VALORES DE LAS FUNCIONES TRIGONOMÉTRICAS PARA LOS ÁNGULOS MULTIPLOS DE 30º Y 45º (π/6 Y π/4)

Función	Ángulos del 1er. cuadrante				Ángulos del 2do. cuadrante				Ángulos del 3er. cuadrante				Ángulos del 4to. cuadrante				
	0º	30º	45º	60º	90º	120º	135º	150º	180º	210º	225º	240º	270º	300º	315º	330º	360º
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
sen	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
tg	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
ctg	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	∞
sec	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	∞	-2	$-\sqrt{2}$	$-\frac{2\sqrt{3}}{3}$	-1	$-\frac{2\sqrt{3}}{3}$	$-\sqrt{2}$	-2	∞	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1
cosec	∞	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	∞	-2	$-\sqrt{2}$	$-\frac{2\sqrt{3}}{3}$	-1	$-\frac{2\sqrt{3}}{3}$	$-\sqrt{2}$	-2	∞

FÓRMULAS FUNDAMENTALES DE LA TRIGONOMETRÍA

Funciones de un ángulo:

1. $\operatorname{sen}^2\alpha+\cos^2\alpha=1$
2. $\sec^2\alpha-\operatorname{tg}^2\alpha=1$
3. $\csc^2\alpha-\operatorname{ctg}^2\alpha=1$
4. $\frac{\operatorname{sen}\alpha}{\cos\alpha}=\operatorname{tg}$
5. $\frac{\cos\alpha}{\operatorname{sen}\alpha}=\operatorname{ctg}\alpha$
6. $\operatorname{sen}\alpha\cdot\csc\alpha=1$
7. $\cos\alpha\cdot\sec\alpha=1$
8. $\operatorname{tg}\alpha\cdot\operatorname{ctg}\alpha=1$

Fórmulas de conversión:

Para cada circunferencia de ángulo central α, se mide por la razón entre del arco l correspondiente a este ángulo y la longitud del radio r de ésta circunferencia:

$$\alpha=\frac{l}{r}$$

$$\alpha^o=\frac{180\alpha}{\pi}(\text{radianes});\ \alpha(\text{radianes})=\frac{\pi\alpha^o}{180}$$

Expresión de una función mediante otra (del mismo ángulo):

1. $\operatorname{sen}\alpha=\pm\sqrt{1-\cos^2\alpha}=\pm\frac{\operatorname{tg}\alpha}{\sqrt{1+\operatorname{tg}^2\alpha}}=\pm\frac{1}{\sqrt{1+\operatorname{ctg}^2\alpha}}=\pm\frac{\sqrt{\sec^2\alpha-1}}{\sec\alpha}=\frac{1}{\csc\alpha}$
2. $\cos\alpha=\pm\sqrt{1-\operatorname{sen}^2\alpha}=\pm\frac{1}{\sqrt{1+\operatorname{tg}^2\alpha}}=\pm\frac{\operatorname{ctg}\alpha}{\sqrt{1+\operatorname{ctg}^2\alpha}}=\frac{1}{\sec\alpha}=\pm\frac{\sqrt{\csc^2\alpha-1}}{\csc\alpha}$
3. $\operatorname{tg}\alpha=\pm\frac{\operatorname{sen}\alpha}{\sqrt{1-\operatorname{sen}^2\alpha}}=\pm\frac{\sqrt{1-\cos^2\alpha}}{\cos\alpha}=\frac{1}{\operatorname{ctg}\alpha}=\pm\sqrt{\sec^2\alpha-1}=\pm\frac{1}{\sqrt{\csc^2\alpha-1}}$
4. $\operatorname{ctg}\alpha=\pm\frac{\sqrt{1-\operatorname{sen}^2\alpha}}{\operatorname{sen}\alpha}=\pm\frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}}=\frac{1}{\operatorname{tg}\alpha}=\pm\frac{1}{\sqrt{\sec^2\alpha-1}}=\pm\sqrt{\csc^2\alpha-1}$

Funciones de la suma y la diferencia de ángulos:

1. $\operatorname{sen}(\alpha \pm \beta) = \operatorname{sen}\alpha\cos\beta \pm \cos\alpha\operatorname{sen}\beta$
2. $\cos(\alpha \pm \beta)\cos\alpha\cos\beta \mp \operatorname{sen}\alpha\operatorname{sen}\beta$
3. $\operatorname{tg}(\alpha \pm \beta) = \dfrac{\operatorname{tg}\alpha \pm \operatorname{tg}\beta}{1 \mp \operatorname{tg}\alpha\operatorname{tg}\beta}$
4. $\operatorname{c\,tg}(\alpha \pm \beta) = \dfrac{\operatorname{c\,tg}\alpha\operatorname{c\,tg}\beta \mp 1}{\operatorname{c\,tg}\beta \pm \operatorname{c\,tg}\alpha}$
5. $\operatorname{sen}(\alpha+\beta+\gamma) = \operatorname{sen}\alpha\cos\beta\cos\gamma + \cos\alpha\operatorname{sen}\beta\cos\gamma + \cos\alpha\cos\beta\operatorname{sen}\gamma - \operatorname{sen}\alpha\operatorname{sen}\beta\operatorname{sen}\gamma$
6. $\cos(\alpha+\beta+\gamma) = \cos\alpha\cos\beta\cos\gamma - \operatorname{sen}\alpha\operatorname{sen}\beta\cos\gamma - \operatorname{sen}\alpha\cos\beta\operatorname{sen}\gamma - \cos\alpha\operatorname{sen}\beta\operatorname{sen}\gamma$

