CSCI 3302
Introduction to Robotics

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Inverse Kinematics

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Inverse Kinematics of Mobile Robots

$$\begin{pmatrix} \dot{x}_I \\ \dot{y}_I \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{r\dot{\phi}_l}{2} + \frac{r\dot{\phi}_r}{2} \\ 0 \\ \frac{\dot{\phi}_r r}{d} - \frac{\dot{\phi}_l r}{d} \end{pmatrix}$$

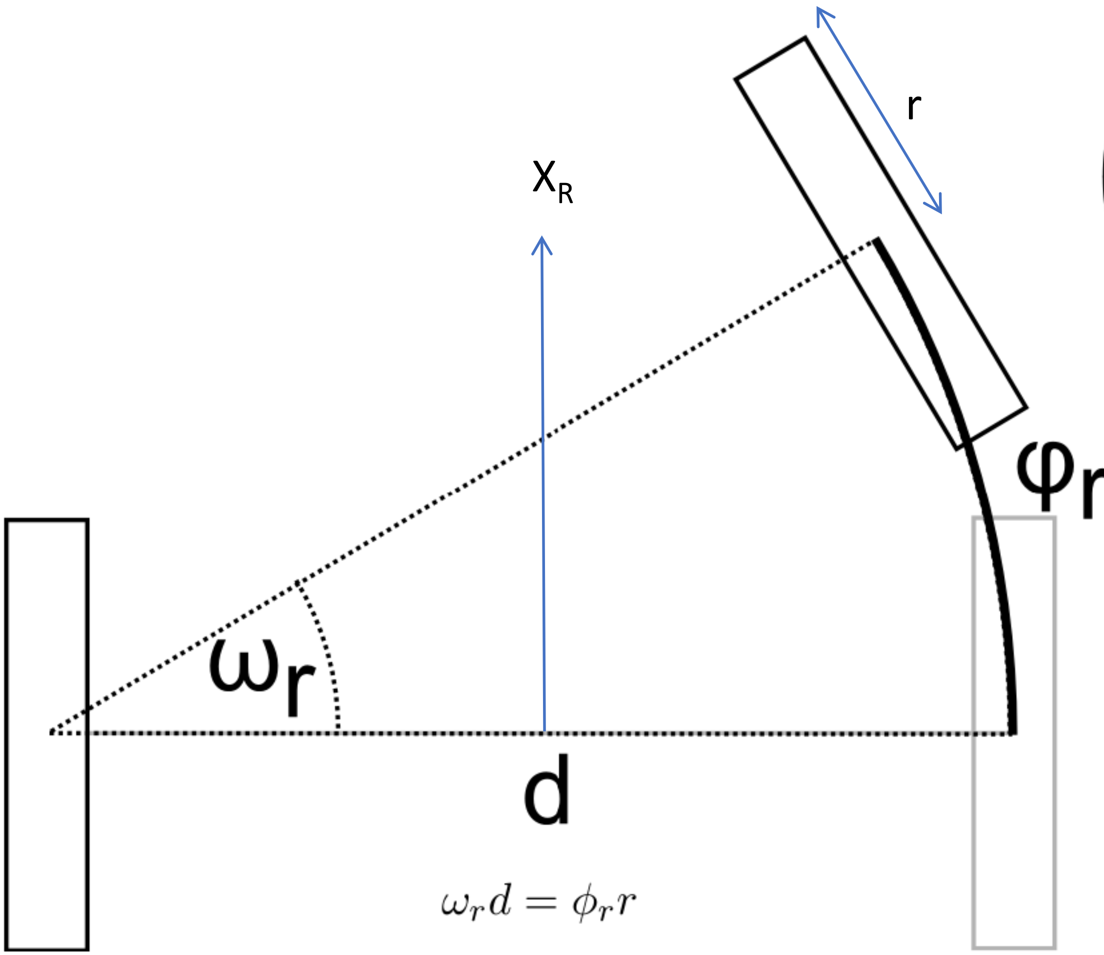
$$\dot{\xi}_I = T(\theta)\dot{\xi}_R$$

$$T^{-1}(\theta)\dot{\xi}_I = T^{-1}(\theta)T(\theta)\dot{\xi}_R$$

$$T^{-1} = \begin{pmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\dot{\xi}_R = T^{-1}(\theta)\dot{\xi}_I$$

