CSCI 3302
Introduction to Robotics

Prof. Alessandro Roncone

aroncone@colorado.edu

https://hiro-group.ronc.one

Lab 5
Path planning with Dijkstra

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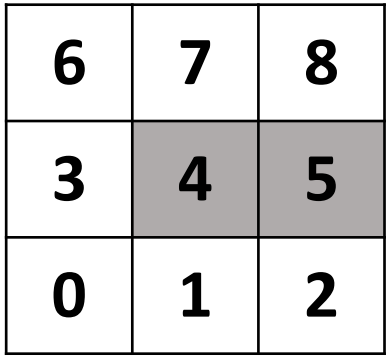
Dijkstra Lab In-Depth

Data Structures

- int dist[num_vertices]
  - Final shortest-distance/lowest cost relative to source
- int prev[num_vertices]
  - Previous node on path back to source
- int Q_cost[num_vertices]
  - Queue of vertices with priority equal to the best-known cost to get to that vertex from source. Used for bookkeeping during Dijkstra function

Important Functions

- Dijkstra(source_vertex)
  - Returns a populated ‘prev’ array, using source_vertex as origin of all paths
- Reconstruct_path(prev, vertex)
  - Iteratively follows prev[vertex] back to the source and returns a list of vertices on the (shortest) path
- get_travel_cost(source_vertex, dest_vertex)
  - Returns the cost (distance) of traveling between two vertices. 1 if neighbors, BIG_NUMBER if not.

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11         Q.add_with_priority(v, dist[v])
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14     while Q is not empty:                             // The main loop
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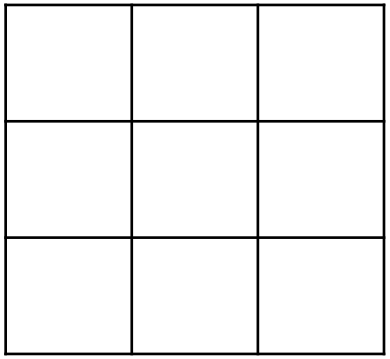

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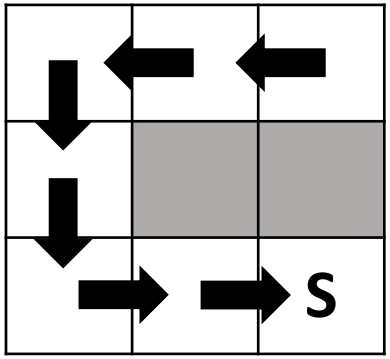

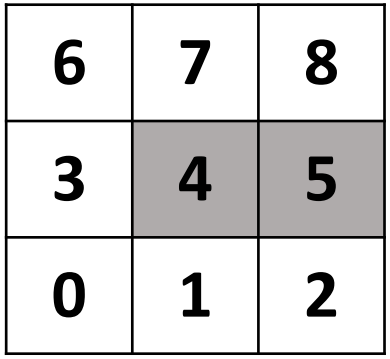


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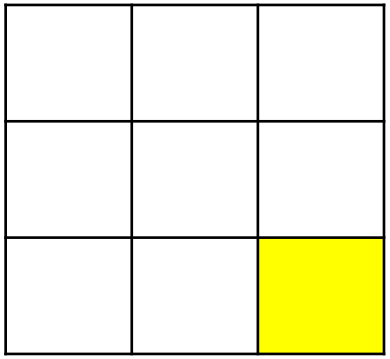
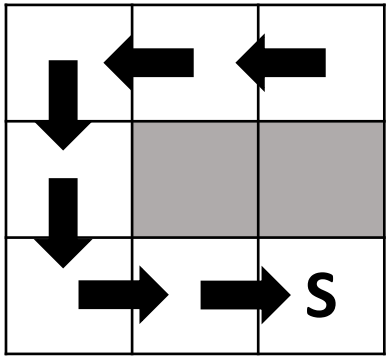
Initialize Source





