Universidad de Guadalajara

CUCEI

Proyecto Modular

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1. Cinemática directa

1.1. Modelado del *youBot Arm*

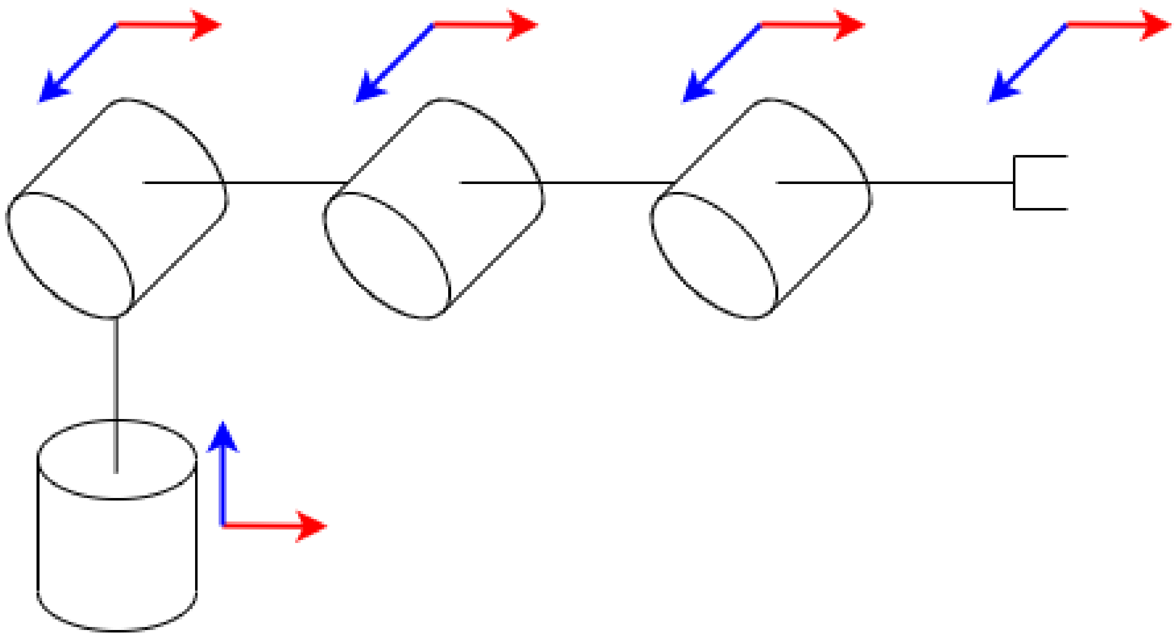

Figura 1: youBot Arm

1.2. Fórmulas y propiedades

$$ {}^{n-1}T_n = \begin{bmatrix} C_{\theta_n} & -S_{\theta_n}C_{\alpha_n} & S_{\theta_n}S_{\alpha_n} & a_nC_{\theta_n} \\ S_{\theta_n} & C_{\theta_n}C_{\alpha_n} & -C_{\theta_n}S_{\alpha_n} & a_nS_{\theta_n} \\ 0 & S_{\alpha_n} & C_{\alpha_n} & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta) $$
$$ \cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta) $$

1.3. Tabla de parámetros *Denavit-Hartenberg*

	α	a	d	θ
1	$\frac{\pi}{2}$	0	d_1	0
2	0	a_2	0	0
3	0	a_3	0	0
4	0	a_4	0	0

Tabla 1: Tabla de parámetros *Denavit-Hartenberg*

1.4. Matrices de transformación

$$\boxed{{}^0T_1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \qquad {}^1T_2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2T_3 = \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^3T_4 = \begin{bmatrix} C_4 & -S_4 & 0 & a_4C_4 \\ S_4 & C_4 & 0 & a_4S_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_4 = {}^0T_1\,{}^1T_2\,{}^2T_3\,{}^3T_4$$

