Homework 9.3 (10.3)

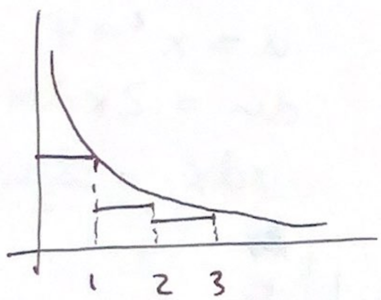
#1 $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} \longrightarrow \int_1^{\infty} \frac{1}{x^2}\,dx$

$\frac{1}{x^2}$ is ...

① Continuous on $x \geq 1$

② positive for all $x \geq 1$

③ decreasing for all $x \geq 1$



$$\int_1^{\infty} \frac{dx}{x^2} = \lim_{t\to\infty} \int_1^{t} \frac{dx}{x^2}$$

$$= \lim_{t\to\infty} -\frac{1}{x}\Big|_1^t = \lim_{t\to\infty} -\frac{1}{t} + 1$$

$$= 1$$ ✓converges

#5 $\sum_{n=1}^{\infty} e^{-2n} \longrightarrow \int_1^{\infty} e^{-2x}\,dx$

$$= \lim_{t\to\infty} \int_1^t e^{-2x}\,dx = \lim_{t\to\infty} -\frac{1}{2}e^{-2x}\Big|_1^t$$

$$= \lim_{t\to\infty} \cancel{-\frac{1}{2}e^{-2t}}^{\,0} + \frac{1}{2}e^{-2} = \frac{1}{2e^2} \quad \checkmark \text{ converges}$$

#7 $\sum_{n=1}^{\infty} \frac{n}{n^2+4} \longrightarrow \int_1^{\infty} \frac{x\,dx}{x^2+4}$

$u = x^2+4$
$du = 2x\,dx$
$x\,dx = \frac{du}{2}$

$$= \lim_{t\to\infty} \int_5^t \left(\frac{1}{u}\right)\left(\frac{du}{2}\right)$$

$$= \lim_{t\to\infty} \frac{\ln|u|}{2}\Big|_5^t$$

$$= \lim_{t\to\infty} \frac{\ln|x^2+4|}{2}\Big|_1^t$$

$$= \frac{\ln|\infty|}{2} - \frac{\ln|5|}{2} = \infty$$

X diverges

#13 $\displaystyle\sum_{n=1}^{\infty} \frac{1}{10^n} = \sum_{n=1}^{\infty} \left(\frac{1}{10}\right)^n \leftarrow$ geometric series where $r = 1/10$, $a = 1$.

✓converges since $|r| < 1$.

#15 $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}$ $\quad \displaystyle\lim_{n\to\infty} \frac{n}{n+1} = 1 \neq 0$

$\uparrow$ diverges X

#17 $\displaystyle\sum_{n=1}^{\infty} \frac{3}{\sqrt{n}} = 3\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \longrightarrow \int_1^{\infty} \frac{dx}{\sqrt{x}}$

(power rule)

$$= \lim_{t\to\infty} \int_1^t \frac{dx}{\sqrt{x}} = \lim_{t\to\infty} 2\sqrt{x}\Big|_1^t = \lim_{t\to\infty} 2\sqrt{t} - 2$$

$$= \infty$$

X diverges

#21 $\displaystyle\sum_{n=2}^{\infty} \frac{\ln|n|}{n} \longrightarrow \int_2^{\infty} \frac{\ln|\cancel{x}|}{\cancel{x}}\,dx$

$u = \ln x$

$du = \frac{dx}{x}$

$$= \lim_{t\to\infty} \int_2^t \frac{\ln|x|}{x}\,dx$$

$$= \lim_{t\to\infty} \frac{(\ln x)^2}{2}\Big|_2^t =$$

$$\lim_{t\to\infty} \frac{(\ln t)^2}{2} - \frac{(\ln 2)^2}{2} = \infty$$

$\uparrow$

∞

X diverges

#23 $\sum_{n=1}^{\infty} \frac{2^n}{3^n} = \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$ geometric series with $r = 2/3$.
✓ converges since $|r| < 1$.

#25 $\sum_{n=0}^{\infty} \frac{-2}{n+1} = \sum_{n=1}^{\infty} \frac{-2}{n} = -2 \sum_{n=1}^{\infty} \frac{1}{n}$

$\longrightarrow \int_1^{\infty} \frac{dx}{x} = \lim_{t \to \infty} \int_1^t \frac{dx}{x} = \lim_{t \to \infty} \underbrace{\ln|t|}_{\infty} - \underbrace{\ln|1|}_{0} = \infty$

X diverges

#27 $\sum_{n=1}^{\infty} \frac{2^n}{n+1}$ $\quad \lim_{n \to \infty} \frac{2^n}{n+1} \xrightarrow{\text{L'Hopitals}} = \lim_{n \to \infty} \frac{2^n \ln 2}{1}$

$= \infty$ X diverges

#37 $\sum_{n=1}^{\infty} \frac{e^n}{10+e^n}$ $\quad \lim_{n \to \infty} \frac{e^n}{10+e^n} = 1 \neq 0$

X diverges