MODELING REAL-TIME HYDRAULIC SYSTEMS WITH POSITION-BASED DYNAMICS

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Abstract. Real-time simulation of various complex physical phenomena is a challenging task, since one must meet the demands of limited computing time for high frame rates and accurate, reproducible physical simulation. One approach to balance efficient and accurate simulation is to model the dynamics with position-based dynamics. In this report, we investigate how one can adapt position-based dynamics to incorporate hydraulic pressures to simulate heavy equipment. The key impact of this work is to lay groundwork for developing standardized virtual training and evaluation of heavy equipment operation and safety.

1. Introduction

With the advent of virtual reality technologies, virtual reality simulation and equipment offer great potential in providing a high degree of realism to simulate a dynamical system. An unavoidable bottleneck for providing this degree of realism is the computing time available for real-time simulation. Visualizing the environment requires fixing a frame rate, and depending on the available computing resources, there is a limited number of calculations that can be done in one frame. When simulating dynamics, using high accuracy solvers and good choice of time step size achieves the desired precision, but at the cost of more computing time. For real-time simulation, the frame rate is known or prescribed, so we are limited by the time complexity of methods we can use to simulate dynamics. Our goal is to devise a real-time simulation framework that simulates both physical bodies (particle-based) and fluid pressures (hydraulics) with the intent of simulating heavy equipment. The prospects of this are to provide operation and safety training in virtual reality.

To develop a hybrid particle-based and fluid pressure simulation, we start from position-based dynamics (PBD) [4]. We provide insight on how PBD can be used to incorporate fluid pressures and provide a sufficiently accurate and stable simulation. The difficulty lies in accounting for the latter. Accuracy is bottlenecked by a computing time budget and stability is affected by the stiffness of a hydraulic system model. In the next sections, we proceed as follows. First, we discuss hydraulic system modeling. Second, we provide an overview of the classic PBD algorithm, which offers a simple and fast algorithm for dynamics simulation. Finally, we briefly discuss our results and future work.

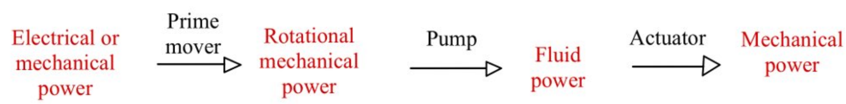

FIGURE 1. Flow diagram of energy transfer through a hydraulic system.

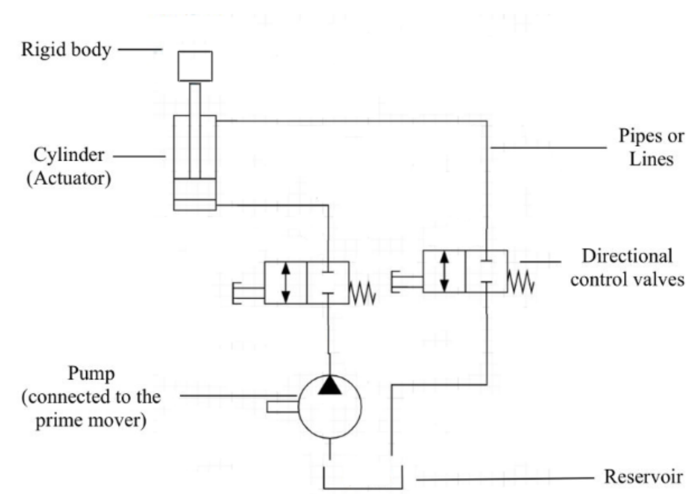

FIGURE 2. Example of a hydraulic system schematic.

2. Related work

2.1. **Hydraulic systems.** A hydraulic system aims to apply a force at a point and transfer it to another point using a liquid. This leverages the underlying physics by having large forces be produced by small input forces. For example, the structure of a hydraulic system can be described as in Figure 1. An electrical or mechanical power can be used to facilitate the movement of a pump. This produces rotational mechanical power which generates fluid flow. The flow is propagated to an actuator, which produces a force to create mechanical movement. This could be, for example, moving a hydraulic cylinder.

Hydraulic systems have numerous uses, and are abundant in the design of construction equipment, vehicles, and manufacturing equipment. In practice, hydraulic systems are described using schematics with a standardized set of notation and symbols. Figure 2 shows a hydraulic system schematic with its component symbols labelled.