1.5. Cinemática directa del *youBot Arm*

$$\boxed{{}^0T_2 = \begin{bmatrix} C_1C_2 & -C_1S_2 & S_1 & a_2C_1C_2 \\ S_1C_2 & -S_1S_2 & -C_1 & a_2S_1C_2 \\ S_2 & C_2 & 0 & a_2S_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$${}^0T_3 = \begin{bmatrix} C_1C_2C_3 - C_1S_2S_3 & -C_1C_2S_3 - C_1S_2C_3 & S_1 & a_3C_1C_2C_3 - a_3C_1S_2S_3 + a_2C_1C_2 \\ S_1C_2C_3 - S_1S_2S_3 & -S_1C_2S_3 - S_1S_2C_3 & -C_1 & a_3S_1C_2C_3 - a_3S_1S_2S_3 + a_2S_1C_2 \\ S_2C_3 + C_2S_3 & -S_2S_3 + C_2C_3 & 0 & a_3S_2C_3 + a_3C_2S_3 + a_2S_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{{}^0T_3 = \begin{bmatrix} C_1C_{23} & -C_1S_{23} & S_1 & a_3C_1C_{23} + a_2C_1C_2 \\ S_1C_{23} & -S_1S_{23} & -C_1 & a_3S_1C_{23} + a_2S_1C_2 \\ S_{23} & C_{23} & 0 & a_3S_{23} + a_2S_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$${}^0T_4 = \begin{bmatrix} C_1C_{23}C_4 - C_1S_{23}S_4 & -C_1C_{23}S_4 - C_1S_{23}C_4 & S_1 & a_4C_1C_{23}C_4 - a_4C_1S_{23}S_4 + a_3C_1C_{23} + a_2C_1C_2 \\ S_1C_{23}C_4 - S_1S_{23}S_4 & -S_1C_{23}S_4 - S_1S_{23}C_4 & -C_1 & a_4S_1C_{23}C_4 - a_4S_1S_{23}S_4 + a_3S_1C_{23} + a_2S_1C_2 \\ S_{23}C_4 + C_{23}S_4 & -S_{23}S_4 + C_{23}C_4 & 0 & a_4S_{23}C_4 + a_4C_{23}S_4 + a_3S_{23} + a_2S_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_4 = \begin{bmatrix} C_1C_{234} & -C_1S_{234} & S_1 & a_4C_1C_{234} + a_3C_1C_{23} + a_2C_1C_2 \\ S_1C_{234} & -S_1S_{234} & -C_1 & a_4S_1C_{234} + a_3S_1C_{23} + a_2C_1C_2 \\ S_{234} & C_{234} & 0 & a_4S_{234} + a_3S_{23} + a_2S_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{{}^0T_4 = \begin{bmatrix} C_1C_{234} & -C_1S_{234} & S_1 & C_1(a_4C_{234} + a_3C_{23} + a_2C_2) \\ S_1C_{234} & -S_1S_{234} & -C_1 & S_1(a_4C_{234} + a_3C_{23} + a_2C_2) \\ S_{234} & C_{234} & 0 & a_4S_{234} + a_3S_{23} + a_2S_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

1.6. Cinemática directa de la plataforma omnidireccional

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -(L+l) \\ 1 & 1 & (L+l) \\ 1 & 1 & -(L+l) \\ 1 & -1 & (L+l) \end{bmatrix}^{\dagger} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$$

1.7. Cinemática inversa de la plataforma omnidireccional

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -(L+l) \\ 1 & 1 & (L+l) \\ 1 & 1 & -(L+l) \\ 1 & -1 & (L+l) \end{bmatrix} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}^{\top} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} \sqrt{2}\sin(\alpha) & -\sqrt{2}\cos(\alpha) & -(L+l) \\ \sqrt{2}\cos(\alpha) & \sqrt{2}\sin(\alpha) & (L+l) \\ \sqrt{2}\cos(\alpha) & \sqrt{2}\sin(\alpha) & -(L+l) \\ \sqrt{2}\sin(\alpha) & -\sqrt{2}\cos(\alpha) & (L+l) \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

$$\alpha = \theta + \frac{\pi}{4}$$

1.8. Cinemática directa desde el origen hasta el efector final

- **w:** Eje coordenado del origen.
- **p:** Eje coordenado adherido a la plataforma.
- **b:** Eje coordenado de la base del manipulador.
- **e:** Eje coordenado del actuador final.

