胡博闻 2016121518

注 * 包含上一周的作业，上次没来得及打印老师让我可以这周一起交上来

49.

$$P(2X + 1 = k) = \begin{cases} 0.1, & k = -1 \\ 0.2, & k = 1 \\ 0.3, & k = 3 \\ 0.4, & k = 5 \end{cases}$$

$$p(X^2 = k) = \begin{cases} 0.2, & k = 0 \\ 0.4, & k = 1 \\ 0.4, & k = 4 \end{cases}$$

50.

$$F(x) = \begin{cases} 0 & x \leqslant 0 \\ 1 - e^{-x} & x > 0 \end{cases}$$

$$\begin{aligned} G(y) &= P(Y \leqslant y) \\ &= P(X^2 \leqslant y) \\ &= P(X \leqslant \sqrt{y}) \\ &= F(\sqrt{y}) \end{aligned}$$

$$G(y) = \begin{cases} 0 & y \leqslant 0 \\ 1 - e^{-\sqrt{y}} & y > 0 \end{cases}$$

$$P(Y) = \begin{cases} \frac{\sqrt{y} e^{-\sqrt{y}}}{2y} & y > 0 \\ 0 & y \leqslant 0 \end{cases}$$

55.

$$p(x) = \begin{cases} \frac{1}{2\pi} & x \in [0, 2\pi] \\ 0 & x \notin [0, 2\pi] \end{cases}$$

$$F(x) = \begin{cases} 0 & x \in (-\infty, 0) \\ \frac{x}{2\pi} & x \in [0, 2\pi] \\ 1 & x \in (2, +\infty) \end{cases}$$

$$\begin{aligned} G(y) &= P(Y \leqslant y) \\ &= P(cosX \leqslant y) \\ &= P(\arccos y \leqslant X \leqslant 2\pi - \arccos y) \end{aligned}$$

$$G(y) = \begin{cases} 0 & y \in (-\infty, -1) \\ 1 - \frac{\arccos y}{\pi} & y \in [-1, 1] \\ 1 & y \in (1, +\infty) \end{cases}$$

$$p(y) = \begin{cases} \frac{1}{\pi \sqrt{1 - y^2}} & y \in [-1, 1] \\ 0 & y \notin [-1, 1] \end{cases}$$

42.

$$p_1 = \Phi(\frac{x - \mu}{\theta}) = \Phi(-1) = 1 - \Phi(1)$$

$$p_2 = 1 - \Phi(\frac{x - \mu}{\theta}) = 1 - \Phi(1) = p_1$$

44.

$$\Delta = \sqrt{16 - 4X}, p(X > 4) = \frac{1}{2} = \Phi(0)$$

$$\mu = 4$$

48.

$$F(X \leqslant 200) = \Phi(\frac{200 - \mu}{\sigma}) = 1 - \Phi(0.8)$$

$$F(X \leqslant 240) = \Phi(\frac{240 - \mu}{\sigma}) = \Phi(0.8)$$

$$\alpha = (1 - \Phi(0.8)) \times 0.1 + (2\Phi(0.8) - 1) \times 0.001 + (1 - \Phi(0.8)) \times 0.2$$
$$= 0.0642$$

$$\beta = \frac{(2\Phi(0.8) - 1)}{\alpha} = 0.009$$

54.

$$p(y) = \begin{cases} \frac{2}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} & x > 0 \\ 0 & x \leqslant 0 \end{cases}$$

2.

	1	2	3	4
1	$\frac{1}{4}$	0	0	0
2	$\frac{1}{8}$	$\frac{1}{8}$	0	0
3	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	0
4	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$

3.

	1	2	3	4	5	$P(X = x_i)$
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0.1	0	0	0	0	0.1
4	0.2	0.1	0	0	0	0.3
5	0.3	0.2	0.1	0	0	0.6
$P(Y = y_i)$	0.6	0.3	0.1	0	0	1

5.

$$\int_\Omega A e^{-(2x + 3y)} = 1$$

$$A = 6$$

$$F(x, y) = \begin{cases} 0 & x \leqslant 0 \text{ or } y \leqslant 0 \\ (1 - e^{-2x})(1 - e^{-2y}) & x > 0, y > 0 \end{cases}$$

$$P(-1 \leqslant X \leqslant 1, \ -2 \leqslant Y \leqslant 2) = (1 - e^{-2})(1 - e^{-6})$$

8.

$0 < x < 1$:

$$p(x) = \int_{-x}^{+x} \frac{3}{2} x \, dy = 3x^2$$

$$p_X(x) = \begin{cases} 3x^2 & 0 < x < 1 \\ 0 & \text{其他} \end{cases}$$

$-1 < y < 1$:

$$p(y) = \int_{|y|}^{1} \frac{3}{2} x \, dx = \frac{3}{4}(1 - y^2)$$

$$p_Y(y) = \begin{cases} \frac{3}{4}(1 - y^2) & -1 < y < 1 \\ 0 & \text{其他} \end{cases}$$

10.

$$P(X + Y \leqslant 1) = \int_0^{\frac{1}{2}} dx \int_x^{1-x} e^{-y} \, dy$$
$$= 1 + e^{-1} - 2e^{-\frac{1}{2}}$$

$$P(X = Y) = 0$$

$x > 0$:

$$p(x) = \int_x^{+\infty} e^{-y} \, dy = e^{-x}$$

$$p_X(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & \text{其他} \end{cases}$$

$y > 0$

$$p(y) = \int_0^y e^{-y} \, dx = ye^{-y}$$

$$p_Y(y) = \begin{cases} ye^{-y} & y > 0 \\ 0 & \text{其他} \end{cases}$$

$$P(X > 2 | Y < 4) = \frac{P(2 < X < Y < 4)}{P(Y < 4)} = \frac{e^{-2} - 3e^{-4}}{1 - 5e^{-4}}$$