Recall: Kinematics of a Mobile Robot



$$\begin{pmatrix} \dot{x_R} \\ \dot{y_R} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \frac{r\dot{\phi_l}}{2} + \frac{r\dot{\phi_r}}{2} \\ 0 \\ \frac{\dot{\phi_r} r}{d} - \frac{\dot{\phi_l} r}{d} \end{pmatrix}$$

$$\omega_r d = \phi_r r$$

Inverse Kinematics of (Non-Holonomic) Mobile Robots

Forward Kinematics Equations

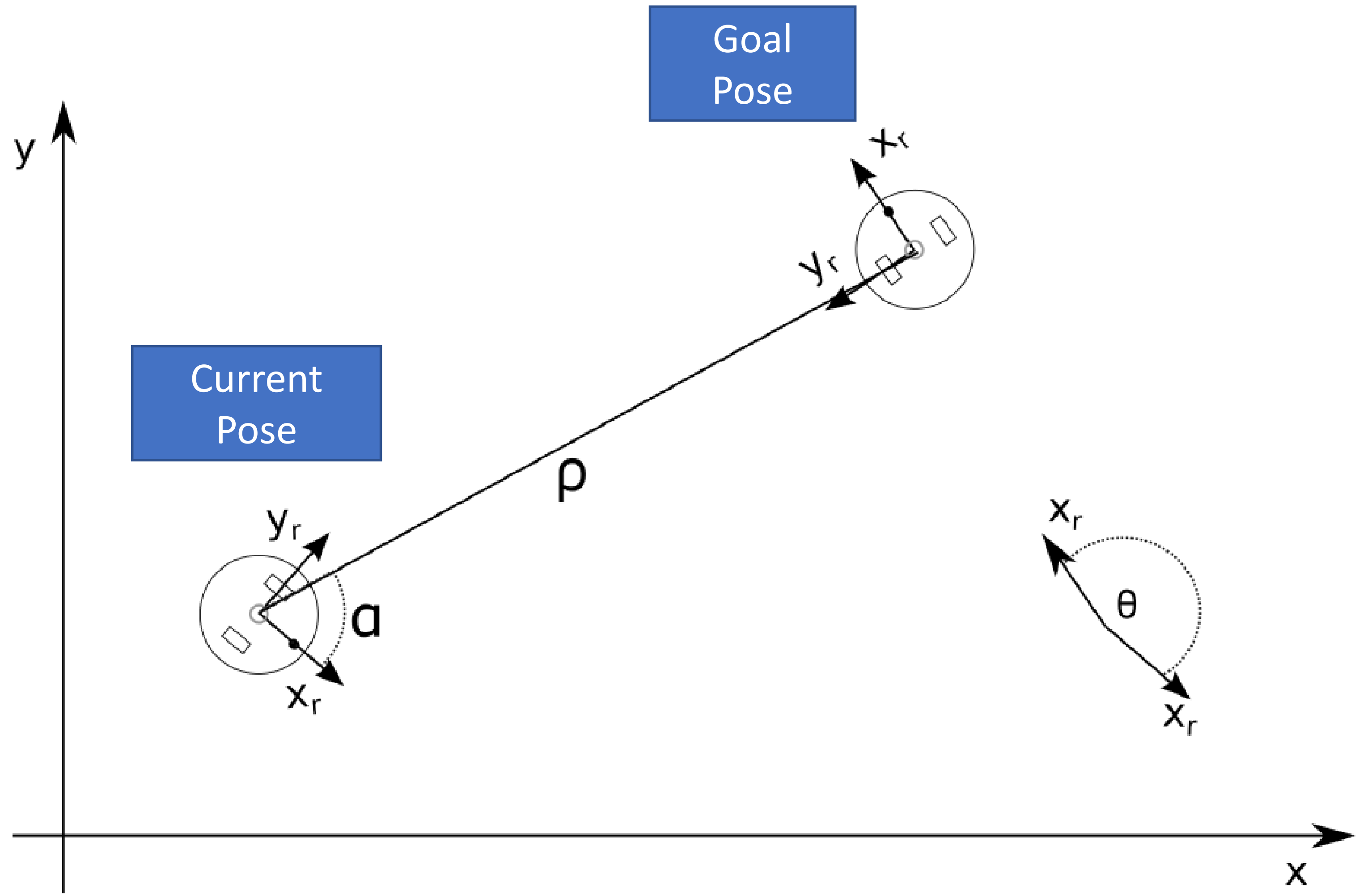
$$\begin{pmatrix} \dot{x_R} \\ \dot{y_R} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \frac{r\dot{\phi_l}}{2} + \frac{r\dot{\phi_r}}{2} \\ 0 \\ \frac{\dot{\phi_r} r}{d} - \frac{\dot{\phi_l} r}{d} \end{pmatrix} \longrightarrow$$