$$^{w}T_e = {^{w}T_p}\,{^{p}T_b}\,{^{b}T_e}$$

$$\boxed{^{w}T_p = \begin{bmatrix} \cos(\theta_p) & -\sin(\theta_p) & 0 & x_p \\ \sin(\theta_p) & \cos(\theta_p) & 0 & y_p \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

Con x_p y y_p como la posición del robot y θ_p como la orientación.

$$\boxed{^{p}T_b = \begin{bmatrix} 1 & 0 & 0 & 0.170 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.060 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$$\boxed{^{b}T_e = {^{0}T_4}}$$

$$\boxed{^{w}T_b = \begin{bmatrix} \cos(\theta_p) & -\sin(\theta_p) & 0 & 0.170\cos(\theta_p) + x_p \\ \sin(\theta_p) & \cos(\theta_p) & 0 & 0.170\sin(\theta_p) + y_p \\ 0 & 0 & 1 & 0.060 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$$\boxed{\begin{aligned} x &= C_\theta C_1(a_4C_{234} + a_3C_{23} + a_2C_2) - S_\theta S_1(a_4C_{234} + a_3C_{23} + a_2C_2) + 0.170C_\theta + x_p \\ y &= S_\theta C_1(a_4C_{234} + a_3C_{23} + a_2C_2) + C_\theta S_1(a_4C_{234} + a_3C_{23} + a_2C_2) + 0.170S_\theta + y_p \\ z &= a_4S_{234} + a_3S_{23} + a_2S_2 + d_1 + 0.060 \end{aligned}}$$

$$\boxed{{}^wT_e = \begin{bmatrix} C_\theta C_1C_{234} - S_\theta S_1C_{234} & -C_\theta C_1S_{234} + S_\theta S_1S_{234} & C_\theta S_1 + S_\theta C_1 & x \\ S_\theta C_1C_{234} + C_\theta S_1C_{234} & -S_\theta C_1S_{234} - C_\theta S_1S_{234} & S_\theta S_1 - C_\theta C_1 & y \\ S_{234} & C_{234} & 0 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$$\boxed{{}^wT_e = \begin{bmatrix} C_{\theta1}C_{234} & -C_{\theta1}S_{234} & S_{\theta1} & C_{\theta1}(a_4C_{234} + a_3C_{23} + a_2C_2) + 0.170C_\theta + x_p \\ S_{\theta1}C_{234} & -S_{\theta1}S_{234} & -C_{\theta1} & S_{\theta1}(a_4C_{234} + a_3C_{23} + a_2C_2) + 0.170S_\theta + y_p \\ S_{234} & C_{234} & 0 & a_4S_{234} + a_3S_{23} + a_2S_2 + d_1 + 0.060 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

1.9. Cinemática diferencial

$$J_v(q) = \begin{bmatrix} \frac{\partial p_1(q)}{\partial q_1} & \frac{\partial p_1(q)}{\partial q_2} & \cdots & \frac{\partial p_1(q)}{\partial q_n} \\ \frac{\partial p_2(q)}{\partial q_1} & \frac{\partial p_2(q)}{\partial q_2} & \cdots & \frac{\partial p_2(q)}{\partial q_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial p_m(1)}{\partial q_1} & \frac{\partial p_m(q)}{\partial q_2} & \cdots & \frac{\partial p_m(q)}{\partial q_n} \end{bmatrix}$$

