Comp 350 assignment 6

Ian Benlolo 260744397
McGill University
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1. (a) **Trapezoid**

$$|I - I_R| = \left| \frac{-1}{12}(b-a)h^2 f''(z) \right|$$

where $h = \frac{b-a}{n}$

$$\left| \frac{-1}{12}\frac{(b-a)^3}{n^2} f''(z) \right| = \left| \frac{-1}{12}\frac{(1-0)^3}{n^2} f''(z) \right| = \frac{1}{12n^2}\left| f''(z) \right|,\ we\ want\ \frac{1}{12n^2} max_{\in[0,1]} \left| f''(z) \right| \leq 10^{-6}$$

$$f(x) = \frac{2}{\sqrt{\pi}} e^{-z^2},\ f'(z) = \frac{2}{\sqrt{\pi}}(-ze^{-2z^2})$$

$$f''(z) = \frac{2}{\sqrt{\pi}}(-2e^{-z^2} + 4z^2 e^{-z^2}) = \frac{2}{\sqrt{\pi}} e^{-z^2}(4z^2 - 2)$$

$$f'''(z) = \frac{2}{\sqrt{\pi}}[4ze^{-z^2} + 8ze^{-z^3} - 8z^3 e^{-z^3}] = \frac{8}{\sqrt{\pi}} ze^{-z^2}(3 - 2z^2) = 0$$

So either $z = 0$ or $3 - 2z^2 = 0 \rightarrow 2z^2 = 3,\ z = \pm\sqrt{\frac{3}{2}}$. The latter is ignored since $\pm\sqrt{\frac{3}{2}}$ are out of the $[0, 1]$ range
Now,

$$\frac{1}{12n^2} \times max\{f \left| f''(0) \right|, \left| f''(1) \right|\} \leq 10^{-6},$$

where, $|f''(0)| = \left| \frac{2}{\sqrt{\pi}} \times -2 \right| = \left| \frac{-4}{\sqrt{\pi}} \right| = \frac{4}{\sqrt{\pi}}$ and $f''(1) = \frac{2}{e\sqrt{\pi}}(4-2) = \frac{4}{e\sqrt{\pi}}$. So $f''(0)$ is the bigger of the two.

$$\frac{1}{12n^2}\frac{4}{\sqrt{\pi}} \leq 10^{-6} \rightarrow n^2 \geq \frac{1}{3\sqrt{\pi}}10^6 \Rightarrow n \geq \sqrt{\frac{1}{3\sqrt{\pi}}10^6} \approx 434$$

(b)

(c) Here are the results of my programs and the code

```
>> run main
ERF(1) = 0.84270079
Recursion Trapezoid: 0.842701
Number of iterations: 513.0
Error: 0.00000026
Adapted Simpson method: 0.842701
Number of iterations: 136.0
Error: -0.00000000420
>>
```

```
function [result, y] = RecursionTrapezoid(a, b)
  %setting h to b-a, and x is the first step
h = b-a;
```

```
x = h/2*(f(0)+f(1));
count=2;
m = 0;
%if the difference between the actual erf(1) and x is
%greater than 10^-6 then recursively do the trapezoid computation
while abs(erf(1)-x) > 10^(-6)
    h = h/2;
    m = m+1;
    sum = 0;
    for i=1:2^(m-1)
        sum = sum + f(a+(2*i-1)*h);
        count = count + 1;
    end
    x = h*sum + x/2;
end
result = x;
y = count;
end

------------------------------------------------------------------------------------------
function soln = adapt_simpson(a,b,ep,lvl, lvl_max, count)
h = b-a;
c = (a+b)/2;
i1=h*(f(a)+4*f(c)+f(b))/6;
count = count + 3;
lvl = lvl + 1;
d=(a+c)/2;
e=(c+b)/2;
i2 = h*(f(a)+4*f(d)+2*f(c)+4*f(e)+f(b))/12;
count = count + 5;
if lvl >= lvl_max
    numbI = i2;
    soln = [numbI, count];
else
    if abs(i2-i1) <= 15*ep
        numbI = i2+(1/15)*(i2-i1);
        soln = [numbI, count];
    else
       soln = adapt_simpson(a,c,ep/2,lvl, lvl_max, count)
        + adapt_simpson(c,b,ep/2,lvl, lvl_max, count); # put on a different
                                                        # line to fit the margins
    end
end
end
```

2. We want $I_G = \alpha f(-\frac{1}{2}) + \beta f(0) + \gamma f(-\frac{1}{2})$. $A_0 = \alpha,\ A_1 = \beta\ A_2 = \gamma$ & $x_0 = -\frac{1}{2},\ x_1 = 0,\ x_2 = \frac{1}{2}$.

$$A_0 = \int_{-1}^{1} \frac{(x-0)(x-\frac{1}{2})}{(-\frac{1}{2}-0)(-\frac{1}{2}-\frac{1}{2})} dx = 2\int_{-1}^{1} x^2 - \frac{1}{2}x dx = 2\left[\frac{x^3}{3} - \frac{x^2}{4}\right]_{-1}^{1} = \cdots = \frac{4}{3}$$

$$A_1 = \int_{-1}^{1} \frac{(x+\frac{1}{2})(x-\frac{1}{2})}{(0+\frac{1}{2})(0-\frac{1}{2})} dx = -4\int_{-1}^{1} x^2 - \frac{1}{4} dx = -4\left[\frac{x^3}{3} - \frac{x}{4}\right]_{-1}^{1} = \cdots = -\frac{2}{3}$$

$$A_2 = \int_{-1}^{1} \frac{(x-0)(x+\frac{1}{2})}{(\frac{1}{2}-0)(\frac{1}{2}+\frac{1}{2})} dx = 2\int_{-1} 1x^2 + \frac{x}{2} dx = \cdots = \frac{4}{3}$$

Therefore

$$I_G = \frac{4}{3} f(-\frac{1}{2}) - \frac{2}{3} f(0) + \frac{4}{3} f(\frac{1}{2})$$