

# Real-Time Bidding

-- Online Advertisement

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# About Me

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- AI Scientist at NetDB Lab
- Ph.D. of NYCU CS
- Specialized in
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# Outline

- Introduction
- Bidding Strategy
  - Second-Price Auction
  - Bid Landscape
  - CTR Prediction
  - Budget Pacing
- First-Price Auction
  - Floor Price
  - Bid Shading
- Takeaways

# Display Ad

Wednesday, May 21, 2025

Today's Paper

S&P 500 -1.61% ↓

# The New York Times

U.S. ▾ World ▾ Business ▾ Arts ▾ Lifestyle ▾ Opinion ▾ | Audio ▾ Games ▾ Cooking ▾ Wirecutter ▾ The Athletic ▾

## Trump Lectures South African President in Televised Oval Office Confrontation

President Trump presented what he said was evidence of racial persecution of white South Africans. The country's president tried to correct the record.

5 MIN READ

## Trump Says the U.S. Is Close to Brokering Peace Between Congo and Rwanda

2 MIN READ



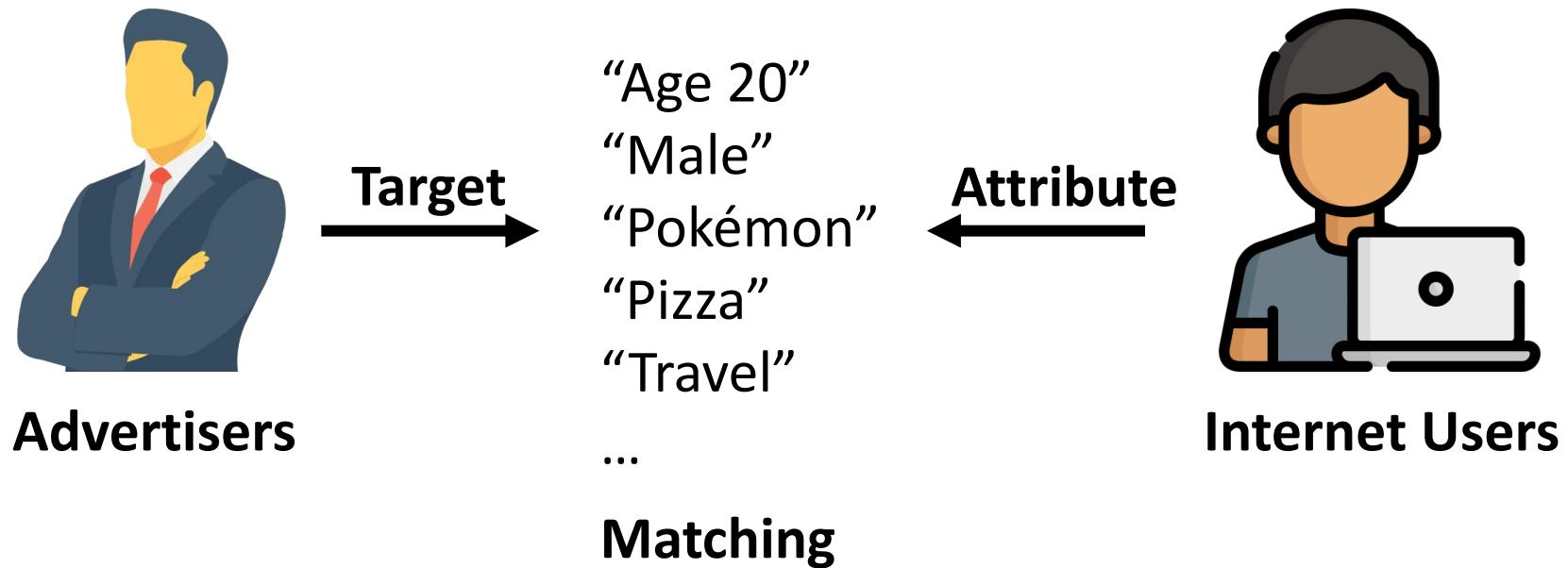
Heather Willensky for The New York Times

## The 22 Best Pizza Places in New York Right Now

Some of the city's most famous pies didn't make the list, while some unexpected spots delivered superlative examples of the form.

10 MIN READ

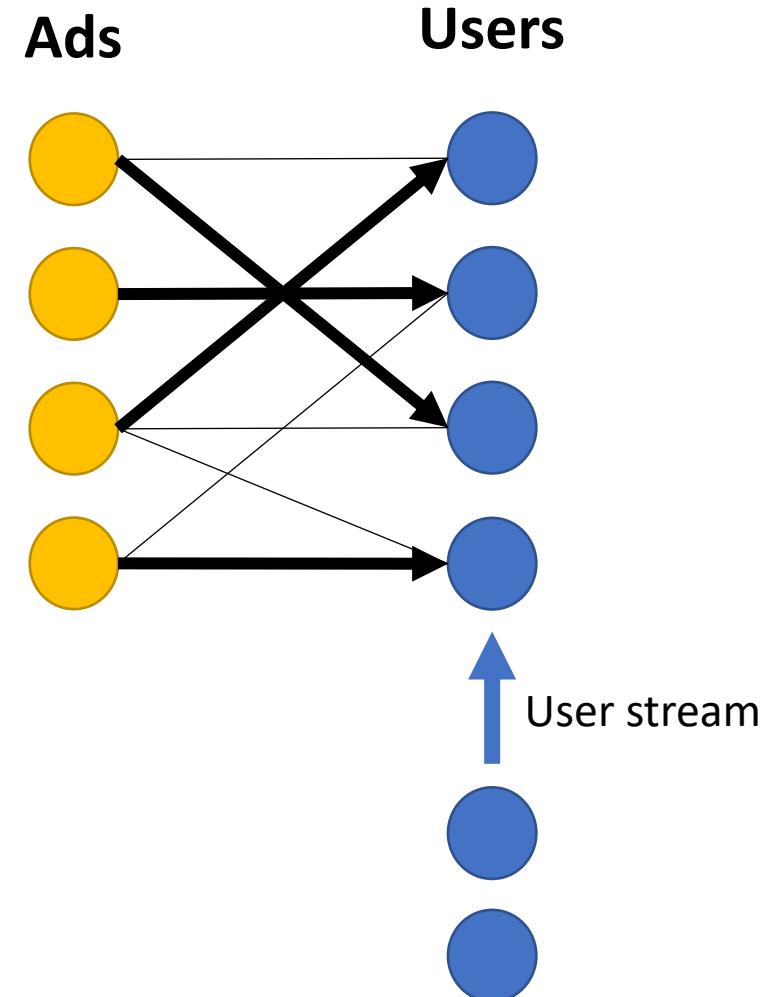
# Display Ad



Publishers (websites) sell their columns to the advertisers

# Online Matching

- **Problem:** find a matching between ads and the user stream
- **Constraint:** budget of each ad (campaign)
- **Goal:** maximize platform revenues
  - Cost Per Million Impression (CPM)
  - **Cost Per Click (CPC)**
  - Cost Per Action/Conversion (CPA)
- GREEDY yields  $1/2$  competitive ratio to OPT
- BALANCE yields  $1 - 1/e \approx 0.63$



# Real-Time Bidding (RTB)

- Difficult to manage users, publishers, and ads explosions
- 2007-2008
- Advertisers **buy individual impressions** via **real-time auctions**
  - rather than purchasing bulk inventory in advance
- Enable **behavioral targeting**
  - target users more precisely and optimize their ads in real time
  - Cookie!