$$\begin{aligned} t_x &= C_{\theta1}(a_4C_{234} + a_3C_{23} + a_2C_2) + 0.170C_\theta + x_p \\ t_y &= S_{\theta1}(a_4C_{234} + a_3C_{23} + a_2C_2) + 0.170S_\theta + y_p \\ t_z &= a_4S_{234} + a_3S_{23} + a_2S_2 + d_1 + 0.060 \end{aligned}$$

$$\frac{\partial t_x}{\partial x_p} = 1$$
$$\frac{\partial t_x}{\partial y_p} = 0$$
$$\frac{\partial t_x}{\partial \theta_p} = -S_{\theta 1}(a_4 C_{234} + a_3 C_{23} + a_2 C_2) - 0.170 S_\theta$$
$$\frac{\partial t_x}{\partial \theta_1} = -S_{\theta 1}(a_4 C_{234} + a_3 C_{23} + a_2 C_2)$$
$$\frac{\partial t_x}{\partial \theta_2} = -C_{\theta 1}(a_4 S_{234} + a_3 S_{23} + a_2 S_2)$$
$$\frac{\partial t_x}{\partial \theta_3} = -C_{\theta 1}(a_4 S_{234} + a_3 S_{23})$$
$$\frac{\partial t_x}{\partial \theta_4} = -a_4 C_{\theta 1} S_{234}$$

$$\frac{\partial t_y}{\partial x_p} = 0$$
$$\frac{\partial t_y}{\partial y_p} = 1$$
$$\frac{\partial t_y}{\partial \theta_p} = C_{\theta 1}(a_4 C_{234} + a_3 C_{23} + a_2 C_2) + 0.170 C_\theta$$
$$\frac{\partial t_y}{\partial \theta_1} = C_{\theta 1}(a_4 C_{234} + a_3 C_{23} + a_2 C_2)$$
$$\frac{\partial t_y}{\partial \theta_2} = -S_{\theta 1}(a_4 S_{234} + a_3 S_{23} + a_2 S_2)$$
$$\frac{\partial t_y}{\partial \theta_3} = -S_{\theta 1}(a_4 S_{234} + a_3 S_{23})$$
$$\frac{\partial t_y}{\partial \theta_4} = -a_4 S_{\theta 1} S_{234}$$

$$\frac{\partial t_z}{\partial x_p} = 0$$
$$\frac{\partial t_z}{\partial x_p} = 0$$
$$\frac{\partial t_z}{\partial x_p} = 0$$
$$\frac{\partial t_z}{\partial \theta_1} = 0$$
$$\frac{\partial t_z}{\partial \theta_2} = a_4C_{234} + a_3C_{23} + a_2C_2$$
$$\frac{\partial t_z}{\partial \theta_3} = a_4C_{234} + a_3C_{23}$$
$$\frac{\partial t_z}{\partial \theta_4} = a_4C_{234}$$

$$J_v = \begin{bmatrix} \frac{\partial t_x}{\partial x_p} & \frac{\partial t_x}{\partial y_p} & \frac{\partial t_x}{\partial \theta_p} & \frac{\partial t_x}{\partial \theta_1} & \frac{\partial t_x}{\partial \theta_2} & \frac{\partial t_x}{\partial \theta_3} & \frac{\partial t_x}{\partial \theta_4} \\ \frac{\partial t_y}{\partial x_p} & \frac{\partial t_y}{\partial y_p} & \frac{\partial t_y}{\partial \theta_p} & \frac{\partial t_y}{\partial \theta_1} & \frac{\partial t_y}{\partial \theta_2} & \frac{\partial t_y}{\partial \theta_3} & \frac{\partial t_y}{\partial \theta_4} \\ \frac{\partial t_z}{\partial x_p} & \frac{\partial t_z}{\partial y_p} & \frac{\partial t_z}{\partial \theta_p} & \frac{\partial t_z}{\partial \theta_1} & \frac{\partial t_z}{\partial \theta_2} & \frac{\partial t_z}{\partial \theta_3} & \frac{\partial t_z}{\partial \theta_4} \end{bmatrix}$$