The process of modeling a hydraulic system is to: first identify all important components of the system, such as pumps, valves, orifices, cylinders, motors, and so on. Next, the components are connected using a schematic, with a volume associated with each component. From there, we set the differential equations governing the system, with the unknown variables consisting of pressures, flow rates, and object positions.

2.2. **Position-based dynamics.** The most popular approaches to real-time simulation of dynamical systems are based on forces. In practice, this is done as follows. Given a fixed frame or time, first identify and calculate all internal and external forces in the system. Next, compute the accelerations of all the objects using Newton's second law of motion. Lastly, perform numerical integration to compute and update the velocities and positions of the objects. However, there are alternative approaches. The one we are interested in is *position-based dynamics (PBD)* [4], which directly works with and manipulates positions. The PBD algorithm offers a robust method for simulation of deformable bodies. PBD is also very simple and computationally efficient, and thus suitable for real-time applications.

The main features of PBD are

- control over explicit integration,
- using and handling purely positional constraints,
- and direct manipulation of vertex positions.

The positional constraints are enforced using a Gauss-Seidel solver, by first predicting future positions of the points from external forces and previous velocity, and then performing a cost-effective projection of the constraints. However, there are a few notable issues. First, this framework results in loss of momentum conservation. Second, PBD's control over stiffness depends nonlinearly on the time step size and number of solver iterations. This means the accuracy of the simulation is very sensitive to perturbations of the hyperparameters.

Despite this, PBD can be used where physical accuracy is less important than computational speed. For our purposes, it suffices to have PBD procure the correct asymptotic behaviour of the dynamical system. For future work, there are extensions and more recent work on PBD [1, 2, 5] that we can try to adapt to simulate fluid pressures and particle positions.

3. Results

3.1. **Modeling hydraulic systems.** A hydraulic system consists of several components, whose formulae are found in [3]. Here we describe the components appearing in Figure 2 and discuss their modeling assumptions with reference to [3]. Here the system schematic consists of a reservoir, fluid, pump, control valves, pipelines and hydraulic cylinder.

A *reservoir* contains a volume of liquid. The hydraulic fluid chosen differs by application but is usually taken to be petroleum or other types of oils. Here, hydraulic fluid is treated as incompressible, but a high-pressure system must account for compressibility.

A *hydraulic pump* is tied to an electric motor to transfer mechanical energy into the system. Given this mechanical part, the pump inlet forms a partial vacuum. This allows atmospheric pressure to push fluid through the inlet and into the pump, which in turn, the pump pushes fluid into the hydraulic system. The pump can be modeled to have constant or variable fluid displacement per cycle. An ideal pump has input torque τ and flow rate Q described by the equations

$$\tau = \frac{D_{\text{pump}}\Delta P}{2\pi}, \qquad Q = D_{\text{pump}} \cdot n.$$

Here D_{pump}, ΔP and n are the pump's volume displacement, pressure differential and shaft speed, respectively.

A *control valve* functions to control pressure, flow rate and direction of a fluid flowing through the hydraulic system. A *directional* control valve controls flow by the area of the orifice for which the fluid moves through. This is modeled by the equation

$$Q_{\text{DV}} = C_D \cdot A \cdot \sqrt{\frac{2\Delta P}{\rho}},$$

where A represents the orifice cross-section area, ΔP is the pressure differential between the input and output of the orifice, ρ is the fluid density, and C_D is a coefficient determined by the shape of the orifice. In simple models, C_D can be treated as constant.

Moving fluid from one component to another is achieved by *pipelines.* Each pipe has a pressure, which is governed by the equation

$$\frac{dP}{dt} = \frac{\beta}{V}(Q_{\text{in}} - Q_{\text{out}}),$$

where P represents the pipe's pressure, β is the fluid's bulk modulus (informally, the resistance to compression), V is the fluid volume of the pipe, and $\Delta Q = Q_{\text{in}} - Q_{\text{out}}$ is the fluid's flow rate differential in the pipe.