# Example of RTB

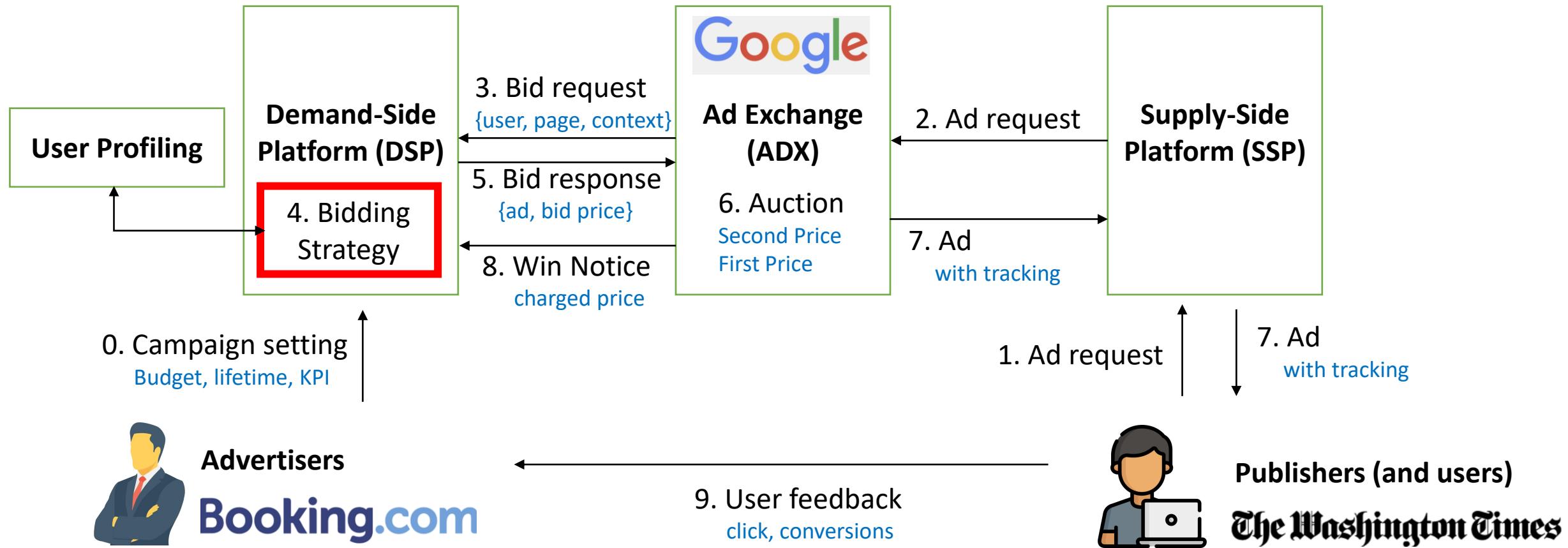
## Search on Booking.com

The screenshot shows the search results for "Rodeway Inn Montrose" on Booking.com. The search interface includes fields for destination, check-in date, and guest count. The results page displays the hotel's name, address (1480 South Townsend Avenue, Montrose, CO 81401), and a rating of 7.0 from 198 reviews. It features a "Great location!" badge and a "Good" rating for food and beverage services. The page includes several thumbnail images of the hotel's exterior and interior rooms, along with a map showing its location.

## Behavioral Targeting

The screenshot shows a Google search result for "seminyak hotels" on the The Washington Times website. The search bar at the top contains the query. The main content area features the newspaper's logo and navigation menu with links for News, Policy, Investigations, Opinion, Sports, Special Reports, and Games. Below the menu, a "TRENDING" section lists Donald Trump, North Korea, Senate, NHL, Russia, China, Vietnam, Congress, NBA, and Tony Evers. A "Booking.com" advertisement is displayed, showing five images of Seminyak villas with "Click" buttons underneath each. At the bottom, there is a "SIGN UP FOR BREAKING NEWS ALERTS" button with a notification icon.

# RTB Architecture and Mechanism



# RTB: Tremendous Data Volume

Daily Request Volume				
Country	RTB		Stock Market	
Taiwan	Tenmax	1B	TAIEX	1.7~3M
US	Total	>100B	Total	80~100M
China	Total	>100B	Shanghai	71M

Query Per Second	
Turn DSP	1.6M
Google Query	40K

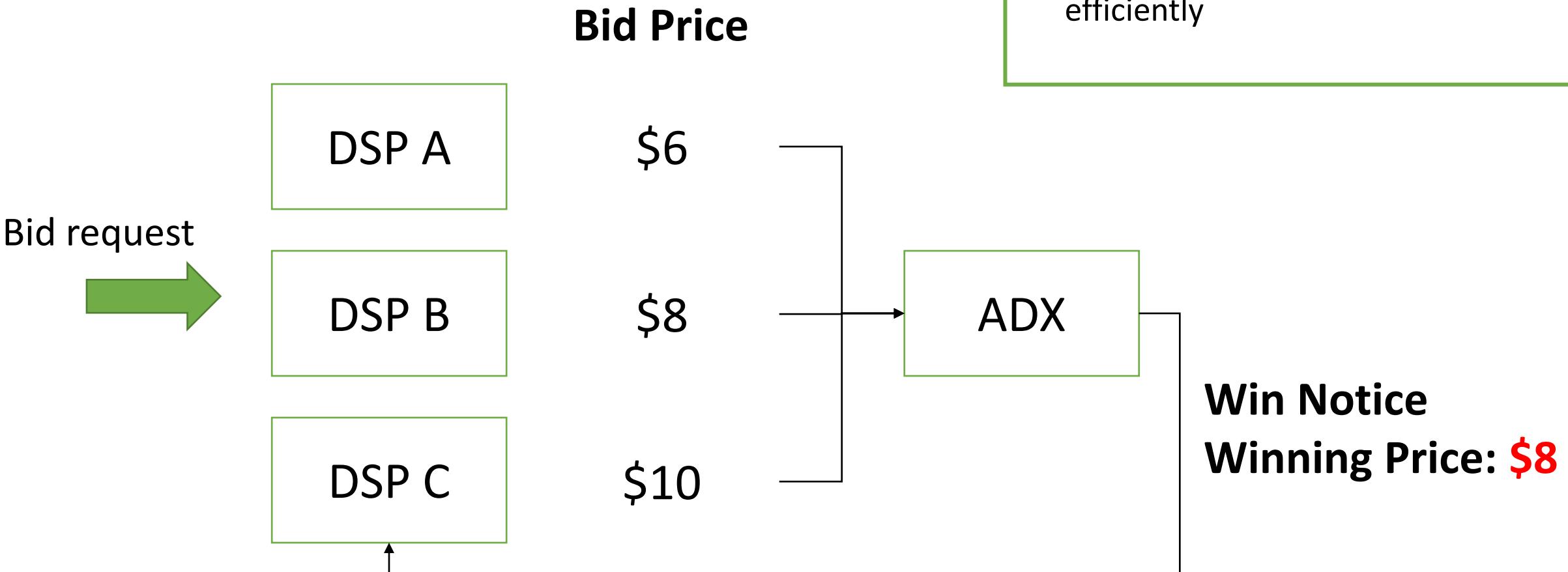
- RTB needs not only fast (<100ms) but also **precise** ← **Trade-off!**

