

Real-Time Bidding

-- Online Advertisement

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About Me

- Hsu-Chao Lai 賴旭昭
- AI Scientist at NetDB Lab
- Ph.D. of NYCU CS
- Specialized in
 - Recommender systems
 - Real-time bidding
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Outline

- Introduction
- Bidding Strategy
 - Second-Price Auction
 - Bid Landscape
 - CTR Prediction
 - Budget Pacing
- First-Price Auction
 - Floor Price
 - Bid Shading
- Takeaways

Display Ad

Wednesday, May 21, 2025

Today's Paper

The New York Times

S&P 500 -1.61% ↓

U.S. ▾ World ▾ Business ▾ Arts ▾ Lifestyle ▾ Opinion ▾ | Audio ▾ Games ▾ Cooking ▾ Wirecutter ▾ The Athletic ▾

Trump Lectures South African President in Televised Oval Office Confrontation

President Trump presented what he said was evidence of racial persecution of white South Africans. The country's president tried to correct the record.

5 MIN READ

Trump Says the U.S. Is Close to Brokering Peace Between Congo and Rwanda

2 MIN READ



Eric Lee/The New York Times



Heather Willensky for The New York Times

The 22 Best Pizza Places in New York Right Now

Some of the city's most famous pies didn't make the list, while some unexpected spots delivered superlative examples of the form.

10 MIN READ



Display Ad

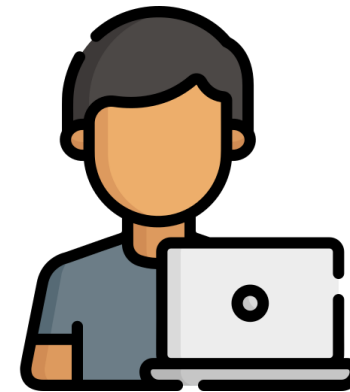


Advertisers

Target →

"Age 20"
"Male"
"Pokémon"
"Pizza"
"Travel"
...

← **Attribute**



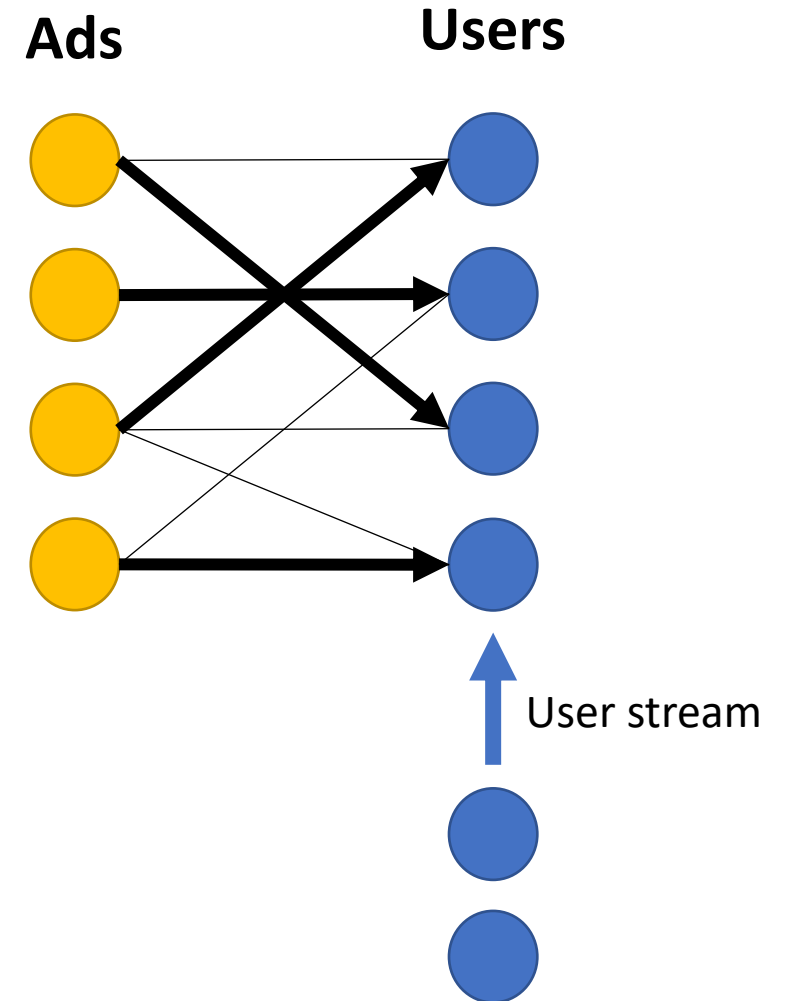
Internet Users

Matching

Publishers (websites) sell their columns to the advertisers

Online Matching

- **Problem:** find a matching between ads and the user stream
- **Constraint:** budget of each ad (campaign)
- **Goal:** maximize platform revenues
 - Cost Per Million Impression (CPM)
 - **Cost Per Click (CPC)**
 - Cost Per Action/Conversion (CPA)
- GREEDY yields $1/2$ competitive ratio to OPT
- BALANCE yields $1 - 1/e \approx 0.63$



Real-Time Bidding (RTB)

- Difficult to manage users, publishers, and ads explosions
- 2007-2008
- Advertisers buy individual impressions via **real-time auctions**
 - rather than purchasing bulk inventory in advance
- Enable behavioral targeting
 - target users more precisely and optimize their ads in real time
 - Cookie!

Example of RTB

Search on Booking.com

Home > All hotels > United States of America > Colorado > Montrose > Rodeway Inn Montrose (Hotel) (USA) Deals

✓ We Price Match

Search
Destination/property name:
Montrose

Check-in date
Check-in Date

Check-out date
Check-out Date

2 adults
No children 1 room

☐ I'm traveling for work

Search

How to get to Rodeway Inn Montrose from Montrose Regional Airport
Car 5 minutes

Info & prices Amenities House rules The fine print Guest reviews (198)

Food and beverage services at this property may be limited or unavailable due to the coronavirus (COVID-19).
[Read more](#)

Hotel Rodeway Inn Montrose
1480 South Townsend Avenue, Montrose, CO 81401, United States of America –
Great location - show map

Good 7.0
198 reviews

Great location! 8.0

+36 photos

Extra health & safety measures

Behavioral Targeting

seminyak hotels - Google Search

https://www.google.com/search?q=seminyak+hotels&rlz=1C1CH8F_en-GBAU736AU736&oq=seminyak+hotels&aqs=chrome..69i57j69...