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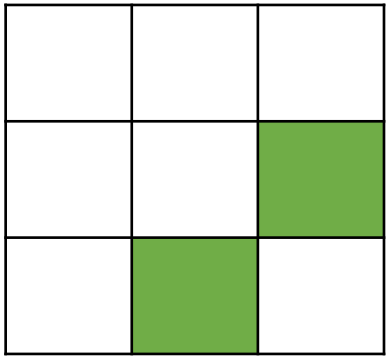

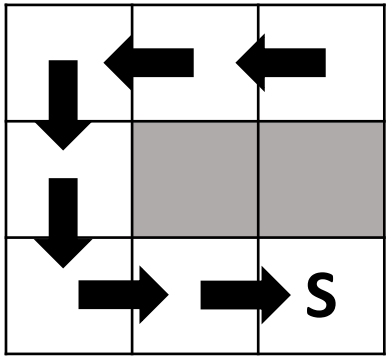


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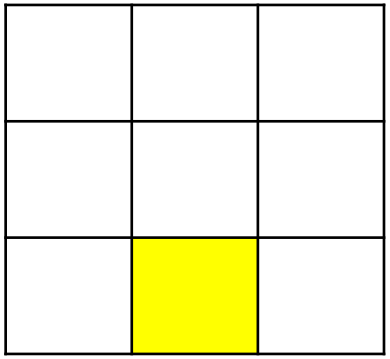

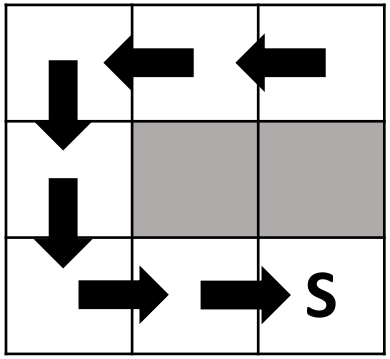


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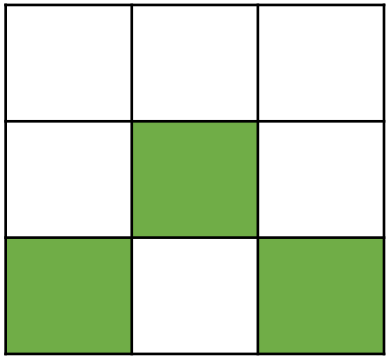

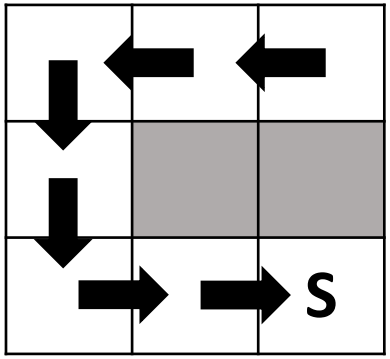


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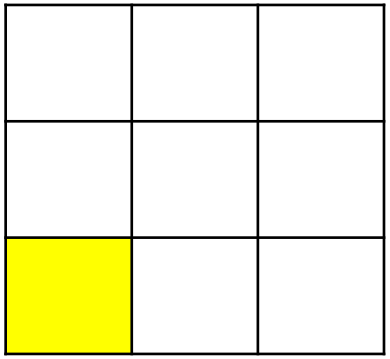

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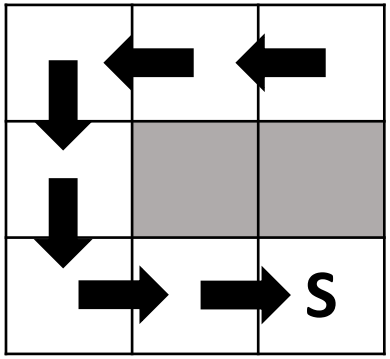

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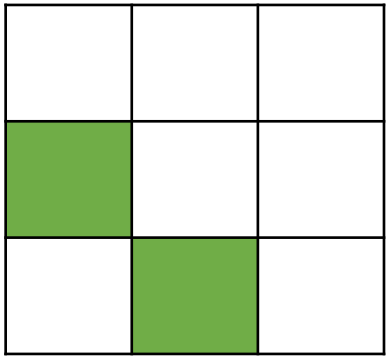