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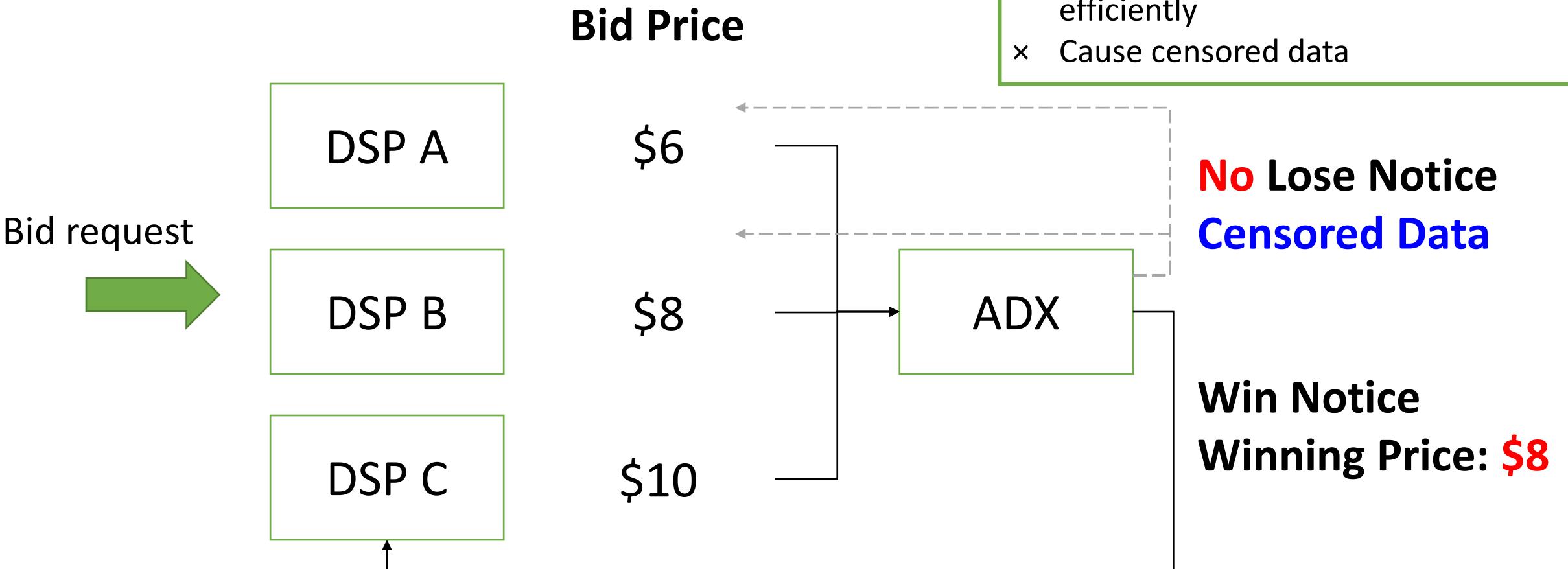
# Second-Price Auction (SPA)

- ✓ Encourage DSPs bid they truly willing to pay
- ✓ Avoid Overpayment
- ✓ Allocate ads to who values it most efficiently



# Censored Data

- ✓ Encourage DSPs bid they truly willing to pay
- ✓ Avoid Overpayment
- ✓ Allocate ads to who values it most efficiently
- ✗ Cause censored data



# Data Format

Bid Request Features	Bid price	Win	Win Price	Click
{banner, 1200x700, nytimes.com, 24, male}	5	1	4	1
{native, 800x800, facebook.com, 26, female}	4	1	3	NA
{video, 1200x1200, udn.com, 36, male}	3	0	NA	NA
{pop-up video, 1260x1260, pixnet.com, 42, male}	1	0	NA	NA

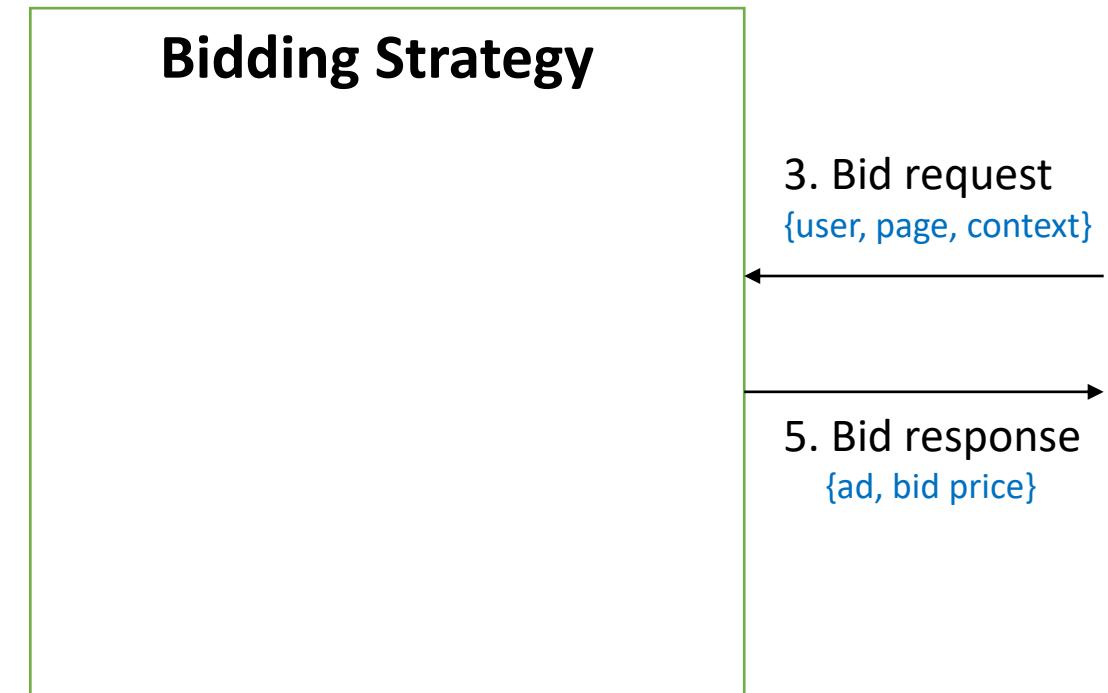
- **Bid Request Feature:** **high-dimensional sparse** binary vectors
  - Publisher information
  - Audience information
  - Impression details
  - Content and context
  - Targeting audiences
  - Etc...