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Booking.com

Seminyak - The Mawar Estate
Click

Canggu - Villa Mana - an elite haven
Click

Canggu - Villa Istana Putih by Nakula Management
Click

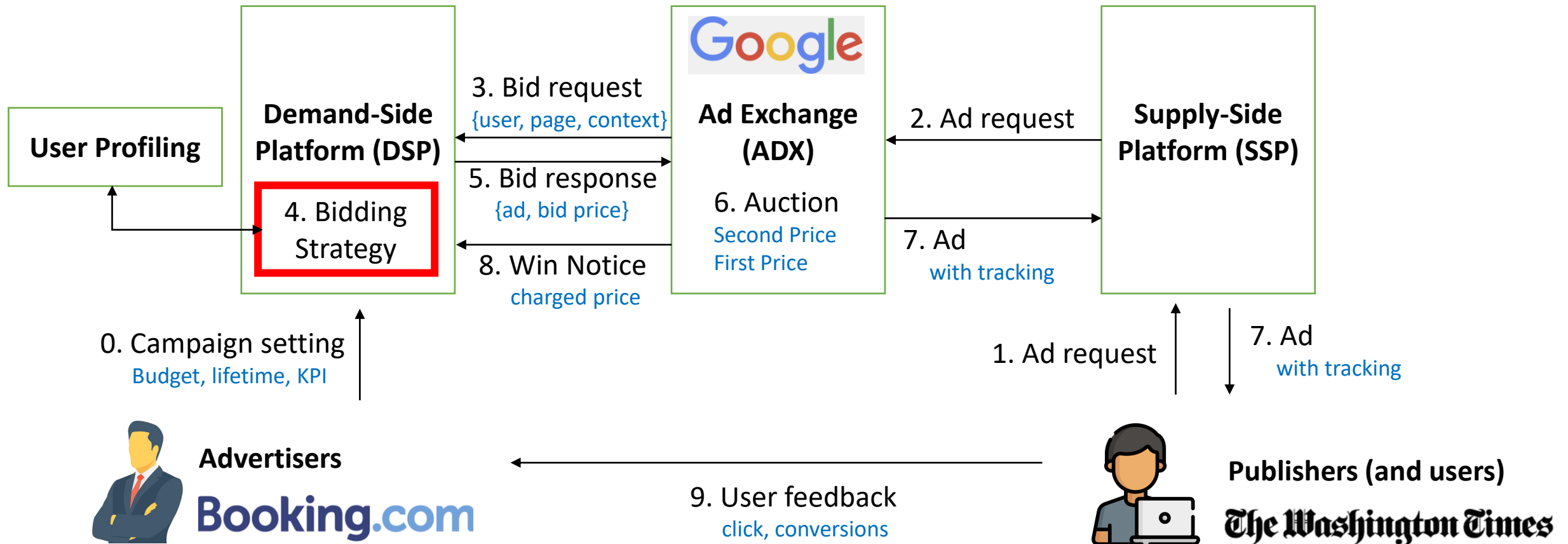
Canggu - Villa Kalyani - an elite haven
Click

Seminyak - Villa Elleo
Click

HOME TOPICS SCOTT DWORKIN

SIGN UP FOR BREAKING NEWS ALERTS

RTB Architecture and Mechanism



RTB: Tremendous Data Volume

Daily Request Volume				
Country	RTB		Stock Market	
Taiwan	Tenmax	1B	TAIEX	1.7~3M
US	Total	>100B	Total	80~100M
China	Total	>100B	Shanghai	71M