2. Control

2.1. Cinemática diferencial

$$\dot{x} = J(q)\dot{q}$$

2.2. Cinemática diferencial inversa

$$\dot{q} = J(q)^{-1}\dot{x}$$

$$\dot{x} = e$$

$$e = (x_d - x_i)$$

$$\dot{q} = J(q)^{-1}\dot{x} \quad \rightarrow \quad \dot{q} = J(q)^{-1}e \quad \rightarrow \quad \dot{q} = J(q)^{-1}(x_d - x_i)$$

$$\dot{q} = J(q)^{-1}(ke)$$

Siendo k una matriz definida positiva.

2.3. Modelo de control del *Kuka YouBot Arm*

$$q = \begin{bmatrix} x_p \\ y_p \\ \theta_p \\ \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} \quad \dot{q} = \begin{bmatrix} \dot{x_p} \\ \dot{y_p} \\ \dot{\theta_p} \\ \dot{\theta_1} \\ \dot{\theta_2} \\ \dot{\theta_3} \\ \dot{\theta_4} \end{bmatrix} \quad \dot{x} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

2.4. Matriz Jacobiana del *Kuka* (Cinemática directa)

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \frac{\partial t_x}{\partial x_p} & \frac{\partial t_x}{\partial y_p} & \frac{\partial t_x}{\partial \theta_p} & \frac{\partial t_x}{\partial \theta_1} & \frac{\partial t_x}{\partial \theta_2} & \frac{\partial t_x}{\partial \theta_3} & \frac{\partial t_x}{\partial \theta_4} \\ \frac{\partial t_y}{\partial x_p} & \frac{\partial t_y}{\partial y_p} & \frac{\partial t_y}{\partial \theta_p} & \frac{\partial t_y}{\partial \theta_1} & \frac{\partial t_y}{\partial \theta_2} & \frac{\partial t_y}{\partial \theta_3} & \frac{\partial t_y}{\partial \theta_4} \\ \frac{\partial t_z}{\partial x_p} & \frac{\partial t_z}{\partial y_p} & \frac{\partial t_z}{\partial \theta_p} & \frac{\partial t_z}{\partial \theta_1} & \frac{\partial t_z}{\partial \theta_2} & \frac{\partial t_z}{\partial \theta_3} & \frac{\partial t_z}{\partial \theta_4} \end{bmatrix} \begin{bmatrix} \dot{x}_p \\ \dot{y}_p \\ \dot{\theta}_p \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix}$$

