8.1 - U-substitution

1. $\int k\ dx = kx + c$
2. $\int x^n\ dx = \frac{x^{n+1}}{n+1} + C$
3. $\int \frac{dx}{x} = \ln|x| + c$
4. $\int e^x\ dx = e^x + c$
5. $\int a^x\ dx = \frac{a^x}{\ln a} + c\ (a > 0, a \neq 1)$
6. $\int \sin x\ dx = -\cos x + c$
7. $\int \cos x\ dx = \sin x + c$
8. $\int \sec^2 x\ dx = \tan x + c$
9. $\int \csc^2 x\ dx = -\cot x + c$
10. $\int \sec x \tan x\ dx = \sec x + c$
11. $\int \csc x \cot x\ dx = -\csc x + c$
12. $\int \tan x\ dx = \ln|\sec x| + c$
13. $\int \cot x\ dx = \ln|\sin x| + c$
14. $\int \sec x\ dx = \ln|\sec x + \tan x| + c$
15. $\int \csc x\ dx = -\ln|\csc x + \cot x| + c$
16. $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\left(\frac{x}{a}\right) + c$
17. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\left|\frac{x}{a}\right| + c$
18. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\text{arcsec}\left|\frac{x}{a}\right| + c$

8.2 - Integration by parts

$$\int u\ dv = uv - \int v\ du$$

8.3 - Trigonometric integrals

Use the following process for evaluating products of powers of sin and cos. The process is similar for powers of tan and sec, except that you will always choose tan to be your substitution.

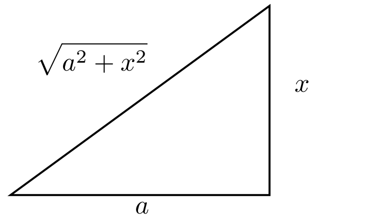
$$\int \sin^m x \cos^n x\ dx$$

Case 1 If m is odd, write m as $2k+1$ and use identity $\sin^2 x = 1 - \cos^2 x$. Then sub $u = \cos x$.

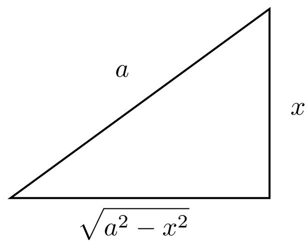
Case 2 If n is odd, write n as $2k+1$ and use the identity $\cos^2 x = 1 - \sin^2 x$. Then sub $u = \sin x$.

Case 3 If both m and n are even sub using half angle formula.

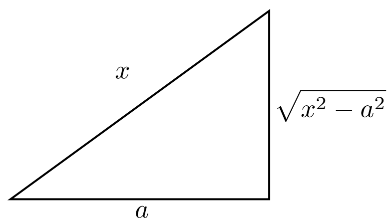
8.4 - Trigonometric substitutions



$x = a\tan\theta$



$x = a\sin\theta$



$x = a\sec\theta$

8.5 - Partial fractions

For when you have a rational function of two polynomials

$$\frac{f(x)}{g(x)}$$

1. Make sure $g(x)$ has the same or higher order than $f(x)$. The order is the number of its highest exponent term, ex: x^2 has order 2, x^3+2x has order 3.
2. If $f(x)$ has the higher order, do polynomial long division first.
3. Factor $g(x)$.
4. If $g(x)$ has a linear factor $(x-r)^n$, then the fraction decomposition will contain
$$\frac{A_1}{(x-r)} + \frac{A_2}{(x-r)^2} + ... + \frac{A_n}{(x-r)^n}$$
5. If $g(x)$ has an unfactorable quadratic factor $(ax^2+bx+c)^n$ (it will have complex roots), then the decomposition will contain
$$\frac{A_1x+B_1}{(ax^2+bx+c)} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + ... + \frac{A_nx+B_n}{(ax^2+bx+c)^n}$$

8.8 - Improper integrals

For when you have:

1. an infinity in the limit, like $\int_1^\infty \frac{dx}{\sqrt{x}}$.
2. an invalid input in your bounds, like $\int_0^1 \frac{dx}{x}$.
3. a discontinuity or asymptote within the integration region, like $\int_{-1}^1 \frac{dx}{x^{2/3}}$.

Situation 1 and 2 can be dealt with by adding a parameter and a limit:

$$\int_1^\infty \frac{dx}{x} = \lim_{t\to\infty} \int_1^t \frac{dx}{x}$$

And situation 3 is the same, except you will end up with two limits from either side:

$$\int_{-1}^1 \frac{dx}{x^{2/3}} = \lim_{t\to 0^-} \int_{-1}^t \frac{dx}{x^{2/3}} + \lim_{t\to 0^+} \int_t^1 \frac{dx}{x^{2/3}}$$

Trigonometric identities

- $\sin^2 x + \cos^2 x = 1$
- $\tan^2 x + 1 = \sec^2 x$
- $\sin\theta = \pm\sqrt{\frac{1-\cos 2\theta}{2}}$
- $\cos\theta = \pm\sqrt{\frac{1+\cos 2\theta}{2}}$
- $\sin 2\theta = \sin\theta\cos\theta$
- $\sin\theta\sin\phi = \frac{\cos(\theta-\phi)-\cos(\theta+\phi)}{2}$
- $\cos\theta\cos\phi = \frac{\cos(\theta+\phi)+\cos(\theta-\phi)}{2}$
- $\sin\theta\cos\phi = \frac{\sin(\theta+\phi)+\sin(\theta-\phi)}{2}$
- $\cos\theta\sin\phi = \frac{\sin(\theta+\phi)-\sin(\theta-\phi)}{2}$

Polynomial long division

To divide $f(x)$ by $g(x)$:

1. Place $f(x)$ inside the division symbol
2. Find what you need to multiply $f(x)$ by to cancel out the highest order term in $g(x)$.
3. Multiply $f(x)$ by this and subtract from $g(x)$
4. Write the new function bellow
5. Repeat until the order of the resulting function is less than the order of $f(x)$

Example:
$x^2 + 2x + 3$ divided by $x - 2$:

$$\begin{array}{rr} & x+4 \\ \hline x-2\,\big) & x^2+2x+3 \\ - & (x^2-2x) \\ \hline & 0+4x+3 \\ - & (4x-8) \\ \hline & 0+11 \end{array} = (x+4) + \frac{11}{x^2+2x+3}$$

L'Hopital's rule

For when you are evaluating a limit in the form:

$$\lim_{t\to a} \frac{f(x)}{g(x)} = \frac{0}{0} \quad \text{OR} \quad \frac{\infty}{\infty}$$

You can take the derivative of both parts:

$$\lim_{t\to a} \frac{f(x)}{g(x)} = \lim_{t\to a} \frac{f'(x)}{g'(x)}$$

And keep taking derivatives until you reach something easy to take the limit of.