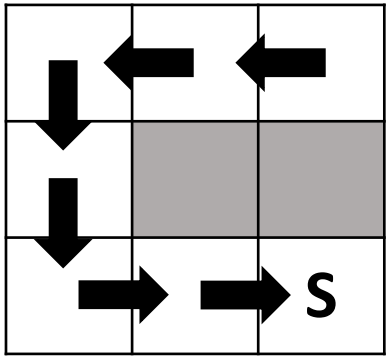


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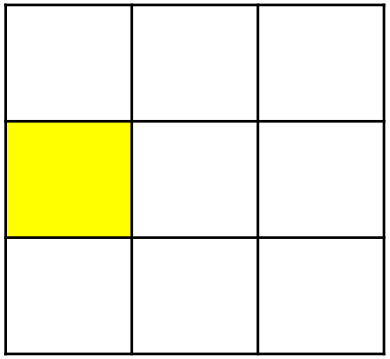

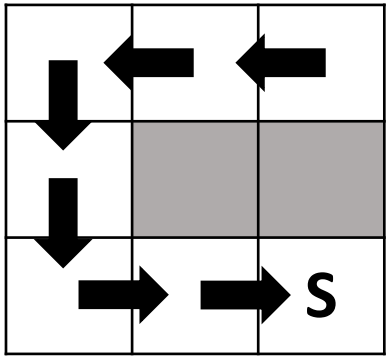


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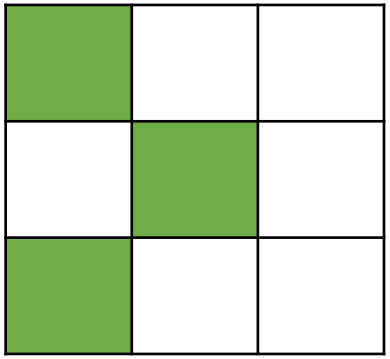

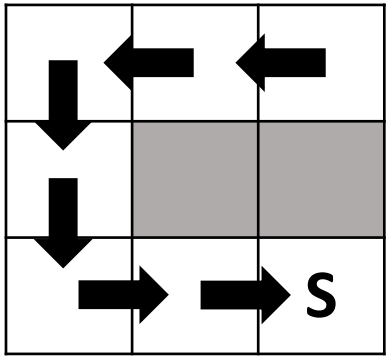


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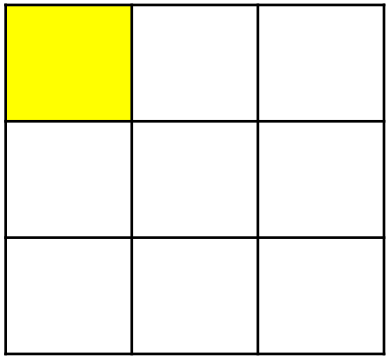

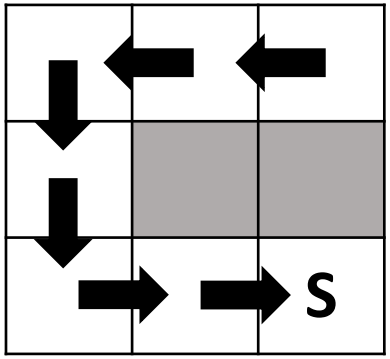


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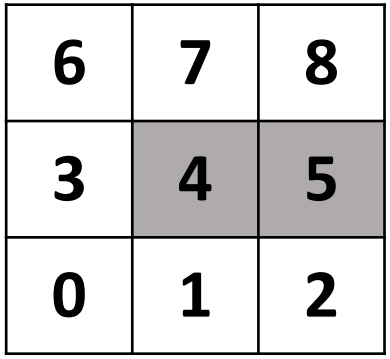


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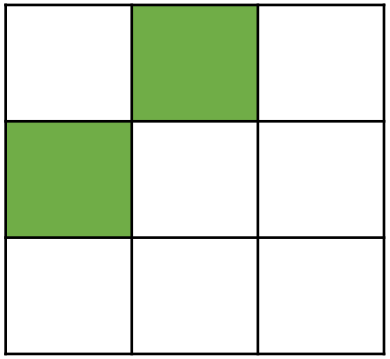

