Normal Distribution

$$E(X) = \mu, Var(x) = \sigma^2$$
$$\psi_X(t) = E(e^{tx}) = e^{\mu t + \frac{\sigma^2 t^2}{2}}$$
$$Y = aX + b \sim N(a\mu + b, a^2\sigma^2)$$
$$\Phi(-x) = 1 - \Phi(x), \Phi^{-1}(P) = -\Phi^{-1}(1-P)$$
$$X \sim N(\mu, \sigma^2), \frac{x-\mu}{\sigma} \sim N(0,1)$$
$$\sum_{i=1}^{n} \sim N(\sum_{i=1}^{n} a_i\mu_i, \sum_{i=1}^{n} a_i^2\sigma^2)$$
$$\text{sample mean : } \overline{\mathrm{x_n}} = \frac{\sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{x_i}}{\mathrm{n}}$$

Gamma Distribution

$$\Gamma(x) = \int_0^\infty x^{\alpha-1}e^{-x}dx$$
$$\Gamma(1) = 1, \Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$$
$$\Gamma(n) = (n-1)!, \gamma(\frac{1}{2}) = \sqrt{\pi}$$
$$\int_0^\infty x^{\alpha-1}e^{-\beta x}dx = \frac{\Gamma(\alpha)}{\beta^\alpha}$$
$$\text{pdf : } \frac{\beta^\alpha}{\gamma(\alpha)}\mathrm{x}^{\alpha-1}\mathrm{e}^{-\beta\mathrm{x}}, \mathrm{x} > 0$$
$$E(X) = \frac{\beta}{\alpha}, Var(x) = \frac{\alpha}{\beta^2}$$
$$\psi_X(t) = (\frac{\beta}{\beta - t})^\alpha$$
$$\sum_{i=1}^{n} x_i \sim Gamma(\sum_{i=1}^{n} \alpha_i, \beta)$$
$$\text{pdf Gamma}(1,\beta) \text{ exp}(\beta) \ \beta\mathrm{e}^{-\beta\mathrm{x}}$$
$$x \sim exp(\beta) \ P(x \geqslant t + h | x \geqslant) = p(x \geqslant h)$$

Law of Large Number

$$\text{mankov inequity :P(x} \geqslant \text{t)} \leqslant \frac{\text{E(x)}}{\text{t}}$$
$$\text{chebyshev inequity :P(|x}-\text{E(x)|} \geqslant \text{t)} \leqslant \frac{\text{Var(x)}}{\text{t}^2}$$

Central limit theorem

$$x_1, x_2, ..x_n \ with \ E(X) = \mu, Var(X) = \sigma^2$$
$$\lim_{n \leftarrow \infty} P(\frac{\overline{x_n} - \mu}{\frac{\sigma}{\sqrt{n}}}) = \Phi(x)$$

continuity correction

Maximum likehood estimator

find parameter when $f(x_1, x_2, x_3 ... | \alpha)$ reaches the maximum value

Properties of MLE

$$\widehat{g(\theta)} = g(\widehat{\theta})$$

M.O.M.E using sample means and variance to estimate parameters

Geometric distribution

$$f(x) = p(1-p)^x$$

sufficient statistics

Improving estimates

$$R(\theta, \delta) = E((\delta^2 - \theta^2))$$

$$= Var(\delta) + (E(\delta) - \theta)^2$$

$$\delta_0(T) = E(\delta(x)|T)\min$$

inadmissible = not a function of T

chi-square distribution

gamma($\frac{n}{2}, \frac{1}{2}$)

$$y \sim N(0,1), y^2 \sim \chi(1)$$

$$\sum x_i \sim \chi(\sum n_i)$$

Joint distribution of Sample mean and sample variance

$$x_i \sim N(\mu, \sigma^2)$$

$$\overline{X}_n \sim N(\mu, \frac{\sigma^2}{n})$$

$$\frac{\sum(X_i - \mu)}{\sigma^2} \sim \chi(n)$$

$$\frac{\sum(X_i - \overline{X}_n)^2}{\sigma^2} \sim \chi(n-1)$$

T distribution

$$X_i \sim N(0,1)$$

$$T(n-1) = \frac{\overline{X} - \mu}{\frac{S}{\sqrt{n}}},\ S = \frac{1}{n-1} \sum(X_i - \overline{X})^2$$

$$T(n-1) = \frac{\overline{X} - \mu}{\frac{S_n}{\sqrt{n-1}}},\ S = \frac{1}{n} \sum(X_i - \overline{X})^2$$

$$T = \frac{\overline{X} - \mu}{\sqrt{\frac{\sum(X_i - X)^2}{n(n-1)}}}$$

$$T(m) = \frac{\sqrt{X}}{\sqrt{\frac{Y}{m}}},\ X \sim N(0,1), Y \sim \chi(m)$$

$$pdf\ \frac{\Gamma(\frac{m+1}{2})}{(m\pi)^{\frac{1}{2}}\Gamma(\frac{m}{2})}(1 + \frac{x^2}{m})^{-\frac{m+1}{2}}$$