# Bidding Strategy

- **Given** a stream of bid requests and their features, ad features (budget, ad content, target audience), etc
- **Determine** bid prices for each bid request
- **Such that**

$$\begin{aligned} & \text{max KPI} & \text{\#clicks} \\ \text{s.t. } & \sum cost \leq Budget \end{aligned}$$

Bid price = Click-Through Rate (CTR)\*Cost-Per-Click (CPC)

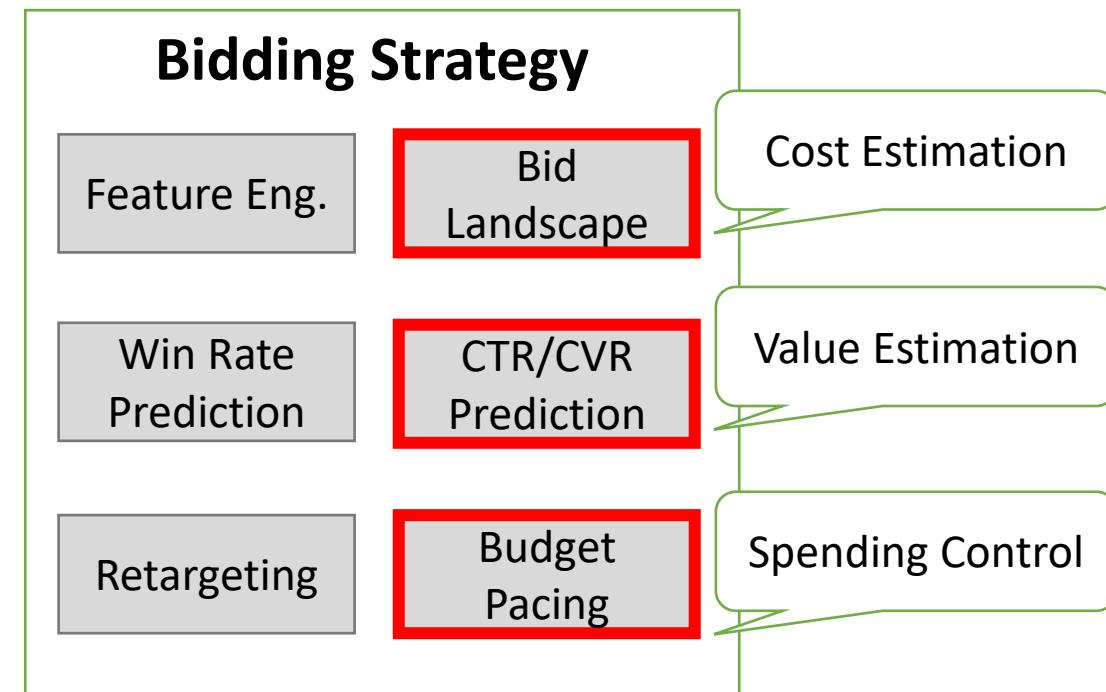


# Bidding Strategy

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# Bid Landscape

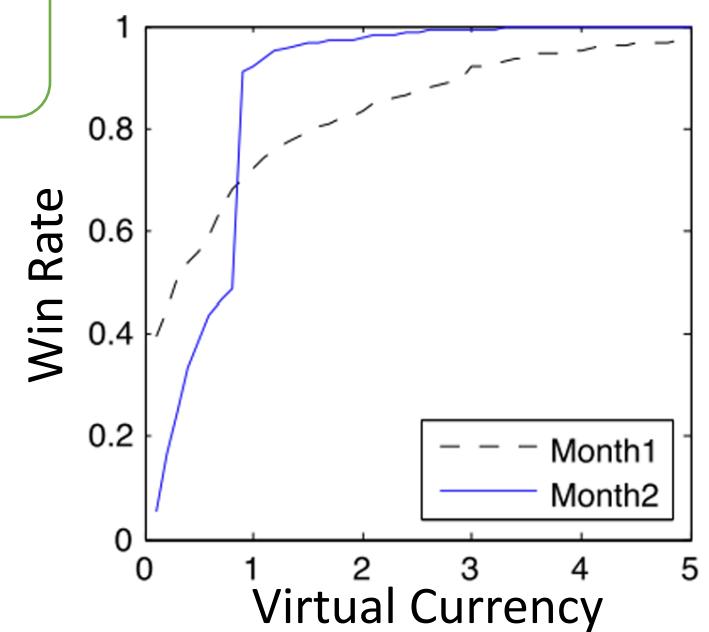
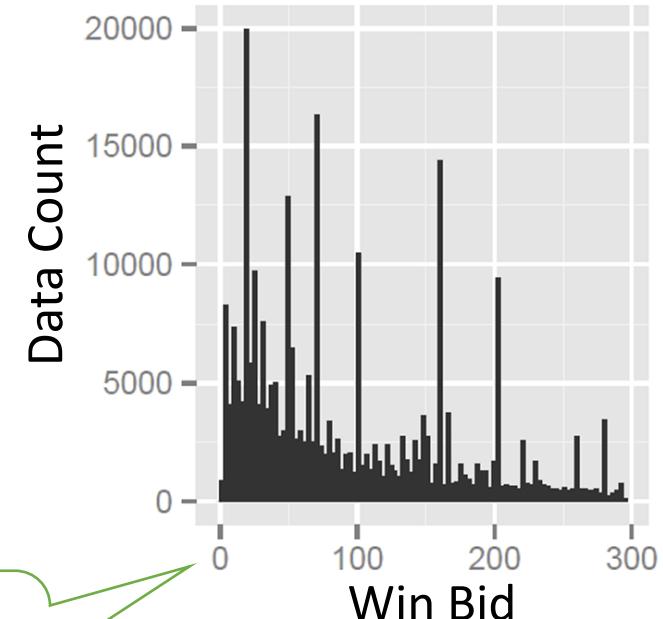
- The distribution (or analysis) of **bid prices** and **outcomes** within an online advertising auction environment

- **Goal:** Depicts market competition

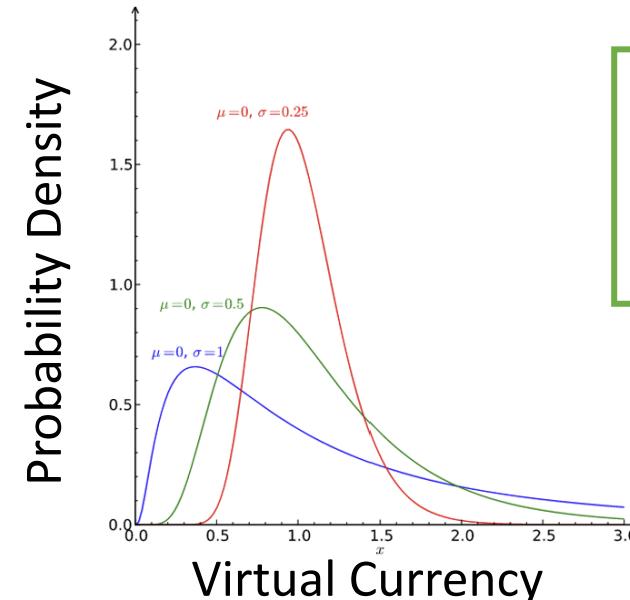
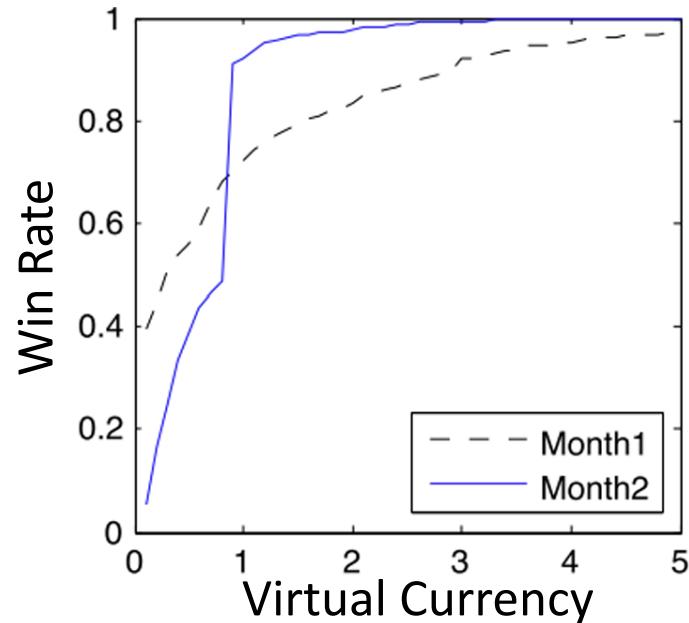
$$\text{• Win rate: } w(b) = \int_{z=0}^b p(z)dz$$