Query Per Second	
Turn DSP	1.6M
Google Query	40K

- RTB needs not only fast (<100ms) but also **precise** ← **Trade-off!**

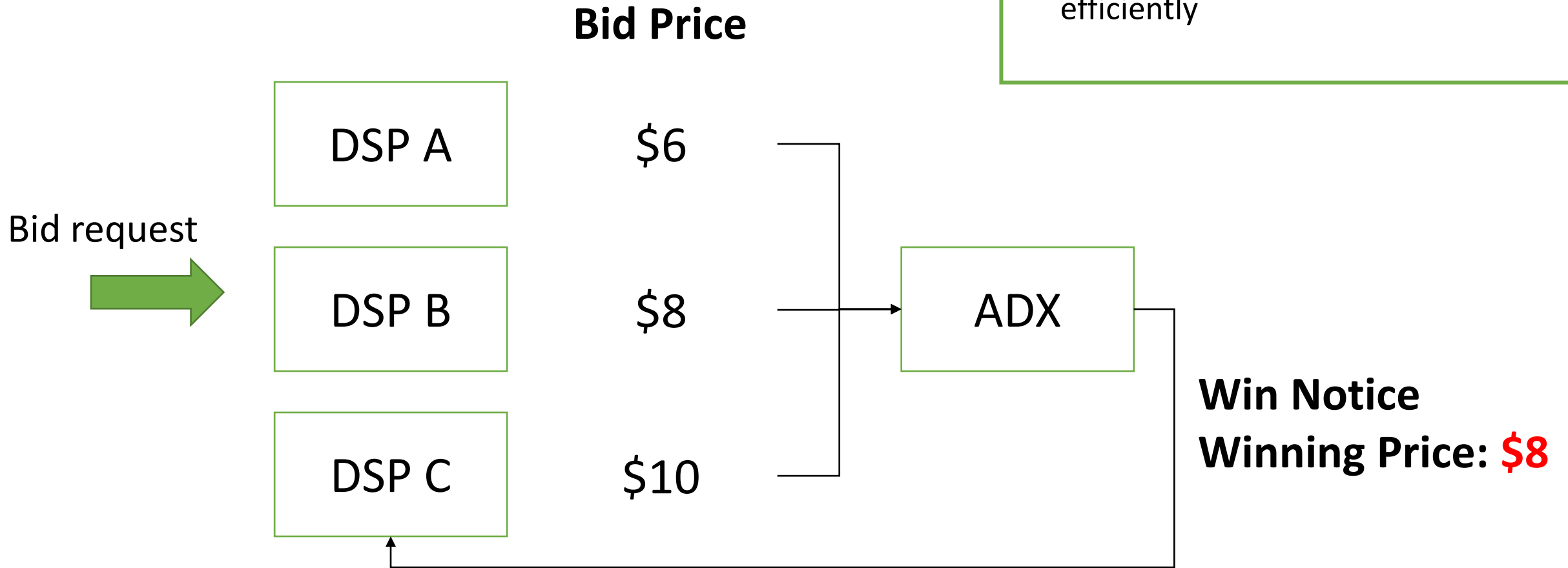


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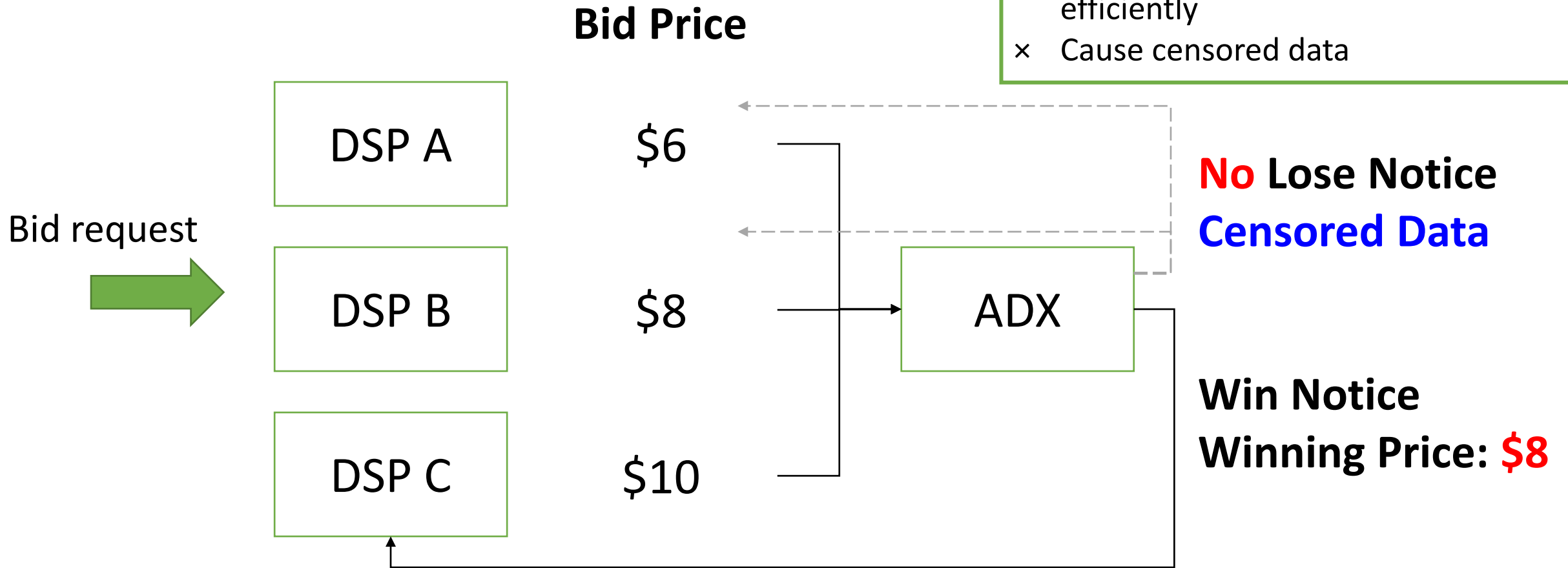
Second-Price Auction (SPA)

- ✓ Encourage DSPs bid they truly willing to pay
- ✓ Avoid Overpayment
- ✓ Allocate ads to who values it most efficiently



Censored Data

- ✓ Encourage DSPs bid they truly willing to pay
- ✓ Avoid Overpayment
- ✓ Allocate ads to who values it most efficiently
- × Cause censored data



Data Format

Bid Request Features	Bid price	Win	Win Price	Click
{banner, 1200x700, nytimes.com, 24, male}	5	1	4	1
{native, 800x800, facebook.com, 26, female}	4	1	3	NA
{video, 1200x1200, udn.com, 36, male}	3	0	NA	NA
{pop-up video, 1260x1260, pixnet.com, 42, male}	1	0	NA	NA

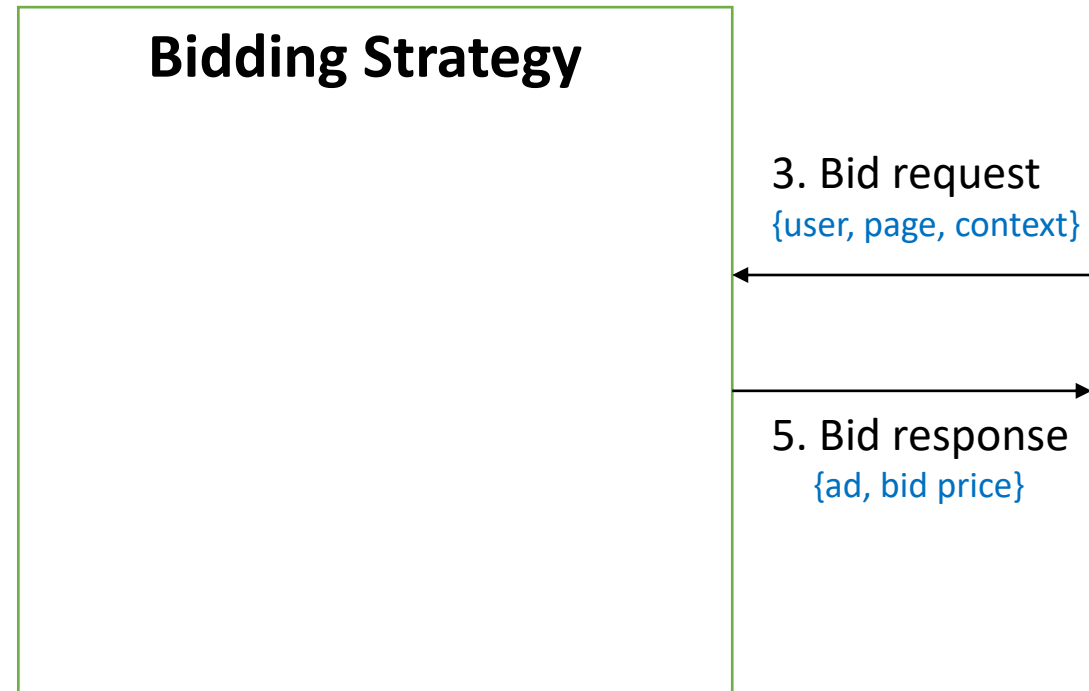
- **Bid Request Feature:** high-dimensional sparse binary vectors
 - Publisher information
 - Audience information
 - Impression details
 - Content and context
 - Targeting audiences
 - Etc...

Bidding Strategy

- **Given** a stream of bid requests and their features, ad features (budget, ad content, target audience), etc
- **Determine** bid prices for each bid request
- **Such that**

$$\begin{aligned} & \max \text{KPI} \quad \text{\#clicks} \\ \text{s. t. } & \sum \text{cost} \leq \text{Budget} \end{aligned}$$

$$\text{Bid price} = \text{Click-Through Rate (CTR)} * \text{Cost-Per-Click (CPC)}$$

