final project

how you get the theoretical distribution

Use the equation on the teacher's handout

scenario 1 :

```
waiting_time_theo_cdf[i] = 1 - rho * np.exp(-mu*(1-rho)*y)
```

```
system_time_theo_cdf[i] = 1 - np.exp(-mu*(1-rho)*y)
```

scenario 2 :

waiting time:

1. caculate $M/H_2/1$'s B*(s)
2. From $B^*(s)$,get waiting Laplace transform $W^*(s)$
3. $W^*(s) \leftrightarrow$ w(y)
4. Integrate w(y) to get CDF

```
s = Symbol('s')
t = Symbol('t')
B_Laplace = p1 * (mu1 / (s + mu1)) + p2 * (mu2 / (s + mu2))
W_Laplace = (1 - rho) / (s - Lambda + Lambda * B_Laplace)
w_y = inverse_laplace_transform(W_Laplace, s, t)

waiting_time_theo_cdf = np.zeros(num_packets)
for i, y in enumerate(waiting_times):
    waiting_time_theo_cdf[i]  = w_y.subs(t, y)
```

system time:

1. caculate $M/H_2/1$'s B*(s)
2. From $B^*(s)$,get waiting Laplace transform $W^*(s)$
3. From $W^*(s)$,get system time Laplace transform $S^*(s)$

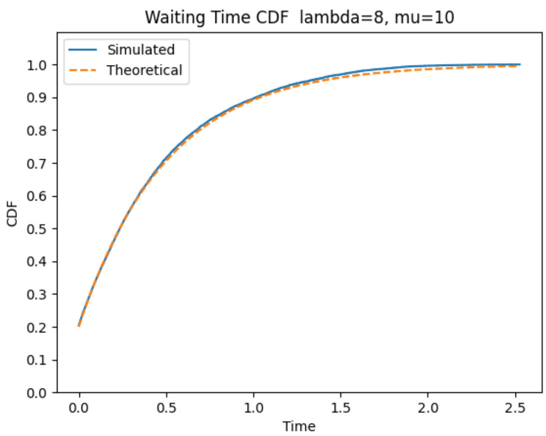
4. $S^*(s) \leftrightarrow$ s(y)

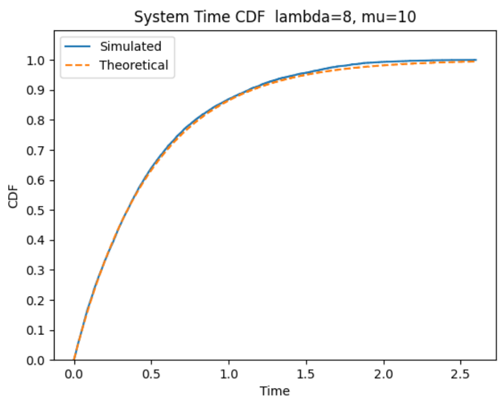
5. Integrate s(y) to get CDF

```
s = Symbol('s')
t = Symbol('t')
B_Laplace_s = p1 * (mu1 / (s + mu1)) + p2 * (mu2 / (s + mu2))
W_Laplace_s = s * (1 - rho) / (s - Lambda + Lambda * B_Laplace_s)
S_Laplace_s = B_Laplace_s * W_Laplace_s
S_y = inverse_laplace_transform(S_Laplace_s, s, t)
S_integrate = integrate(S_y)

system_time_theo_cdf = np.zeros(num_packets)

for i in range(len(system_times)):
    y = system_times[i]
    system_time_theo_cdf[i] = S_integrate.subs(t, y)
```

plot figures for two scenarios, each with two cases & compare simulation and theoretical results for each case.

Scenario 1

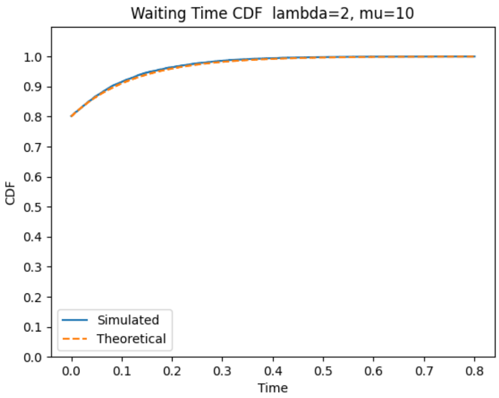


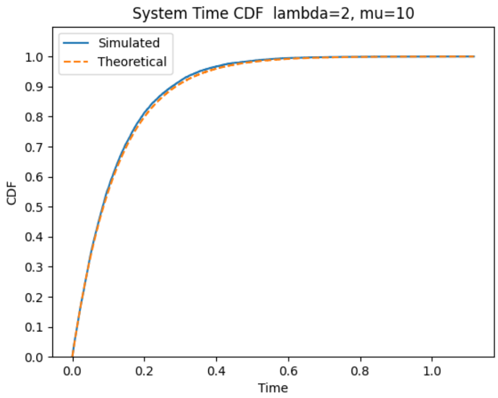


Waiting Time MSE: 1.0696434348479214e-06
System Time MSE: 4.5573588823791814e-05

Because the system parameter p_0=1−p_0=1−ρ, it implies that there is a 20% probability of the queue being empty in this scenario, resulting in a waiting time of 0.