$$\text{• Expected cost: } c(b) = \frac{\int_{z=0}^b zp(z)dz}{\int_{z=0}^b p(z)dz}$$

How can DSP collect  
so many bid prices  
for one bid request?



# Bid Landscape



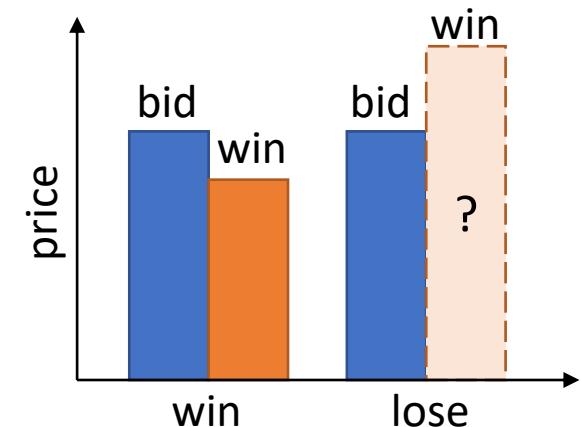
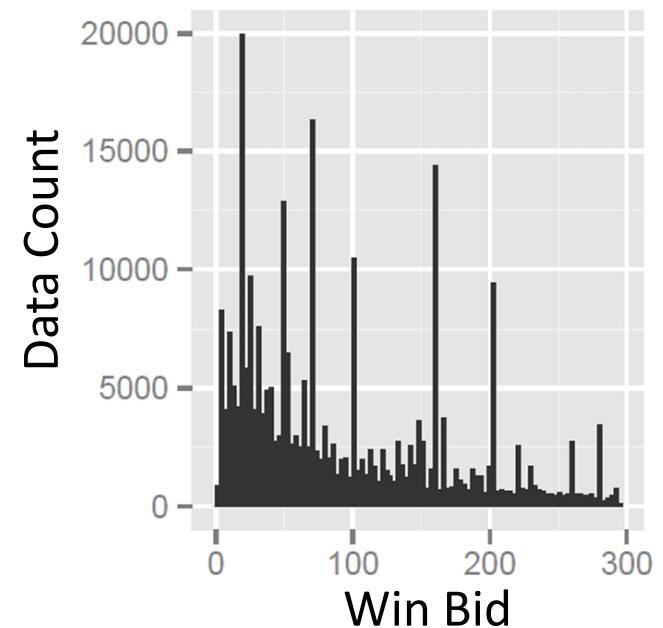
- ✓ Long-tail
- ✓ Non-negative bid price
- ✓ Normality

- Log-normal distribution (PDF)

$$f(b; \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln b - \mu)^2}{2\sigma^2}}, b > 0$$

# Underestimation of Bid Landscape

- Using win bid only is **over-optimistic**
  - Ignore bid prices you lost
- For DSP, impossible to know empirical win prices due to **censorship!**
- **Survival Analysis** in Medical/Financial domains
  - Study not-yet-happening events
  - Patient death in the future (death vs. time)
  - Win bid at a higher price (win vs. price)



# Mixture Model with Censored Data

- For winning data, use **PDF  $\phi$**  of standard normal distribution

$$\hat{z}_i = \beta x_i + \varepsilon; \max_{\beta_w} \sum_{i \in W} \underbrace{\log(\phi(\frac{z_i - \beta_w x_i}{\sigma}))}_{\text{Log-likelihood}}$$

- For censored (losing) data, use **CDF  $\varphi$**  of standard normal distribution
  - $\varphi$  gives *the probability your bid is not enough to win*

$$\underline{P(b_i < \hat{z}_i)} = \varphi(\frac{\beta_l x_i - b_i}{\sigma})$$

The probability that your bid  $b_i$  is less than the (unobserved) true winning price  $\hat{z}_i$ .

Why CDF but not PDF?

# Mixture Model with Censored Data

- Overall objective

$$\max_{\beta_w, \beta_l} \sum_{i \in W} \frac{\log(\phi(\frac{z_i - \beta_w x_i}{\sigma}))}{\text{maximize precision}} + \sum_{i \in L} \frac{\log(\varphi(\frac{\beta_l x_i - b_i}{\sigma}))}{\text{maximize prob. of } b_i < \hat{z}_i} \Rightarrow \text{raise } \hat{z}_i$$

- Win price prediction

$$\hat{z}_i = [P(z_i < b_i)\beta_w + (1 - P(z_i < b_i))\beta_l]x_i,$$

$$\text{where } P(z_i < b_i) = \frac{1}{1 + \exp(-\beta_{logit}x_i)}$$

Isn't this weird? Are separated  $\beta_w$  and  $\beta_l$  necessary?

Additional logistic regression to classify if the price is winnable, independent to  $\varphi$

# Mixture Model with Censored Data

- Overall objective

$$\max_{\beta} \sum_{i \in W} \log(\phi(\frac{z_i - \beta x_i}{\sigma})) + \sum_{i \in L} \log(\varphi(\frac{\beta x_i - b_i}{\sigma}))$$

- Win price prediction

$$\hat{z}_i = \beta x_i$$



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# Optimal Bid Price

- Reward for a given bid:  $R(b) = \int_0^b (\underline{r} - z) \underline{p(z)} dz$   
True value of  
this request      Win rate at bid price =  $z$
- Optimal bid price  $b^* = \max_b R(b)$   
 $\Rightarrow \frac{\partial R}{\partial b} = (r - b)p(b) = 0$   
 $\Rightarrow b^* = r$   
 $\Rightarrow$  *The optimal bid price is exactly the value we believe*

# Click-Based Value

- Strategy: bid  $\begin{cases} \text{value of click}, & \text{if clicked} \\ 0, & \text{otherwise} \end{cases}$
- Average value of click:  $(\text{value of click}) \times \frac{\#click}{\#\text{impression}}$

- Click as KPI:  $\textcolor{red}{CPC} \times \textcolor{red}{CTR}$

Cost-Per-Click (CPC):

1. Optimal reward  $b^* = r$
2. Manually-set maximum bid
3. Total spend/Total (expected) clicks

Click-Through-Rate (CTR):

1.  $\#click/\#\text{impression}$
2.  $P(\text{click} | x)$

**Accurate CTR predictions  
=> Precise data-driven bidding strategy**

# ▶ CTR Prediction

- **Given**:  $n$  impressions consisting of feature vector  $x$  and click label  $y \in \{0,1\}$
- **Goal**: train a CTR prediction model  $f$  such that  $\hat{y} = f(x)$
- **Challenges**:
  - High-dimensional and extremely sparse feature space
  - Feature interactions
  - Imbalance label ( $\#\text{clicked}/\#\text{non-clicked} \approx 0.001$ )

Logistic regression

# Field-Aware Features

- Bid Request Feature: high-dimensional sparse binary vectors

- Publisher information
- Audience information
- Impression details
- Content and context
- Targeting audiences
- Etc...