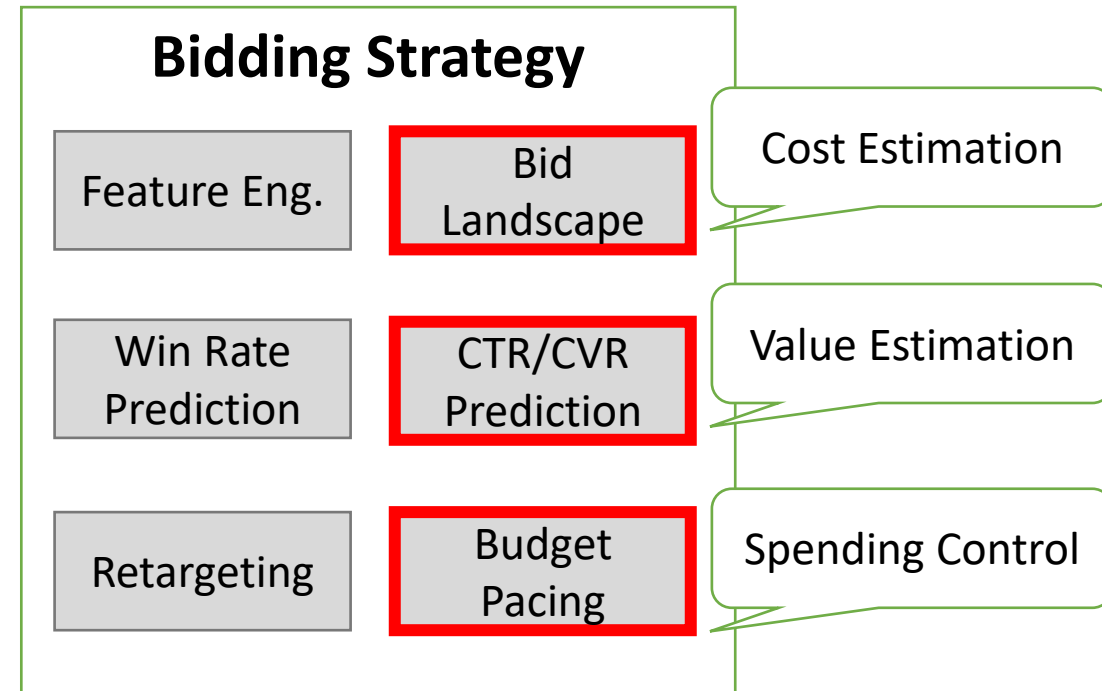


Bidding Strategy

- **Given** a stream of bid requests and their features, ad features (budget, ad content, target audience), etc
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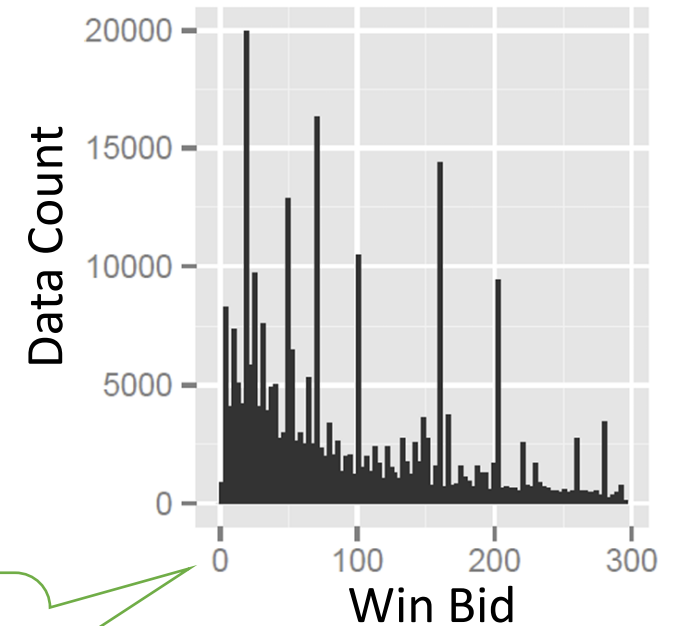
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Bid Landscape

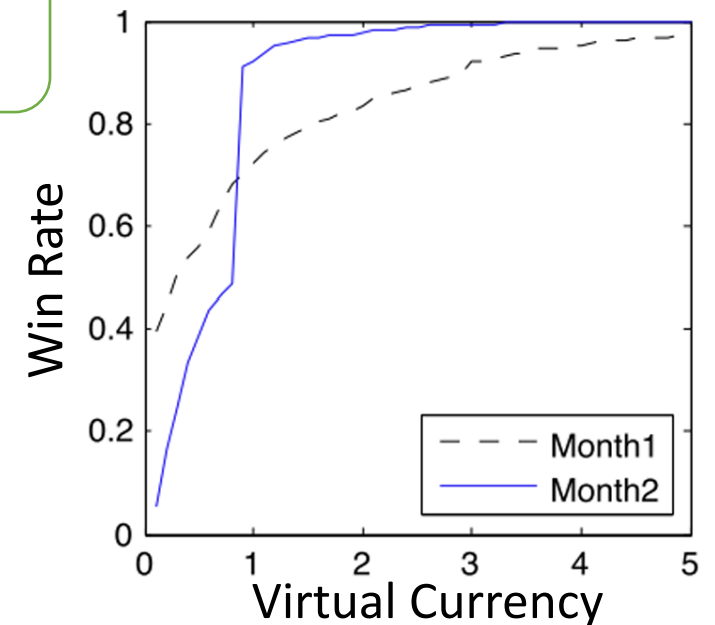
- The distribution (or analysis) of **bid prices** and **outcomes** within an online advertising auction environment

- **Goal**: Depicts market competition

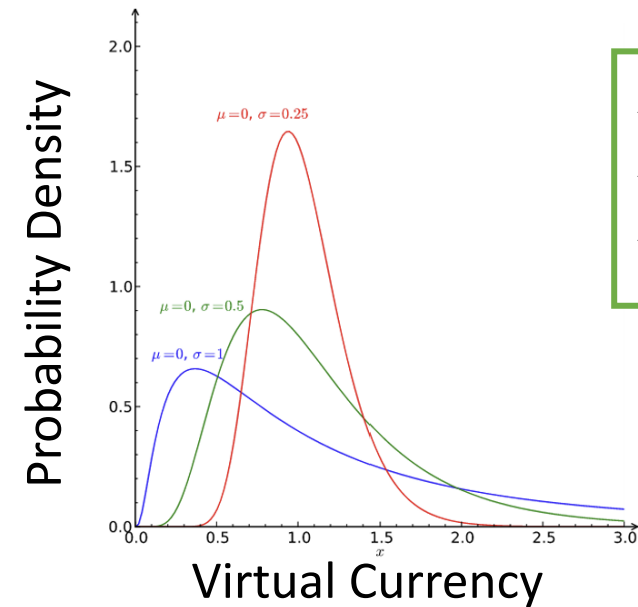
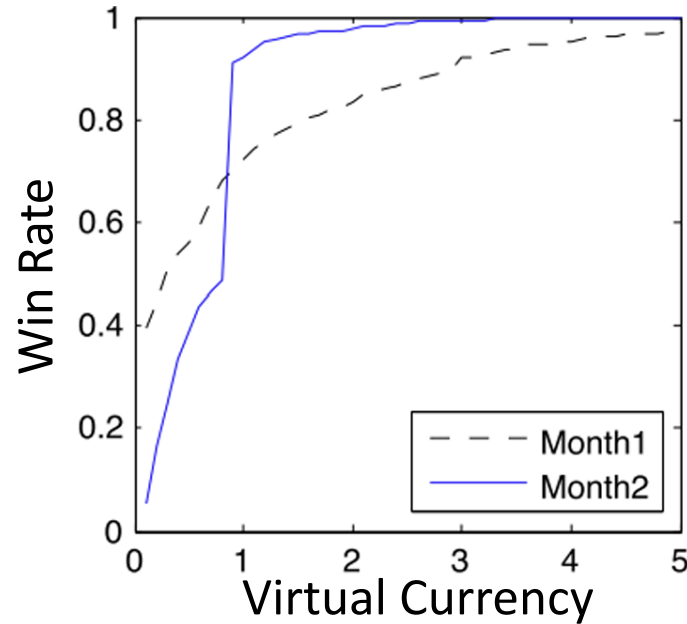
- Win rate: $w(b) = \int_{z=0}^b p(z) dz$
- Expected cost: $c(b) = \frac{\int_{z=0}^b zp(z) dz}{\int_{z=0}^b p(z) dz}$



How can DSP collect so many bid prices for one bid request?