Funciones de ángulos múltiples:

1. $\operatorname{sen}2\alpha = 2\operatorname{sen}\alpha\cos\alpha$
2. $\operatorname{sen}3\alpha = 3\operatorname{sen}\alpha - 4\operatorname{sen}^3\alpha$
3. $\operatorname{sen}4\alpha = 8\cos^3\alpha\operatorname{sen}\alpha - 4\cos\alpha\operatorname{sen}\alpha$
4. $\cos2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha$
5. $\cos3\alpha = 4\cos^3\alpha - 3\cos\alpha$
6. $\cos4\alpha = 8\cos^4 - 8\cos^2\alpha + 1$
7. $\operatorname{tg}2\alpha = \dfrac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}$
8. $\operatorname{tg}3\alpha = \dfrac{3\operatorname{tg}\alpha - \operatorname{tg}^3\alpha}{1-3\operatorname{tg}^2\alpha}$
9. $\operatorname{tg}4\alpha = \dfrac{4\operatorname{tg}\alpha - 4\operatorname{tg}^3\alpha}{1-6\operatorname{tg}^2\alpha + \operatorname{tg}^2\alpha}$
10. $\operatorname{c\,tg}2\alpha = \dfrac{\operatorname{c\,tg}^2\alpha - 1}{2\operatorname{c\,tg}\alpha}$
11. $\operatorname{c\,tg}3\alpha = \dfrac{\operatorname{c\,tg}^3\alpha - 3\operatorname{c\,tg}\alpha}{3\operatorname{c\,tg}^2\alpha - 1}$
12. $\operatorname{c\,tg}\alpha = \dfrac{\operatorname{c\,tg}^4\alpha - 6\operatorname{c\,tg}^2\alpha + 1}{4\operatorname{c\,tg}^3\alpha - 4\operatorname{c\,tg}\alpha}$

Funciones de ángulo mitad:

1. $\operatorname{sen}\left(\dfrac{\alpha}{2}\right) = \pm\sqrt{\dfrac{1-\cos\alpha}{2}}$
2. $\cos\left(\dfrac{\alpha}{2}\right) = \pm\sqrt{\dfrac{1+\cos\alpha}{2}}$
3. $\operatorname{tg}\left(\dfrac{\alpha}{2}\right) = \pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}} = \dfrac{1-\cos\alpha}{\operatorname{sen}\alpha} = \dfrac{\operatorname{sen}\alpha}{1+\cos\alpha}$
4. $\operatorname{c\,tg}\left(\dfrac{\alpha}{2}\right) = \pm\sqrt{\dfrac{1+\cos\alpha}{1-\cos\alpha}} = \dfrac{1+\cos\alpha}{\operatorname{sen}\alpha} = \dfrac{\operatorname{sen}\alpha}{1-\cos\alpha}$

Suma y diferencia de funciones:

1. $\operatorname{sen}\alpha + \operatorname{sen}\beta = 2\operatorname{sen}\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right)$
2. $\operatorname{sen}\alpha - \operatorname{sen}\beta = 2\cos\left(\dfrac{\alpha+\beta}{2}\right)\operatorname{sen}\left(\dfrac{\alpha-\beta}{2}\right)$
3. $\cos\alpha + \cos\beta = 2\cos\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right)$
4. $\cos\alpha - \cos\beta = 2\operatorname{sen}\left(\dfrac{\alpha+\beta}{2}\right)\operatorname{sen}\left(\dfrac{\alpha-\beta}{2}\right)$
5. $\operatorname{tg}\alpha \pm \operatorname{tg}\beta = \dfrac{\operatorname{sen}(\alpha \pm \beta)}{\cos\alpha\cos\beta}$
6. $\operatorname{c\,tg}\alpha \pm \operatorname{c\,tg}\beta = \pm\dfrac{\operatorname{sen}(\alpha \pm \beta)}{\operatorname{sen}\alpha\operatorname{sen}\beta}$
7. $\operatorname{tg}\alpha + \operatorname{c\,tg}\beta = \dfrac{\cos(\alpha-\beta)}{\cos\alpha\operatorname{sen}\beta}$
8. $\operatorname{c\,tg}\alpha - \operatorname{tg}\beta = \dfrac{\cos(\alpha+\beta)}{\operatorname{sen}\alpha\cos\beta}$

Producto de funciones:

1. $\operatorname{sen}\alpha\operatorname{sen}\beta = \dfrac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right)$
2. $\cos\alpha\cos\beta = \dfrac{1}{2}\left(\cos(\alpha-\beta) + \cos(\alpha+\beta)\right)$
3. $\operatorname{sen}\alpha\cos\beta = \dfrac{1}{2}\left(\operatorname{sen}(\alpha-\beta) + \operatorname{sen}(\alpha+\beta)\right)$