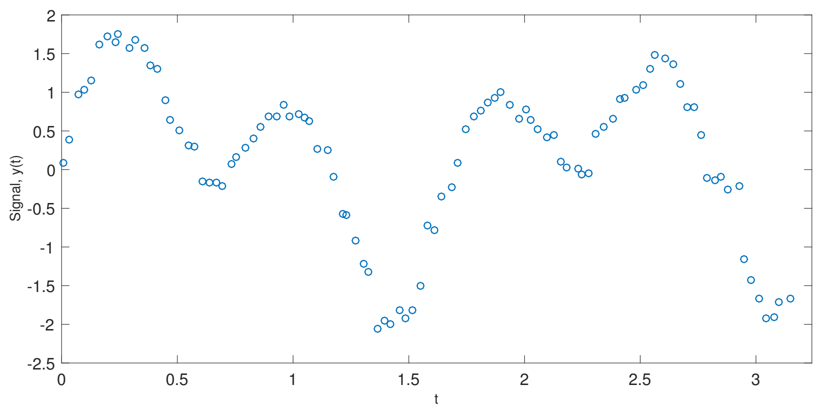
Math 365 **Homework 5** **Due Nov. 2, 2016**
Grady Wright

1. (**Oscillatory data fitting**, 15pts) NCM, Chapter 5, Problem 5.8

2. (**Longley data set**, 20pts) NCM, Chapter 5, Problem 5.11

3. (**Fitting planetary orbits**, 20pts) NCM, Chapter 5, Problem 5.12

4. (**Separable least squares**, 15pts) Download the file `PeriodicData.mat` from the course webpage. This file contains 100 noisy samples of a signal $y(t)$ at various times t over the interval $[0,\ \pi]$; see the plot below.



Your goal is to fit this data with a non-linear model of the form:

$$y(t) = a_1 \sin(\omega_1 t) + a_2 \sin(\omega_2 t).$$

Fortunately, this is can be formulated as a separable least squares problem, similar to what we did with the exponential model in class (see also Section 5.8 of the book). Use the separable least squares technique to determine the amplitudes a_1 and a_2 and the frequencies ω_1 and ω_2 in the model. Use `fminsearch` to find a minimum of the non-linear problem with an initial guess of 3 for ω_1 and 7 for ω_2. Report the values for a_1, a_2, ω_1, and ω_2 and make a plot of the resulting data and the model.

5. (**Simpson's rule**, 30pts) In this exercise you will use Simpson's rule to approximate the following integrals:

$$\begin{aligned} I(f_1) &= \int_{-1}^{1} (x-1)^2 e^{-x^2} dx\ , \\ I(f_2) &= 2\int_{-1}^{1} \frac{1}{1+x^2} dx\ , \end{aligned}$$

Please answer all parts of the exercises a–c below.

(a) Use MATLAB's symbolic tool box to obtain exact answers to $I(f_1)$ (and $I(f_2)$ if you can't figure it out directly).

(b) Write a MATLAB function for approximating the general integral

$$\int_a^b f(x)dx$$

using Simpson's rule. Your function should take as input a function representing the integrand $f(x)$, the end points of the interval a and b and the number of subintervals n to use in making the approximation. Put this code in your write-up and the corresponding m-file in your dropbox folder.

(c) Using your Simpson's rule method from part (a), compute an approximation to $I(f_1)$ and $I(f_2)$ for $n = 4, 8, 16, 32, 64, 128$.

i. Report the magnitude of the error in these approximations for each n in a nice table.
ii. Produce a plot of the magnitude of the error vs. $1/n$ (on a log-log scale).
iii. For the $I(f_1)$ verify that the error is decreasing like $O(1/n^4)$.
iv. Does this rate of decrease in the error appear to be true for $I(f_2)$? What rate does the error appear to decrease for this integral?

If you are curious about why this happens please come and talk to me.