Hydraulic cylinders convert the energy stored in the hydraulic system into mechanical energy. As noted before, the cylinder can be used to displace a rigid body. For an ideal cylinder, the 'outgoing' flow corresponds to the rate at which more room is being created in the cylinder chamber for fluid. An increase in pressure pushes the cylinder piston, resulting in more room. This is expressed by

$$Q_{\text{cyl}} = A \cdot \frac{dx}{dt},$$

where A is the area of the piston base pushing the cylinder and x is the displacement of the rigid body. The displacement $x = 0$ corresponds to when the cylinder is fully retracted. Consequently, the cylinder experiences pressure from the flow, which is governed by the equation

$$\frac{dP}{dt} = \frac{\beta}{V_{\text{cyl}}}(Q_{\text{in}} - A \cdot \frac{dx}{dt}).$$

As before, β is the fluid bulk modulus, $V_{\text{cyl}} = A \cdot x$ is the cylinder fluid volume.

3.2. **Model example.** Given the equations of components, we can derive a system of differential equations. Consider the system in Figure 3, which is an ideal cylinder

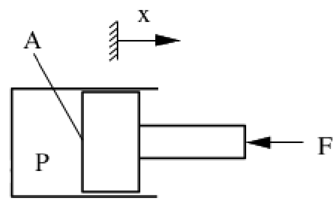

FIGURE 3. A hydraulic system consisting of an ideal cylinder with no flow (image from [3]).

with no incoming flow. This means that $Q = 0$, so the change in pressure of the cylinder is given by

$$\frac{dP}{dt} = -\frac{\beta}{x} \cdot \frac{dx}{dt}.$$

Recall the cylinder piston position is modeled so that $x \geq 0$. Thus if x is decreasing then the pressure is increasing, which matches our intuition. To be able to solve for x and P, we can apply Newton's second law to the piston, so that

$$m\frac{d^2x}{dt^2} = P \cdot A - F.$$

Now we have two equations with two unknowns, x and P, so we can solve this numerically provided we have some initial conditions.

3.3. **Numerical solution with PBD.** Continuing with our Figure 3 example, we took a few different approaches to approximate the system using PBD. One naive approach is to treat the pressures as positions. This does not work well since P and x are described by different physical units. We instead treat pressure and position update steps independently, where in the absence of damping and other modifications in PBD, the algorithm resembles Euler's method. The downside of this is that Euler's method is inaccurate with stiff equations, which worsens from the limit of how small the time step size can be for real-time simulation. Even with our small example, the equations are stiff, since β can be several orders of magnitude larger than other quantities, such as the position, volume or flow.

This is reflected in Figure 4, which plots exact and PBD solutions of the cylinder with no flow. Here $t \in [0, 0.1]$, $\beta = 2.2 \cdot 10^9$, which is the estimated bulk modulus of water, the piston mass is $m = 1$ kilogram, and the base is $A = 0.03^2$ meters squared. The initial conditions are $x(0) = 1$, $x'(0) = 0$ and $P(0) = \beta$. The exact solution in orange is computed using SciPy's `odeint` with very small time step, whereas the PBD solution is computed for a time step of 1200 frames per second (that is, $\Delta t = 0.1/120$). Asymptotically, the pressure appears to be dampening, and as a result the cylinder position and velocity diverge. The asymptotic behaviour is not stable as we predicted.

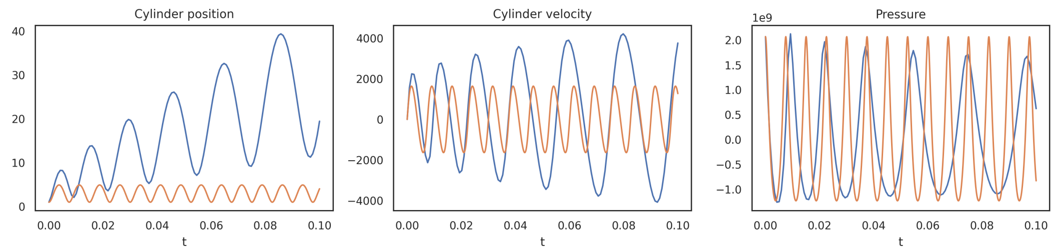

FIGURE 4. Hydraulic cylinder dynamics from Figure 3 with some set of initial conditions. The orange is the exact solution and blue is a position-based dynamics solution.

4. Conclusion

In this project, we explored the use of position-based dynamics for simulating hydraulic systems. A systemic procedure was demonstrated for obtaining governing equations of simple hydraulic system models. Moreover, PBD was implemented and experimentally tuned to examine the asymptotic behaviour of PBD for hydraulic systems. We hope this work provides a foundation for those actively working on this in the future, that is, developing stable algorithms for real-time simulation of hydraulic systems.

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