COMP 350 Numerical Computing
Assignment #4. Solving a nonlinear equation

Date given: Wed, Oct 18. Date due: 5pm, Wed, Nov 1, 2017.

(To be marked by Sitao Luan (sitao.luan@mail.mcgill.ca))

1. (a) (8 points) The **Steffensen method** for solving the equation $f(x) = 0$ uses the following iteration formula

$$x_{n+1} = x_n - \frac{f(x_n)}{g(x_n)},$$

where

$$g(x_n) = \frac{f(x_n + f(x_n)) - f(x_n)}{f(x_n)}.$$

If this method converges for an initial point x_0, show it usually has quadratic convergence rate. What is the advantage of this method over Newton's method?

Hint: To establish the quadratic convergence, in addition to expand the Taylor series of $f(r)$ about x_n, which was used in our convergence analysis of Newton's method, expand the Taylor series of $f(x_n + f(x_n))$ about x_n.

(b) (8 points) Use Steffensen's method, Newton's method and the secant method to find a root of $x^3 - 2x^2 + x - 3 = 0$. For both Steffensen's method and Newton's method, use the initial points $x_0 = 3$ and $x_0 = 2.5$. For the secant method, use $x_0 = 3$ and $x_1 = 2.5$ as the two initial points. In your test, you may take $xtol = 1.e{-}14$, $ftol = 0$ and $n_{max} = 40$. Comment on the test results.

2. (4 points) (**Simultaneous nonlinear equations**) Using the Taylor series in two variables (x, y) of the form

$$f(x + h, y + k) = f(x, y) + hf_x(x, y) + kf_y(x, y) + \cdots$$

where $f_x = \partial f/\partial x$ and $f_y = \partial f/\partial y$, establish that Newton's method for solving the two simultaneous equations

$$\begin{aligned} f(x, y) &= 0, \\ g(x, y) &= 0 \end{aligned}$$

can be described with the formulas

$$x_{n+1} = x_n - \frac{fg_y - gf_y}{f_x g_y - g_x f_y}, \quad y_{n+1} = y_n - \frac{f_x g - g_x f}{f_x g_y - g_x f_y}.$$

Here the functions f, f_x, f_y, g, g_x, g_y are evaluated at (x_n, y_n).

Hint: Use the idea we used in class to derive Newton's method for solving a single nonlinear equation.

Note: The iteration can also be written in the matrix-vector form:

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} x_n \\ y_n \end{bmatrix} - J^{-1} \begin{bmatrix} f \\ g \end{bmatrix}, \quad J \equiv \begin{bmatrix} f_x & f_y \\ g_x & g_y \end{bmatrix} \text{ is called the Jacobian matrix.}$$