondecreasing or nonincreasing

- **Bounded above**

  means that $a_n < M$ for all n for some M

- **Bounded bellow**

  means that $a_n > M$ for all n for some M

- **Monotonic Sequence Theorem**

  If a sequence is both <u>bounded</u> bellow and above, and <u>monotonic</u>, then it converges