COMP 350 Numerical Computing
Assignment #6, Numerical Integration

Date given: Wed, Nov 15. Date due: 5:00pm, Wed, Nov 29, 2017.

1. (15 points) The function $\text{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-x^2} dx$ is called an error function in statistics.

   (a) (3 points) If we want to ensure the absolute error of the trapezoid rule for computing erf(1) to be bounded by 10^{-6}, how many points should be used?

   (b) (6 points) Compute erf(1) by the recursive trapezoid rule. Stop the iteration when the difference between two consecutive computed integrals is smaller than or equal to 10^{-6}. Let the computed result be denoted by I_T. Compute the error $I - I_T$, where I can be obtained by the MATLAB built-in function `erf`. Report the number of function evaluations, and print the final result, the error, and the MATLAB codes as well.

   (c) (6 points) Compute erf(1) by the adaptive Simpson's method. Try to avoid redundant function evaluations. Take $\epsilon = 10^{-6}$ and `level_max=100`. Let the computed result be denoted by I_S. Compute the error $I - I_S$.

   Report the number of function evaluations, and print the final result, the error, and the MATLAB codes as well.

2. (5 points) Construct a rule of the form

$$\int_{-1}^{1} f(x)dx \approx \alpha f\left(-\frac{1}{2}\right) + \beta f(0) + \gamma f\left(\frac{1}{2}\right)$$

   such that it is exact for all polynomials of degree ≤ 2.

   Hint: Use one of the approaches we used in class to derive the two-point Gaussian quadrature rule.