M fields

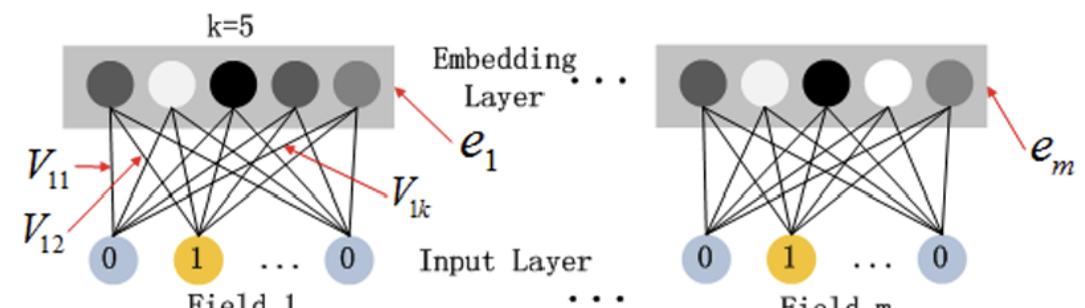
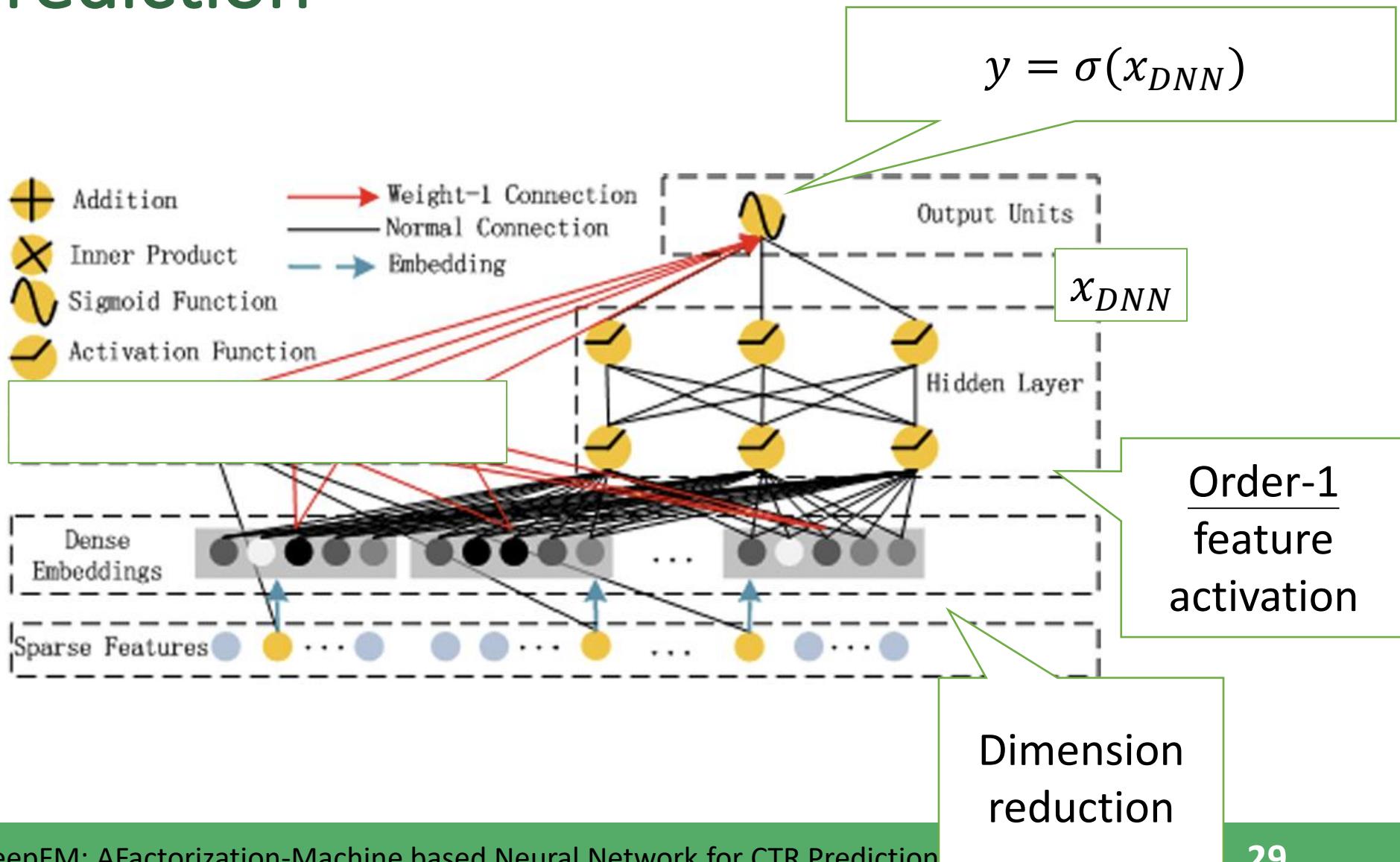