Bid Landscape



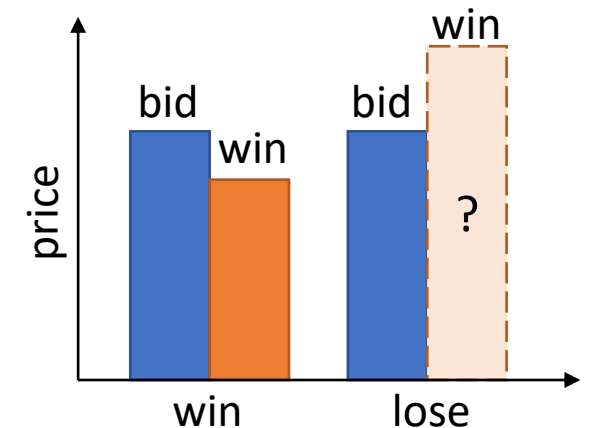
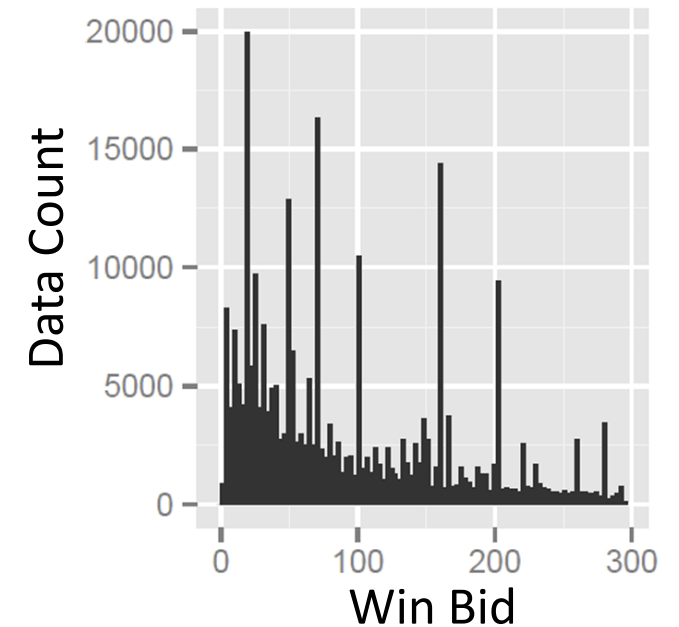
- ✓ Long-tail
- ✓ Non-negative bid price
- ✓ Normality

- Log-normal distribution (PDF)

$$f(b; \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{\frac{-(\ln b - \mu)^2}{2\sigma^2}}, b > 0$$

Underestimation of Bid Landscape

- Using win bid only is **over-optimistic**
 - Ignore bid prices you lost
- For DSP, impossible to know empirical win prices due to **censorship**!
- **Survival Analysis** in Medical/Financial domains
 - Study not-yet-happening events
 - Patient death in the future (death vs. time)
 - Win bid at a higher price (win vs. price)



Mixture Model with Censored Data

- For **winning data**, use **PDF ϕ** of standard normal distribution

$$\hat{z}_i = \beta x_i + \varepsilon; \max_{\beta_w} \sum_{i \in W} \underbrace{\log(\phi(\frac{z_i - \beta_w x_i}{\sigma}))}_{\text{Log-likelihood}}$$

- For **censored (losing) data**, use **CDF φ** of standard normal distribution
 - φ gives the probability your bid is not enough to win

$$\underline{P(b_i < \hat{z}_i)} = \varphi(\frac{\beta_l x_i - b_i}{\sigma})$$

The probability that your bid b_i is less than the (unobserved) true winning price \hat{z}_i .

Why CDF but not PDF?

Mixture Model with Censored Data

- Overall objective

$$\max_{\beta_w, \beta_l} \sum_{i \in W} \log(\phi(\frac{z_i - \beta_w x_i}{\sigma})) + \sum_{i \in L} \log(\varphi(\frac{\beta_l x_i - b_i}{\sigma}))$$

maximize preciseness maximize prob. of $b_i < \hat{z}_i$
 \Rightarrow raise \hat{z}_i

- Win price prediction

$$\hat{z}_i = [P(z_i < b_i)\beta_w + (1 - P(z_i < b_i))\beta_l]x_i,$$

$$\text{where } P(z_i < b_i) = \frac{1}{1 + \exp(-\beta_{\text{logit}} x_i)}$$

Additional logistic regression to classify if the price is winnable, independent to φ

Isn't this weird? Are separated β_w and β_l necessary?

Mixture Model with Censored Data

- Overall objective

$$\max_{\beta} \sum_{i \in W} \log(\phi(\frac{z_i - \beta x_i}{\sigma})) + \sum_{i \in L} \log(\varphi(\frac{\beta x_i - b_i}{\sigma}))$$

- Win price prediction

$$\hat{z}_i = \beta x_i$$



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Optimal Bid Price

- Reward for a given bid: $R(b) = \int_0^b (\underbrace{r}_{\text{True value of this request}} - \underbrace{z}_{\text{Win rate at bid price = z}}) p(z) dz$
- Optimal bid price $b^* = \max_b R(b)$
 - $\Rightarrow \frac{\partial R}{\partial b} = (r - b)p(b) = 0$
 - $\Rightarrow b^* = r$
 - \Rightarrow *The optimal bid price is exactly the value we believe*

Click-Based Value

- Strategy: bid $\begin{cases} \text{value of click, if clicked} \\ 0, \text{ otherwise} \end{cases}$
- Average value of click: $(\text{value of click}) \times \frac{\#click}{\#impression}$

- Click as KPI: **$CPC \times CTR$**

Cost-Per-Click (CPC):

1. Optimal reward $b^* = r$
2. Manually-set maximum bid
3. Total spend/Total (expected) clicks

Click-Through-Rate (CTR):

1. $\#click/\#impression$
2. $P(click | x)$

**Accurate CTR predictions
=> Precise data-driven bidding strategy**

CTR Prediction

- **Given**: n impressions consisting of feature vector x and click label $y \in \{0,1\}$
- **Goal**: train a CTR prediction model f such that $\hat{y} = f(x)$
- **Challenges**:
 - High-dimensional and extremely sparse feature space
 - Feature interactions
 - Imbalance label ($\# \text{clicked} / \# \text{non-clicked} \approx 0.001$)

Logistic regression

Field-Aware Features

- **Bid Request Feature:** high-dimensional **sparse** binary vectors

- Publisher information
- Audience information
- Impression details
- Content and context
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- Etc...