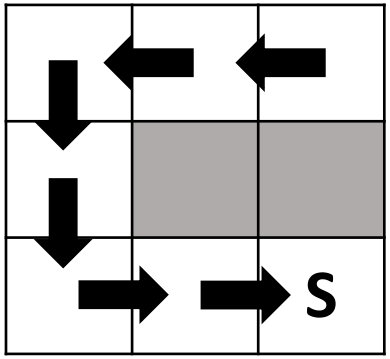

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6	7	8
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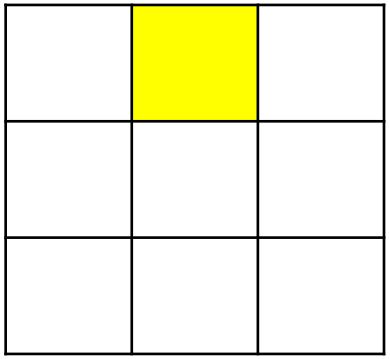
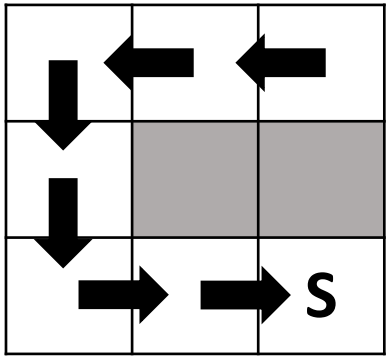
0	1	2