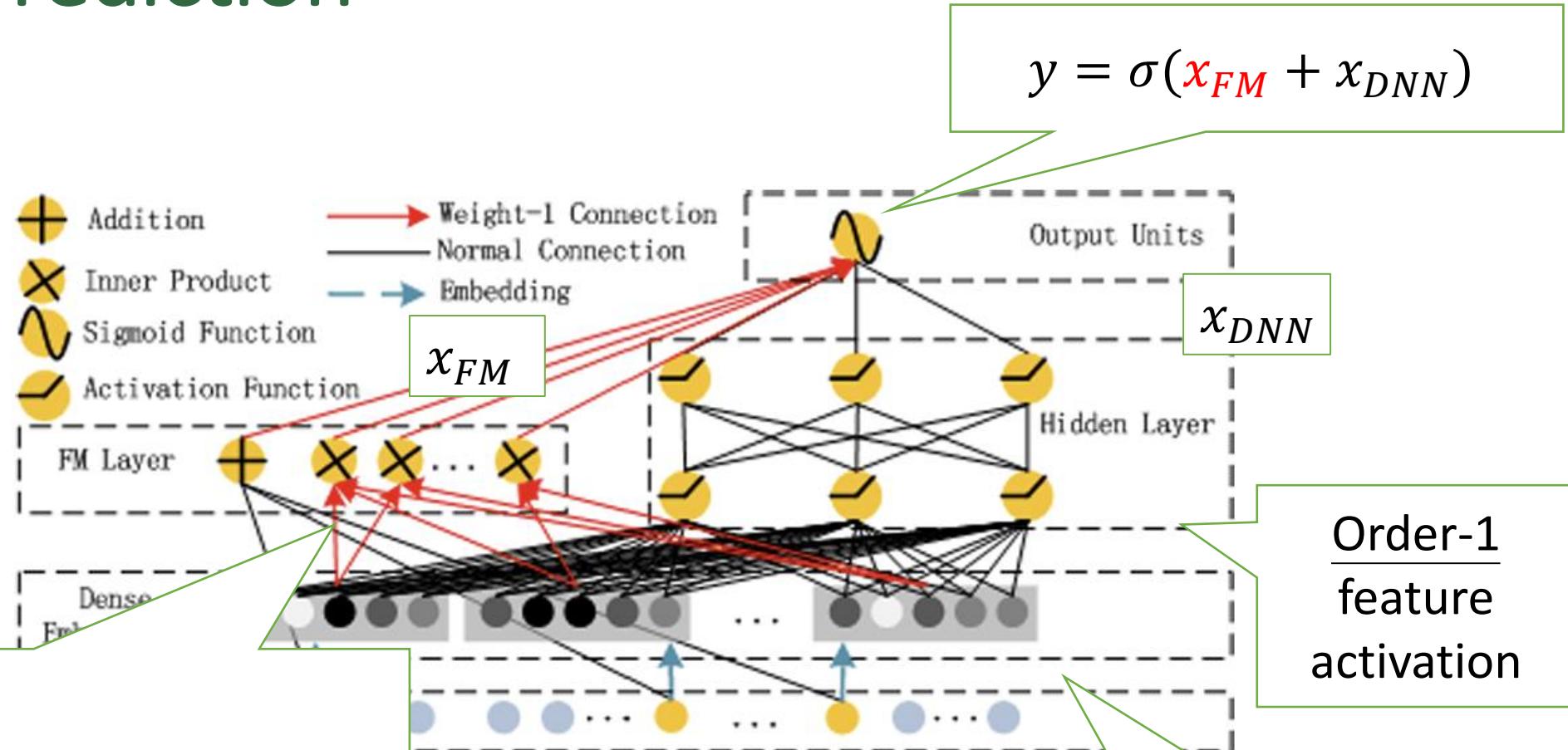
Figure 4: The structure of the embedding layer

- Dimension reduction for each field => field-aware features
  - PCA, LDA, etc
  - Embedding in deep models

# Order-1 Prediction



# Order-2 Prediction



**Order-2 feature:**

$$x_{FM} = \langle w, x \rangle + \sum_{i=1}^d \sum_{j=i+1}^d \langle V_i, V_j \rangle x_i x_j,$$

where  $V_i, V_j$  are latent vectors

## Label Imbalance Issue

- Imbalance label ( $\#\text{clicked}/\#\text{non-clicked} \approx 0.001$ )
- Always predict non-click yields 99.9% accuracy
- Downsample #non-clicked
- #clicked augmentation (upsample)
- Upweight importance of clicked samples
- Etc...

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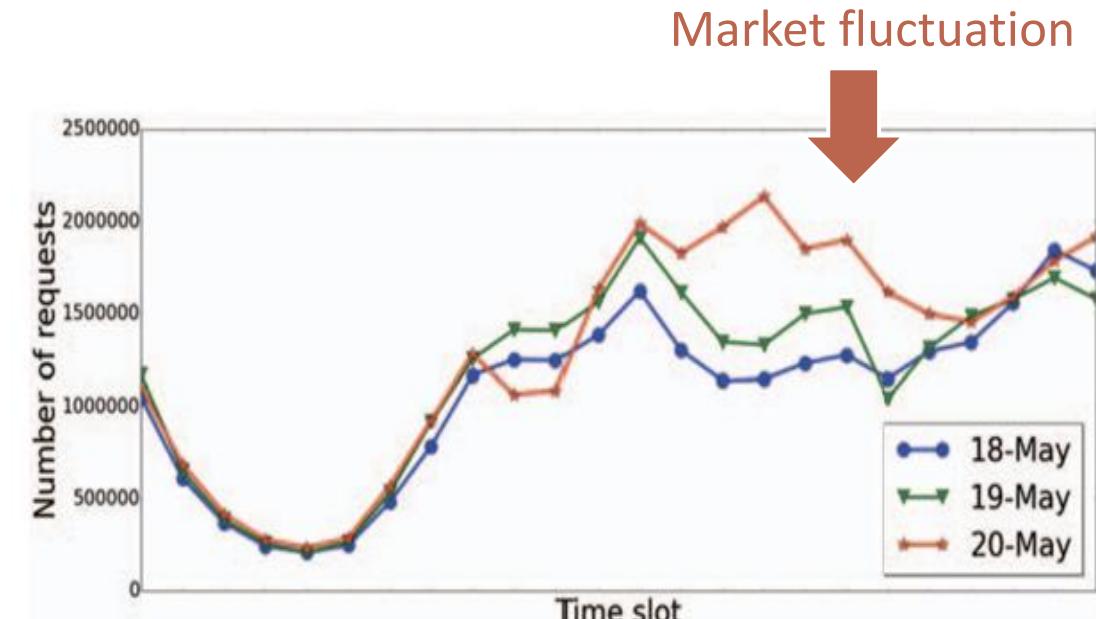
# What We Have Now

- Win price prediction:  $\hat{z}_i = \beta x_i$
- Value prediction:  $\hat{v}_i = CTR \times CPC$
- **Naïve bidding strategy:**
  - If  $\hat{v}_i > \hat{z}_i$ : offer bid price  $b_i = \hat{z}_i + 1$
  - Else: offer bid price  $b_i = \{\hat{v}_i, 0\}$

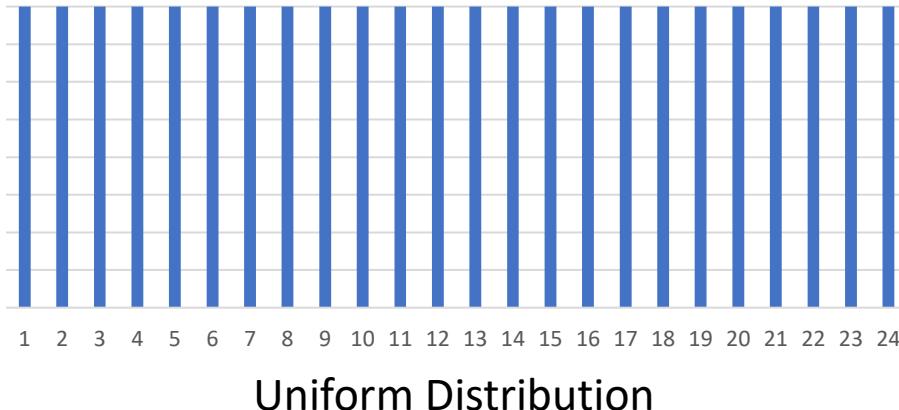
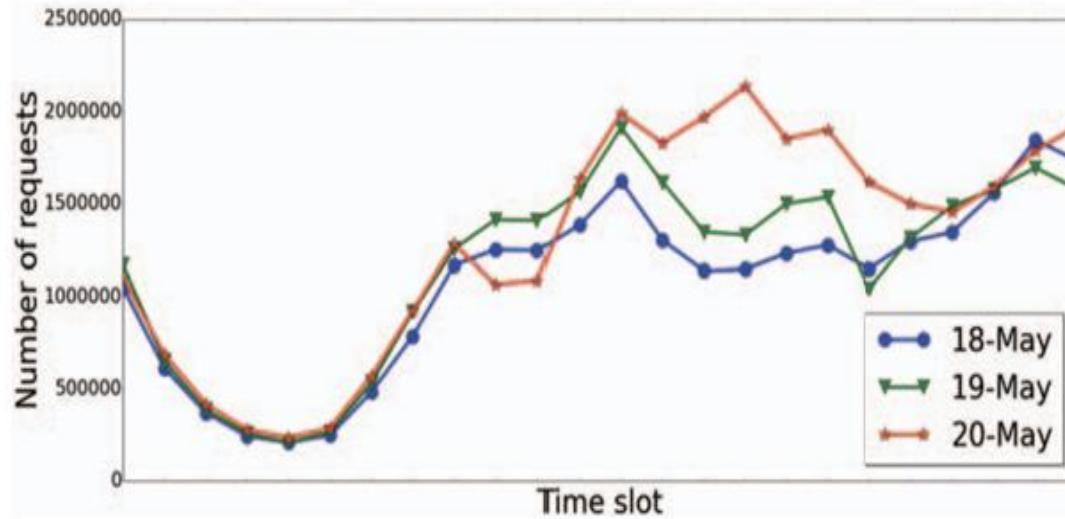
This strategy prevents us from overpayment