M fields

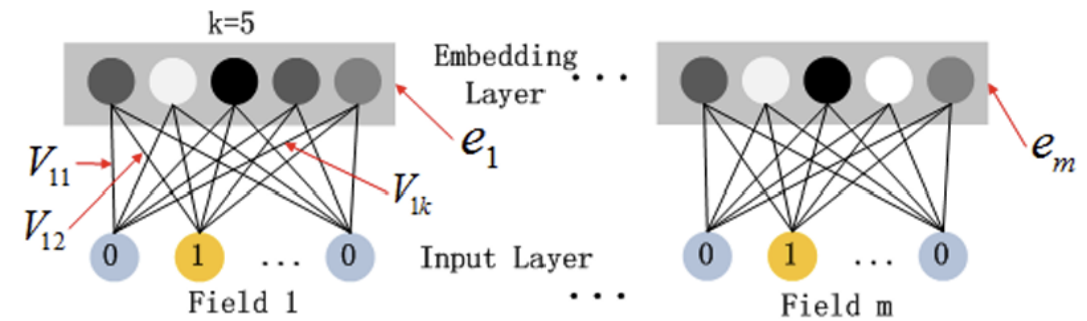
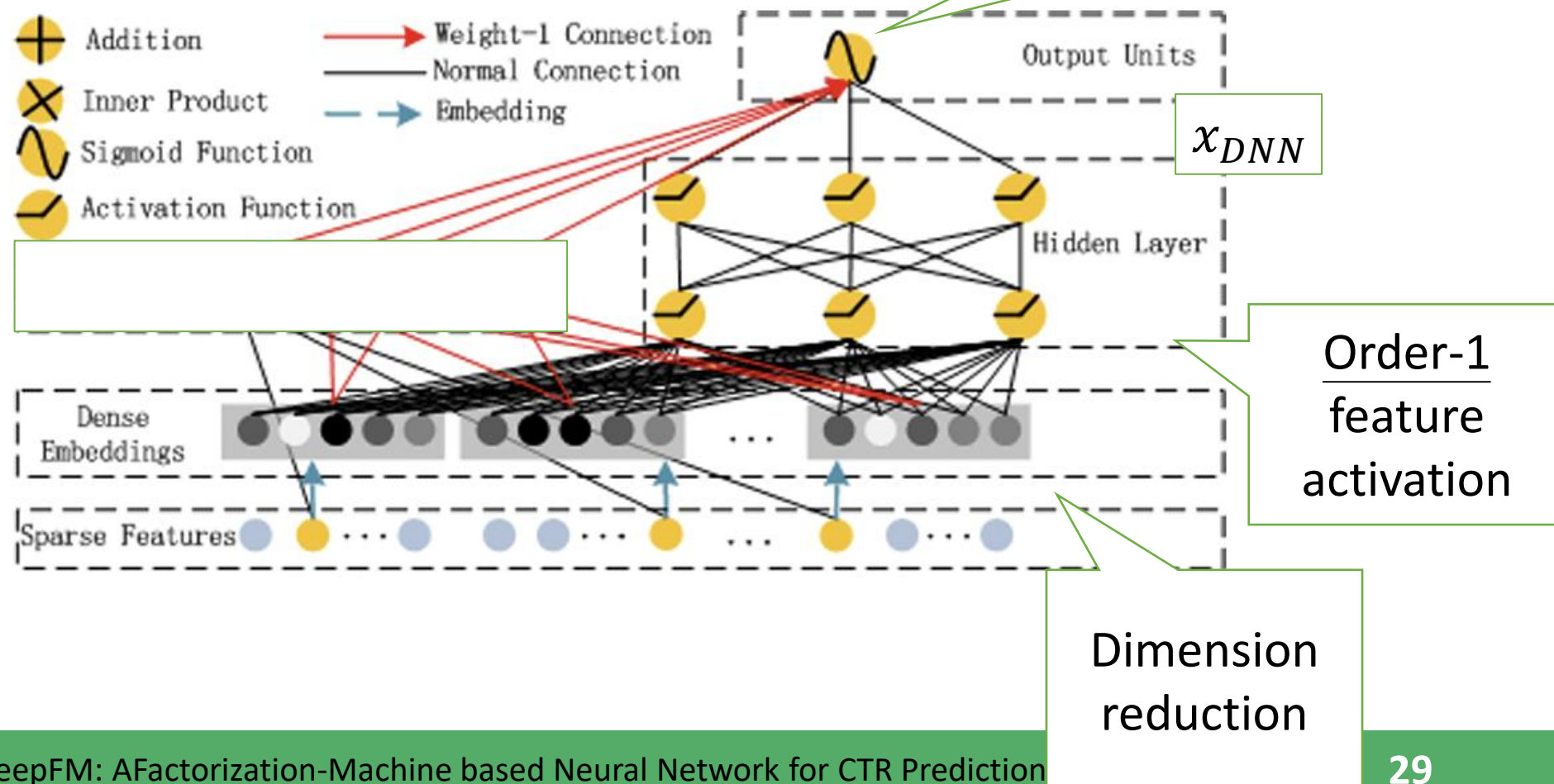


Figure 4: The structure of the embedding layer

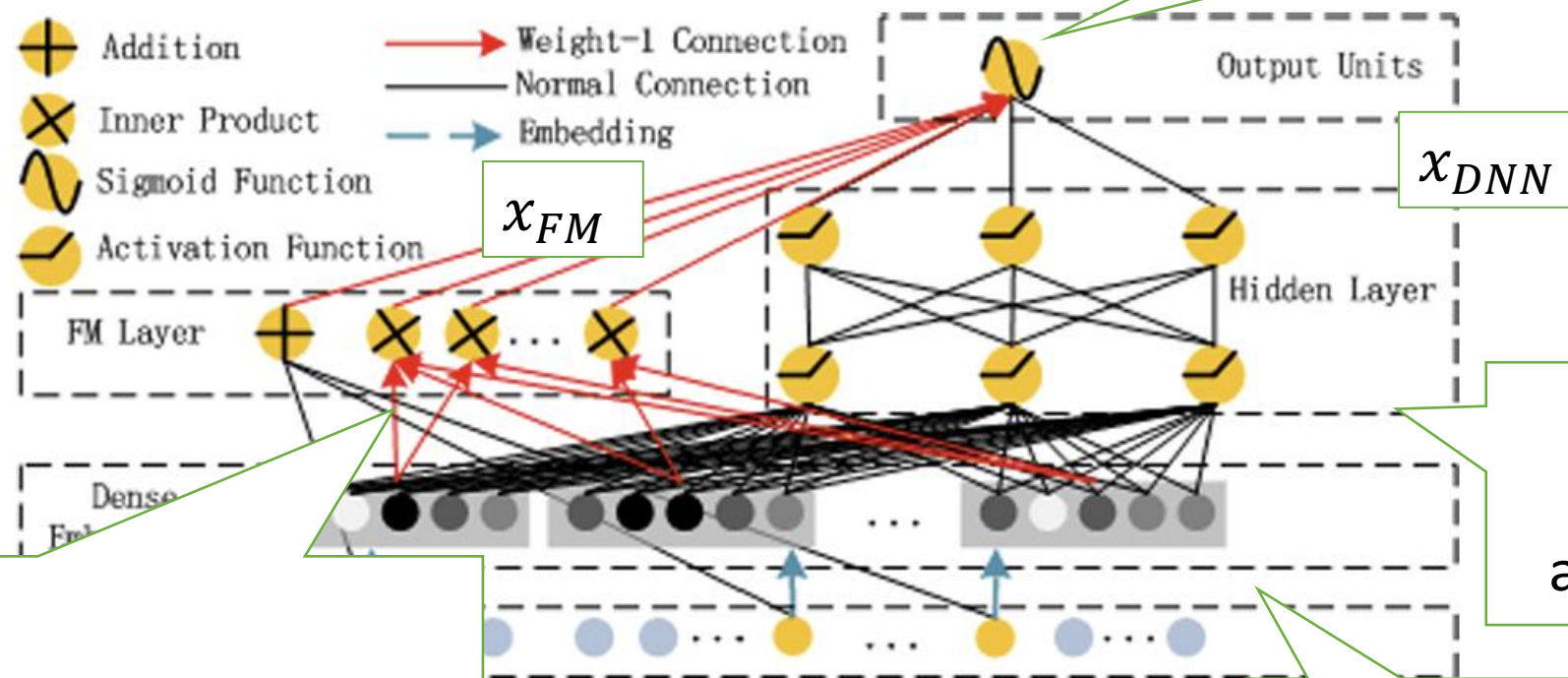
- **Dimension reduction** for each field => **field-aware features**

