Assignment# 4 comp 302 | 11/04/18

2. a) $R_{b:T}\ \frac{}{true : boolean}$, $R_{b:F}\ \frac{}{false : boolean}$, $R_{==}\ \frac{n:\tau,\ m:\tau}{n==m;\ boolean}$

$R_\tau\ \frac{X:\bullet\ \ X:!\bullet\ \ x:!!\bullet}{x:\tau}$ $R_{cond}\ \frac{C:boolean,\ A:\tau,\ B:\tau}{(if\ C\ then\ A\ else\ B):\tau}$ $R_\S\ \frac{V:\bullet}{\S V:!\bullet}$

$R_{let}\ \frac{\Gamma\vdash expr:\tau \quad \Gamma,x:\tau\vdash body:\tau}{\Gamma\vdash let\ x=expr\ in\ body:\tau}$ $R_*\ \frac{V:\bullet}{*\S V:\bullet}$

$R_\S\ \frac{y:!\vdash x=\S y:!\bullet}{}$ $R_\S\ \frac{x:!\bullet\vdash z=\S x:!!\bullet}{}$ $R_*\ \frac{z:!!\bullet\vdash w=*z:!\bullet}{}$

$R_{let}\ \frac{\Gamma_1\vdash x=\S y:!\bullet \quad \Gamma_2\vdash z=\S x:!!\bullet \quad \Gamma_3\vdash w=*z:!\bullet}{\Gamma_1\vdash let\ x=\S y\ in\ \{let\ z=\S x\ in\ \{let\ w=*z\ in\ \{if\ (w==x)\ then\ \{let\ r=*\S y\ in\ r\}\ else\ \{let\ r=*\S *z\ in\ r\}\}\}\}}$

$\Gamma_1=\{y:\bullet\}\quad \Gamma_2=\Gamma_1\cup\{x:!\bullet\}\quad \Gamma_3=\Gamma_2\cup\{z:!!\bullet\}\quad \Gamma_4=\Gamma_3\cup\{w=!\bullet\}$

$R_\tau\ \frac{w:!\bullet}{R_{==}\ w:\tau\ \ x:\tau}\ \frac{x:!\bullet}{}$ $R_\S\ \frac{y:\bullet}{\S y:!\bullet}$ $R_*\ \frac{}{}$ $R_{let}\ \frac{\Gamma:\bullet}{}$ $R_\tau\ \frac{r:\bullet}{r:\tau}$

$R_{let}\ \frac{w:!\bullet,\ x:!\bullet\vdash (w==x):boolean \quad \Gamma_4\vdash *\S y:\bullet \quad \Gamma_4\vdash let\ r=*\S y\ in\ r:\bullet}{in\ \{if\ (w==x)\ then\ \{let\ r=*\S y\ in\ r\}\ else\ \{let\ r=*\S *z\ in\ r\}\}\}:\tau}$

$R_*\ \frac{z:!!\bullet}{*z:!\bullet\ \ **z:\bullet}$

$R_{let}\ \frac{\Gamma_4\vdash r=**z:\bullet}{let\ r=**z\ in\ r:\bullet}$