# Issues

- **Naïve bidding strategy:**
  - If  $\hat{v}_i > \hat{z}_i$ : offer bid price  $b_i = \hat{z}_i + 1$
  - Else: offer bid price  $b_i = \{\hat{v}_i, 0\}$
- Suppose the lifetime is one week long
  - Naïve method may spend all the budget on the first day
  - No ad delivery in the remaining six days
- Sensitive to abnormal traffic patterns (e.g., market fluctuations)
- Advertisers prefer **smooth delivery over lifetime**
- Need to **foresee future revenue**



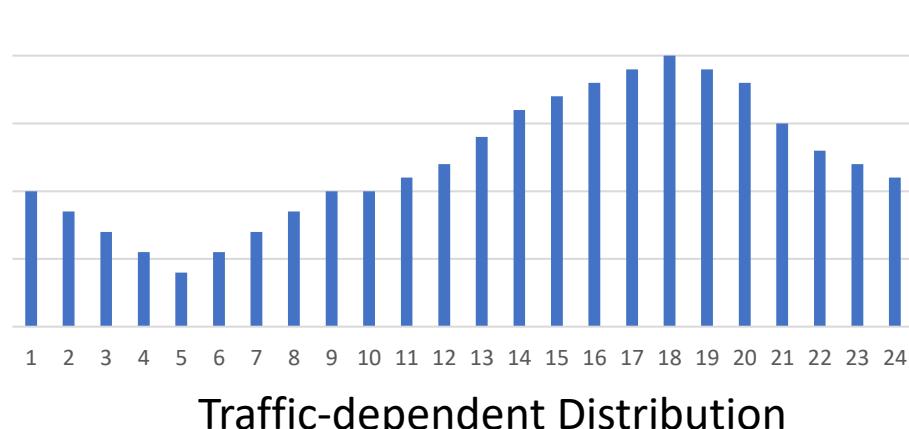
# Budget Over Time



For  $B_t \in \{B_1, B_2, \dots, B_T\}$   
Run naïve bidding strategy

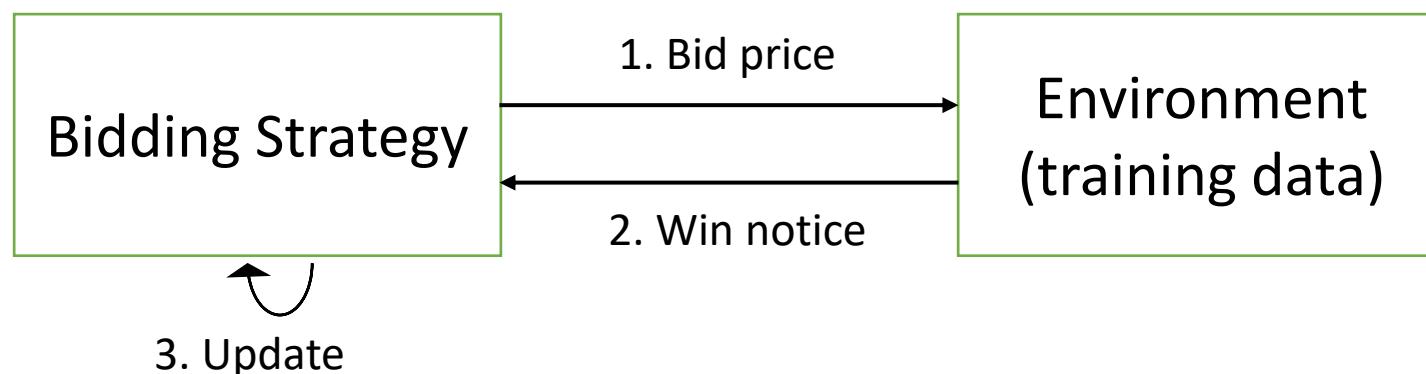
※ traffic prediction steps in

Might not spend all the budget  
⇒ Revenue loss



# Deep Q-Network (DQN)

- Reinforcement learning (RL)
  - An intelligent brute-force approach
  - Learn complex and dynamic relations through experience
  - DQN is often used in RTB due to efficiency issue



## Elements (MDP Process)

- **State  $S$ :** {feature, pCTR, pWinPrice, remaining budget, time left, ....}
- **Action  $A$ :** bid price or bid price adjustment (e.g.,  $ratio * constant\ price$ )
- **Reward  $R$ :** utilities earned per bid request won
  - Click, impression, revenue, ...
- **Transition  $T = P(s'|s, a)$ :** given current state  $s$ , select action  $a$ , transit to state  $s'$ , and gain reward  $r$

# What Is DQN Optimizing

- DQN minimizes the difference between the predicted Q-value (the expected return for a state-action pair) and the target Q-value

$$\bullet L(\underline{\theta}_i) = E_{s,a,r,s' \sim \rho} [(y_i - Q(s,a;\theta_i))^2]$$

The i-th round DQN                      Target              Predicted Q-value by current DQN

$$\bullet \quad y_i = \underline{r} + \gamma \cdot \max_{a'} Q(s', a'; \theta_{i-1})$$

Immediate reward  
from transiting from  
 $s$  to  $s'$  with action  $a$

The maximum Q-value of the next state-action pair, estimated by last-iteration DQN  
⇒ Future benefit!

$\gamma$  is the discount factor of future benefit,  
commonly use {0.9, 0.95, 0.99}

# DQN Output

- Given the current state  $s$
- Predict Q-value on all action candidates  $a \in A$
- Output the action with highest Q-value
- $a^* = \arg \max_{a \in A} Q(s, a)$

$s = \{ \dots, \text{remaining budget}, \text{time left}, \dots \}$



DQN: if I don't raise bid price, reward will be low => bid price raised

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# ► References and Further Reading

- A Collection of RTB Papers: [wnzhang/rtb-papers: A collection of research and survey papers of real-time bidding \(RTB\) based display advertising techniques.](#)