- PCA, LDA, etc
- Embedding in deep models

Order-1 Prediction



Order-2 Prediction



$$y = \sigma(x_{FM} + x_{DNN})$$

x_{DNN}

x_{FM}

Order-1
feature
activation

Dimension
reduction

Order-2 feature:

$$x_{FM} = \langle w, x \rangle + \sum_{i=1}^d \sum_{j=i+1}^d \langle V_i, V_j \rangle x_i x_j,$$

where V_i, V_j are latent vectors

Label Imbalance Issue

- Imbalance label ($\text{\#clicked}/\text{\#non-clicked} \approx 0.001$)
- Always predict non-click yields 99.9% accuracy
- Downsample \#non-clicked
- \#clicked augmentation (upsample)
- Upweight importance of clicked samples
- Etc...



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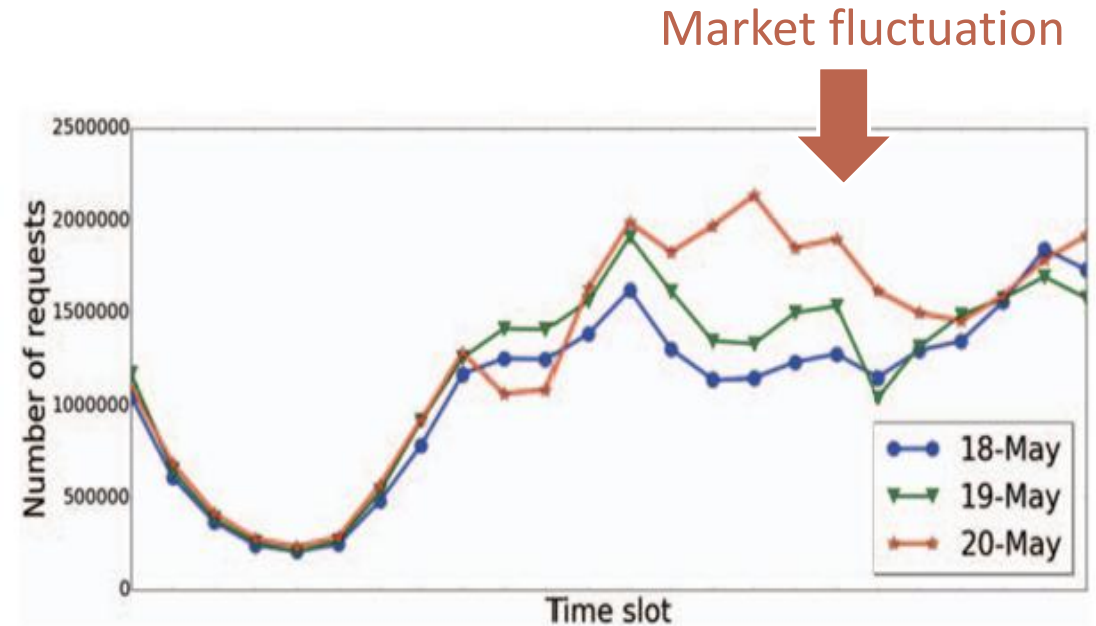
What We Have Now

- Win price prediction: $\hat{z}_i = \beta x_i$
- Value prediction: $\hat{v}_i = CTR \times CPC$
- **Naïve bidding strategy:**
 - If $\hat{v}_i > \hat{z}_i$: offer bid price $b_i = \hat{z}_i + 1$
 - Else: offer bid price $b_i = \{\hat{v}_i, 0\}$

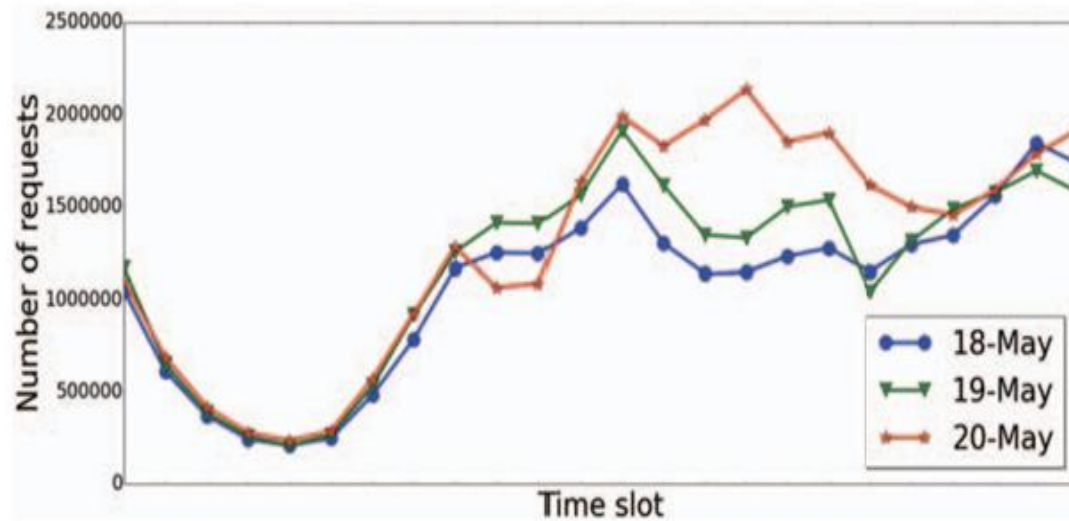
This strategy prevents us from overpayment