For the system, since service time needs to be added, the probability of the system time being 0 is 0





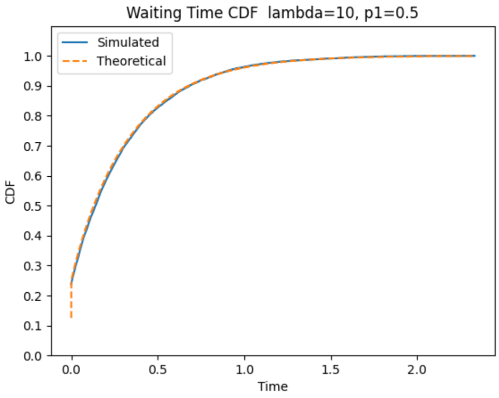
Waiting Time MSE: 3.557395826664209e-05
System Time MSE: 3.087146547356855e-05

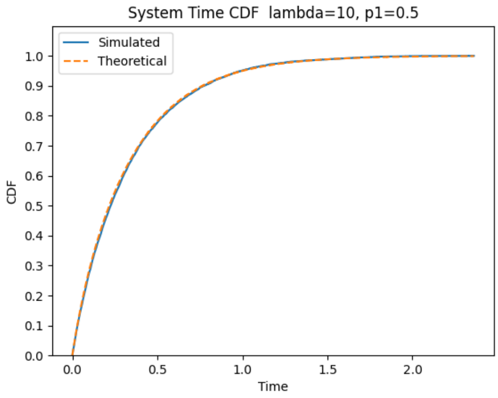
Because the system parameter p_0=1−p_0=1−ρ, it implies that there is a 80% probability of the queue being empty in this scenario, resulting in a waiting time of 0.

For the system, since service time needs to be added, the probability of the system time being 0 is 0

Compared to the first scenario, the probability of waiting time being 0 is much higher in this case. Consequently, the system exhibits a faster ascent in the regions with lower x-axis values.

Scenario 2

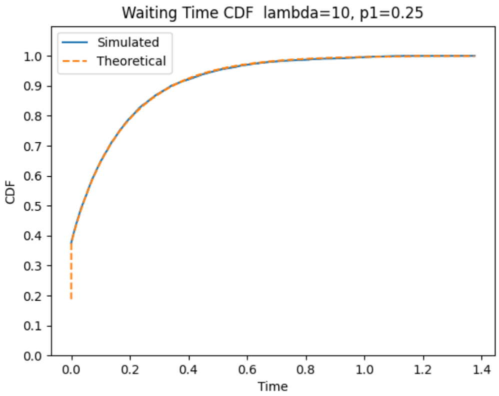


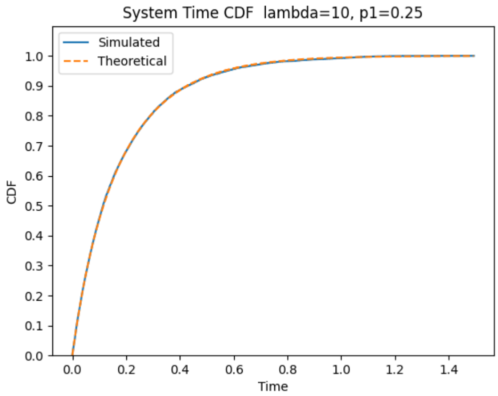


Waiting Time MSE: 0.0031499082585408258
System Time MSE: 6.946649948341226e-05

Because the system parameter p_0=1−p_0 =1−(0.5×1+0.5×0.5)=0.25*p0=1−ρ=1−(0.5×1+0.5×0.5)=0.25, it means that in this scenario, there is a 25% probability of the queue being empty, resulting in a waiting time of 0.

For the system, since service time needs to be added, the probability of the system time= 0 is 0.





Waiting Time MSE: 5.3923912001367455e-06
System Time MSE: 7.258197587938811e-05

Because the system parameter p_0=1−p_0=1−(0.5×1+0.5×0.5)=0.25*p0=1−ρ=1-(0.25*1+0.75*0.5) = 0.375, it means that in this scenario, there is a 25% probability of the queue being empty, resulting in a waiting time of 0.

For the system, since service time needs to be added, the probability of the system time= 0 is 0.