Improper Integrals
Calculus II §8.8

Ian Turner
(irturner@mailbox.sc.edu)

February 6, 2024

Main Ideas

- Improper integrals are definite integrals with infinities or asymptotes within their bounds.
- This can be dealt with by replacing the infinity or asymptote with a limit.
- If the limit does not converge, the integral does not exist.

Problems

1.

$$\int_0^\infty \frac{dx}{x^2+1}$$

$$= \lim_{b\to\infty} \int_0^b \frac{dx}{x^2+1}$$ step 1: use a limit to remove infinity

$$= \lim_{b\to\infty} \arctan x\Big|_0^b$$ step 2: evaluate the integral

$$= \lim_{b\to\infty} \arctan b = \frac{\pi}{2}$$ step 3: evaluate bounds and limit

3.

$$\int_0^1 \frac{dx}{\sqrt{x}}$$ this integral has an asymptote

$$= \lim_{t\to 0} \int_t^1 \frac{dx}{\sqrt{x}}$$ step 1: place a limit at the asymptote

$$= \lim_{t\to 0} 2\sqrt{x}\Big|_t^1$$ step 2: evaluate the integral

$$= \lim_{t\to 0} 2 - 2\sqrt{t} = 2 - 0 = 2$$ step 3: evaluate bounds and limit

5.

$$\int_{-1}^1 \frac{dx}{x^{2/3}}$$ the issue here is the asymptote in the middle

$$\int_{-1}^0 \frac{dx}{x^{2/3}} + \int_0^1 \frac{dx}{x^{2/3}}$$ step 1: split integral into parts

$$= \lim_{t\to 0^-} \int_{-1}^t \frac{dx}{x^{2/3}} + \lim_{t\to 0^+} \int_t^1 \frac{dx}{x^{2/3}}$$ step 2: place limit at asymptote

$$= \lim_{t\to 0^-} 3x^{1/3}\Big|_{-1}^t + \lim_{t\to 0^+} 3x^{1/3}\Big|_t^1$$ step 3: evaluate integral

$$= \lim_{t\to 0^-} 3t^{1/3} + -(-3) + \lim_{t\to 0^+} 3 - 3t^{1/3} = 6$$ step 4: evaluate bounds and limit