Issues

- **Naïve bidding strategy:**
 - If $\hat{v}_i > \hat{z}_i$: offer bid price $b_i = \hat{z}_i + 1$
 - Else: offer bid price $b_i = \{\hat{v}_i, 0\}$
- Suppose the lifetime is one week long
 - Naïve method may spend all the budget on the first day
 - No ad delivery in the remaining six days
- Sensitive to abnormal traffic patterns (e.g., market fluctuations)
- Advertisers prefer **smooth delivery over lifetime**
- Need to **foresee future revenue**



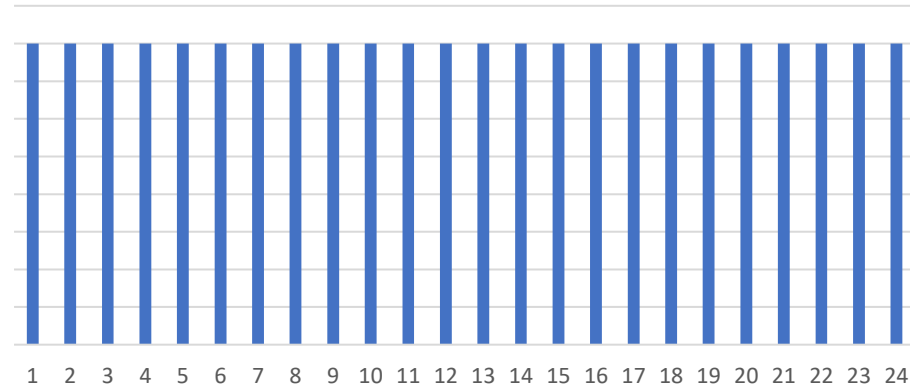
Budget Over Time



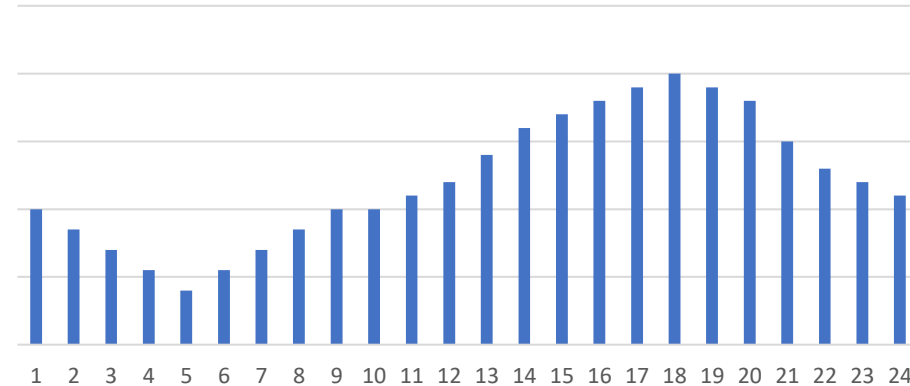
For $B_t \in \{B_1, B_2, \dots, B_T\}$
Run naïve bidding strategy

✗ traffic prediction steps in

Might not spend all the budget
⇒ Revenue loss



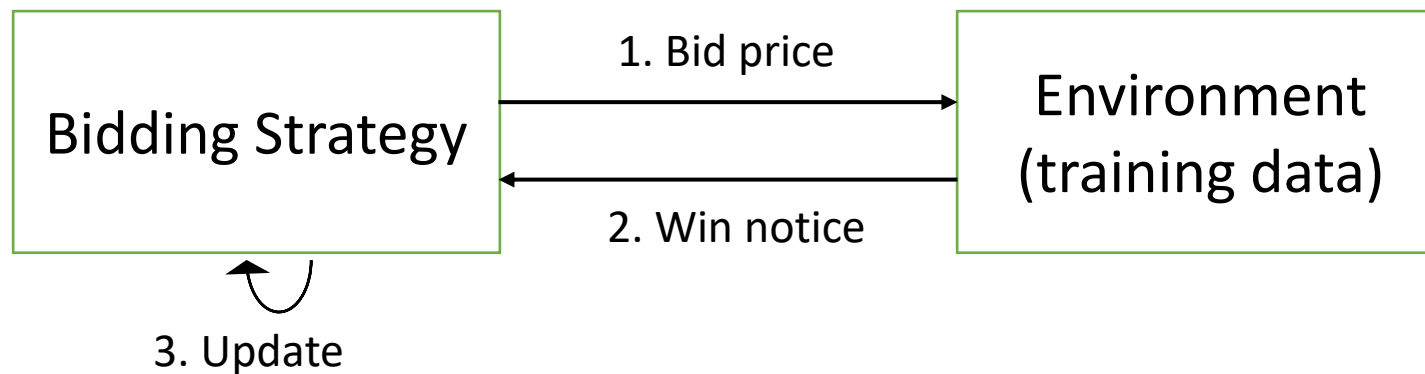
Uniform Distribution



Traffic-dependent Distribution

Deep Q-Network (DQN)

- Reinforcement learning (RL)
 - An intelligent brute-force approach
 - Learn complex and dynamic relations through experience
 - DQN is often used in RTB due to efficiency issue



Elements (MDP Process)

- **State S** : {feature, pCTR, pWinPrice, remaining budget, time left,}
- **Action A** : bid price or bid price adjustment (e.g., *ratio* * *constant price*)
- **Reward R** : utilities earned per bid request won
 - Click, impression, revenue, ...
- **Transition $T = P(s'|s, a)$** : given current state s , select action a , transit to state s' , and gain reward r

What Is DQN Optimizing

- DQN minimizes the difference between the predicted Q-value (the expected return for a state-action pair) and the target Q-value

$$L(\theta_i) = E_{s,a,r,s' \sim \rho} [\underbrace{(y_i}_{\text{Target}} - \underbrace{Q(s, a; \theta_i)}_{\text{Predicted Q-value by current DQN}})^2]$$

The i-th round DQN

Target

Predicted Q-value by current DQN

$$y_i = \underbrace{r}_{\text{Immediate reward from transiting from } s \text{ to } s' \text{ with action } a} + \gamma \cdot \max_{a'} Q(s', a'; \theta_{i-1})$$

Immediate reward
from transiting from
s to s' with action a

The maximum Q-value of the next state-
action pair, estimated by last-iteration DQN
⇒ Future benefit!

γ is the discount factor of future benefit,
commonly use {0.9, 0.95, 0.99}

DQN Output

- Given the current state s
- Predict Q-value on all action candidates $a \in A$
- Output the action with highest Q-value
- $a^* = \arg \max_{a \in A} Q(s, a)$

$s = \{\dots, \text{remaining budget, time left, } \dots\}$



DQN: if I don't raise bid price, reward will be low => bid price raised



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References and Further Reading

- A Collection of RTB Papers: [wnzhang/rtb-papers: A collection of research and survey papers of real-time bidding \(RTB\) based display advertising techniques.](https://wnzhang.github.io/rtb-papers/)