If new path is cheaper than old path, update and add to Q



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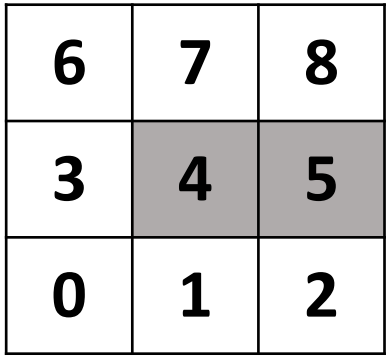
```

Q = [-1, -1, -1, -1, -1, -1, -1, **5**, -1]
dist = [2, 1, 0, 3, inf, inf, 4, **5**, inf]
prev = [1, 2, -1, 0, -1, -1, 3, 6, -1]

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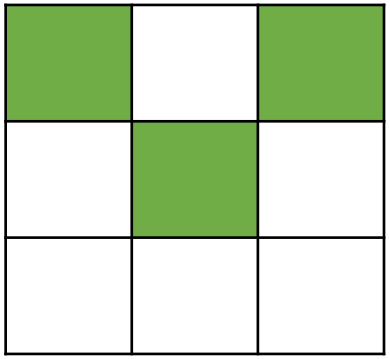
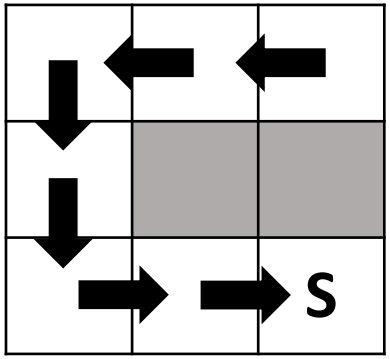
Expand Neighbors





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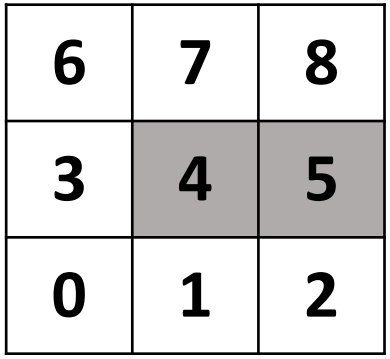

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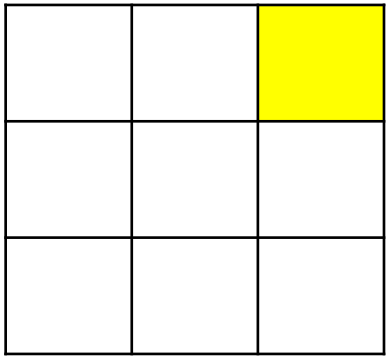

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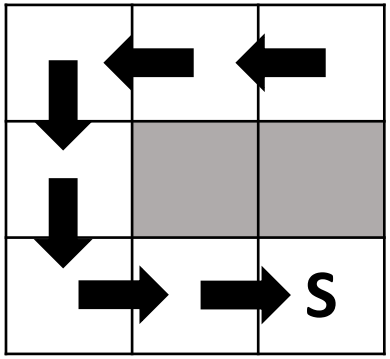

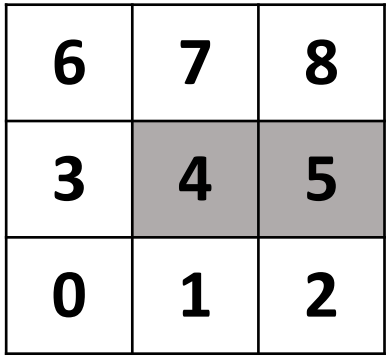

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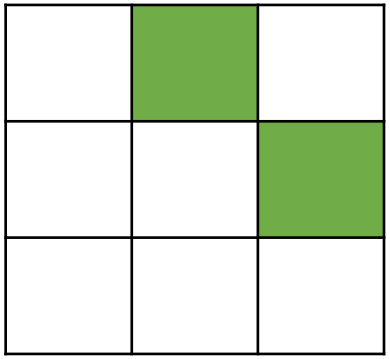

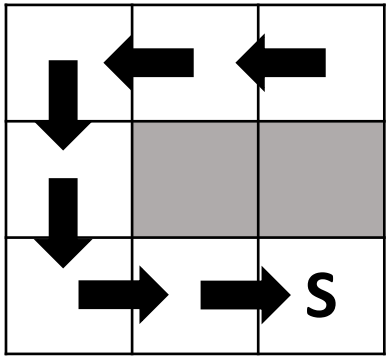


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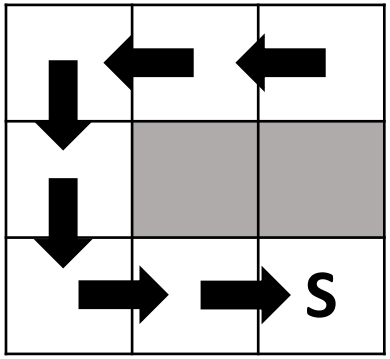
3	4	5
0	1	2

Final Dist costmap

4	5	6
3	∞	∞
2	1	0

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Dijkstra Lab In-Depth

Data Structures

- int dist[num_vertices] → [2, 1, 0, 3, inf, inf, 4, 5, 6]
  - Final shortest-distance/lowest cost relative to source
- int prev[num_vertices] → [1, 2, -1, 0, -1, -1, 3, 6, 7]
  - Previous node on path back to source
- int Q_cost[num_vertices] →
  - Queue of vertices with priority equal to the best-known cost to get to that vertex from source. Used for bookkeeping during Dijkstra function

Important Functions

- Dijkstra(source_vertex)
  - Returns a populated 'prev' array, using source_vertex as origin of all paths
- Reconstruct_path(prev, vertex)
  - Iteratively follows prev[vertex] back to the source and returns a list of vertices on the (shortest) path
- get_travel_cost(source_vertex, dest_vertex)
  - Returns the cost (distance) of traveling between two vertices. 1 if neighbors, BIG_NUMBER if not.

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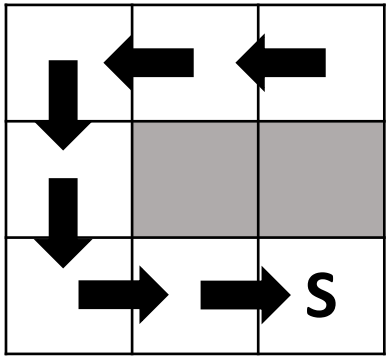
3	4	5
0	1	2

Final Dist costmap

4	5	6
3	∞	∞
2	1	0

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```
1  S ← empty sequence
2  u ← target
3  while prev[u] is defined:                  // Construct the shortest path with a stack S
4      insert u at the beginning of S         // Push the vertex onto the stack
5      u ← prev[u]                            // Traverse from target to source
6  insert u at the beginning of S             // Push the source onto the stack
```