Confidence interval

μ known, δ unknown

$$\frac{\overline{X} - \mu}{\frac{\delta}{\sqrt{n}}} \sim N(0,1)$$

$$\mu \sim [\overline{X} - \frac{\delta}{\sqrt{n}}\Phi^{-1}, \overline{X} + \frac{\delta}{\sqrt{n}}\Phi^{-1}]$$

μ unknown, δ known

$$\frac{\sum(X_i - \mu)^2}{\sigma^2} \sim \chi(n)$$

$$\delta^2 \sim [\frac{\sum(X_i - \mu)^2}{\chi_n^{-1}(\alpha)}, \frac{\sum(X_i - \mu)^2}{\chi_n^{-1}(\alpha)}]$$

both unknown, δ^2 remains the same calculation

$$\frac{\overline{X} - \mu}{\frac{\sigma^‘}{\sqrt{n}}} \sim T(n-1)$$

$$\mu \in [\overline{X} - \frac{\sigma^‘}{\sqrt{n}}T_{n-1}^{-1}, \overline{X} + \frac{\sigma^‘}{\sqrt{n}}T_{n-1}^{-1}]$$

Unbiased Estimator

for estimator $\delta(x)$ and parameter θ, for every θ, $E(\delta) = \theta$: unbiased estimator

$$MSE = Var(\delta(x)) + \text{bias}^2$$

$$\text{bias} = \text{E}(\delta(\text{x})) - \theta$$

for mean μ, $\overline{X}$; for σ^2, $\delta`$

$$E(\widehat{\sigma}^2) = \frac{n-1}{n}\sigma^2$$

$$\sigma`^2 = \frac{n}{n-1}\widehat{\sigma}^2$$

$$E((\sigma` - \sigma)^2) = Var(\sigma'^2)$$

$$= Var(\frac{\sigma^2}{n-1}\frac{n-1}{\sigma^2}\sigma^{2'})$$

$$= \frac{\sigma^4}{(n-1)^2} \times 2(n-1) = \frac{2\sigma^2}{n-1}$$

$$E((\widehat{\sigma^2} - \sigma^2)) = Var() + \text{bias}^2$$

$$Var = \frac{\sigma^2}{n^2} \times 2(n-1)$$

$$\text{bias} = \frac{\sigma^2}{\text{n}^2}$$

$$= \frac{2(n-1)\sigma^2}{n^2}$$

Testing Hypotheses

$$H_0, H_1$$

simple / composite hypothesis, critical region

type I / II error, power function $\pi(\delta|\theta)$

size of a test

$$\alpha(\delta) = \sup\pi(\delta|\theta),\ \theta \in {}_0$$

H_0 is simple, $\alpha(\delta) = \pi(\delta|theta_0)$

Testing simple Hypotheses

$H_0 : \theta = \theta_0$, $H_1 : \theta = \theta_1$

$$\alpha(\delta), \beta(\delta)$$

with smallest $a\alpha + b\beta$ when reject H_0 when $af(x|theta_0) < bf(x|theta_1)$ and accept when $af(x|theta_0) > bf(x|theta_1)$

find smallest β with known size or α, reject when $f(x|\theta_0) < kf(x|\theta_1)$, accept the averse

UMP test

MLR: $\frac{f(x_i|\theta_2)}{f(x_i|\theta_1)}$increasing with $T = r(x)$ when $\theta_2 > \theta_1$

For $H_0 : \theta \leqslant \theta_0$ and $H_1 : \theta > \theta_1$, with test $T > C$, T is the MLR. This is the UMP test. In addition, power function is increasing of θ. If Hypotheses reverse, the test is $T < C$ and decreasing function of θ

two sided alternatives

no UMP

usually normal

using $\overline{X} < C_1, \overline{X} > C_2$

if C_1, C_2 is symetrical respect to μ_0, then power function will be symetrical respect to μ_0

Content

MSE

unbiased

randomized test (b test)

confidence interval μ σ^2

t distribution