2.5. Matriz Jacobiana del *Kuka* (Cinemática inversa)

$$\begin{bmatrix} \dot{x}_p \\ \dot{y}_p \\ \dot{\theta}_p \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} \frac{\partial t_x}{\partial x_p} & \frac{\partial t_x}{\partial y_p} & \frac{\partial t_x}{\partial \theta_p} & \frac{\partial t_x}{\partial \theta_1} & \frac{\partial t_x}{\partial \theta_2} & \frac{\partial t_x}{\partial \theta_3} & \frac{\partial t_x}{\partial \theta_4} \\ \frac{\partial t_y}{\partial x_p} & \frac{\partial t_y}{\partial y_p} & \frac{\partial t_y}{\partial \theta_p} & \frac{\partial t_y}{\partial \theta_1} & \frac{\partial t_y}{\partial \theta_2} & \frac{\partial t_y}{\partial \theta_3} & \frac{\partial t_y}{\partial \theta_4} \\ \frac{\partial t_z}{\partial x_p} & \frac{\partial t_z}{\partial y_p} & \frac{\partial t_z}{\partial \theta_p} & \frac{\partial t_z}{\partial \theta_1} & \frac{\partial t_z}{\partial \theta_2} & \frac{\partial t_z}{\partial \theta_3} & \frac{\partial t_z}{\partial \theta_4} \end{bmatrix}^{\dagger} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_p \\ \dot{y}_p \\ \dot{\theta}_p \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} \frac{\partial t_x}{\partial x_p} & \frac{\partial t_x}{\partial y_p} & \frac{\partial t_x}{\partial \theta_p} & \frac{\partial t_x}{\partial \theta_1} & \frac{\partial t_x}{\partial \theta_2} & \frac{\partial t_x}{\partial \theta_3} & \frac{\partial t_x}{\partial \theta_4} \\ \frac{\partial t_y}{\partial x_p} & \frac{\partial t_y}{\partial y_p} & \frac{\partial t_y}{\partial \theta_p} & \frac{\partial t_y}{\partial \theta_1} & \frac{\partial t_y}{\partial \theta_2} & \frac{\partial t_y}{\partial \theta_3} & \frac{\partial t_y}{\partial \theta_4} \\ \frac{\partial t_z}{\partial x_p} & \frac{\partial t_z}{\partial y_p} & \frac{\partial t_z}{\partial \theta_p} & \frac{\partial t_z}{\partial \theta_1} & \frac{\partial t_z}{\partial \theta_2} & \frac{\partial t_z}{\partial \theta_3} & \frac{\partial t_z}{\partial \theta_4} \end{bmatrix}^{\dagger} \begin{bmatrix} x_d - x \\ y_d - y \\ z_d - z \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_p \\ \dot{y}_p \\ \dot{\theta}_p \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} \frac{\partial t_x}{\partial x_p} & \frac{\partial t_x}{\partial y_p} & \frac{\partial t_x}{\partial \theta_p} & \frac{\partial t_x}{\partial \theta_1} & \frac{\partial t_x}{\partial \theta_2} & \frac{\partial t_x}{\partial \theta_3} & \frac{\partial t_x}{\partial \theta_4} \\ \frac{\partial t_y}{\partial x_p} & \frac{\partial t_y}{\partial y_p} & \frac{\partial t_y}{\partial \theta_p} & \frac{\partial t_y}{\partial \theta_1} & \frac{\partial t_y}{\partial \theta_2} & \frac{\partial t_y}{\partial \theta_3} & \frac{\partial t_y}{\partial \theta_4} \\ \frac{\partial t_z}{\partial x_p} & \frac{\partial t_z}{\partial y_p} & \frac{\partial t_z}{\partial \theta_p} & \frac{\partial t_z}{\partial \theta_1} & \frac{\partial t_z}{\partial \theta_2} & \frac{\partial t_z}{\partial \theta_3} & \frac{\partial t_z}{\partial \theta_4} \end{bmatrix}^{\dagger} \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix} \begin{bmatrix} x_d - x \\ y_d - y \\ z_d - z \end{bmatrix}$$

2.6. Modelo omnidireccional

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -(L+l) \\ 1 & 1 & (L+l) \\ 1 & 1 & -(L+l) \\ 1 & -1 & (L+l) \end{bmatrix} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}^{\top} \begin{bmatrix} \dot{x}_p \\ \dot{y}_p \\ \dot{\theta}_p \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} \sqrt{2}\sin(\alpha) & -\sqrt{2}\cos(\alpha) & -(L+l) \\ \sqrt{2}\cos(\alpha) & \sqrt{2}\sin(\alpha) & (L+l) \\ \sqrt{2}\cos(\alpha) & \sqrt{2}\sin(\alpha) & -(L+l) \\ \sqrt{2}\sin(\alpha) & -\sqrt{2}\cos(\alpha) & (L+l) \end{bmatrix} \begin{bmatrix} \dot{x}_p \\ \dot{y}_p \\ \dot{\theta}_p \end{bmatrix}, \quad \alpha = \theta + \frac{\pi}{4}$$