Inverse Kinematics Equations

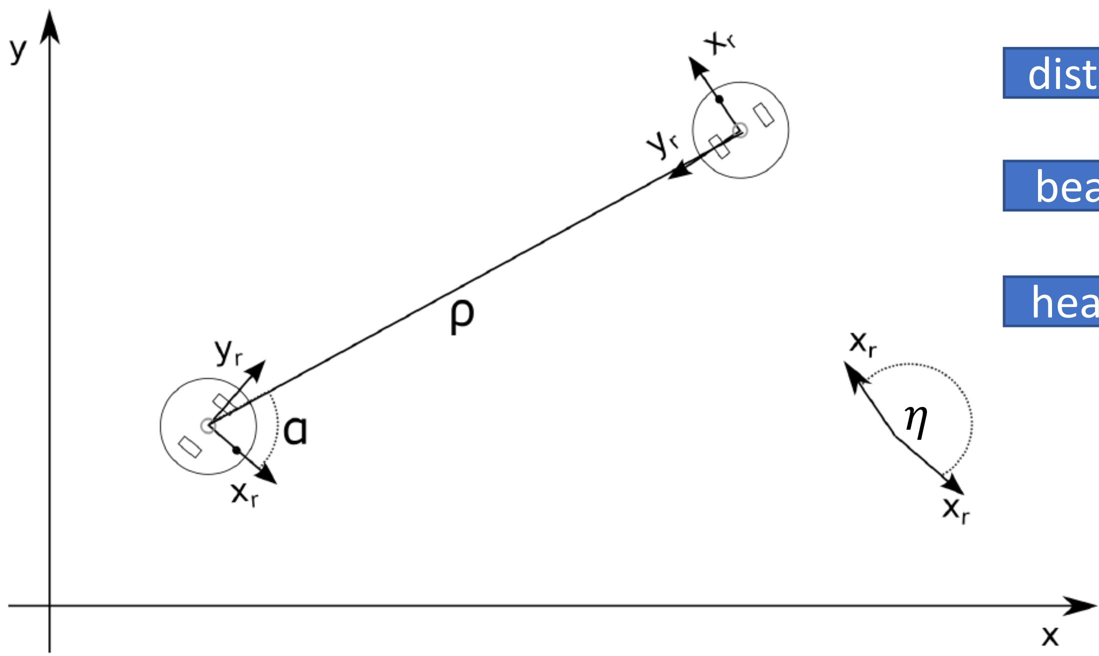
$$\dot{\phi}_L = \frac{\dot{X}_R - \frac{\dot{\theta} d}{2}}{r}$$

$$\dot{\phi}_R = \frac{\dot{X}_R + \frac{\dot{\theta} d}{2}}{r}$$

Position Change Using Feedback Control



Position Change Using Feedback Control



Error Terms

distance $\rho = \sqrt{(x_r - x_g)^2 + (y_r - y_g)^2}$

bearing $\alpha = \tan^{-1}\left(\frac{y_g - y_r}{x_g - x_r}\right) - \theta_r$

heading $\eta = \theta_g - \theta_r$

Note about (3.65):

Uses different rotation axis

(Uses Z going into the plane,

not out of the plane like we use)

Update Rules

translation $\dot{x} = p_1 \rho$

rotation $\dot{\theta} = p_2 \alpha + p_3 \eta$

p_1, p_2, p_3 are controller gains

Todays's Lab: IK on Mobile Robot!

Implement a feedback controller to allow the

robot to navigate to a given pose.

(Will use sparki.motorRotate(·,·) instead of sparki.move*() functions)

Inverse Kinematics Lab

First challenge

Derive the Inverse Kinematics for Sparki
and the Position/Bearing/Heading Error equations

Second challenge

Implement a three command IK solver to move your robot to a desired pose

Third challenge

Implement a feedback controller to continuously move your robot to a desired pose

Inverse Kinematics Lab Challenges

Three Problems to overcome:

- ϕ_L' and ϕ_R' saturate over low Δ_t
- $X_R' \gg \theta'$ causes angle to be ignored
- α and β both influence θ'

$$\alpha = \tan^{-1}\left(\frac{y_g - y_r}{x_g - x_r}\right) - \theta_r$$

Assumes axis coming out of the table (correct)

$$\alpha = \theta_r - \tan^{-1}\frac{y_r - y_g}{x_r - x_g}$$

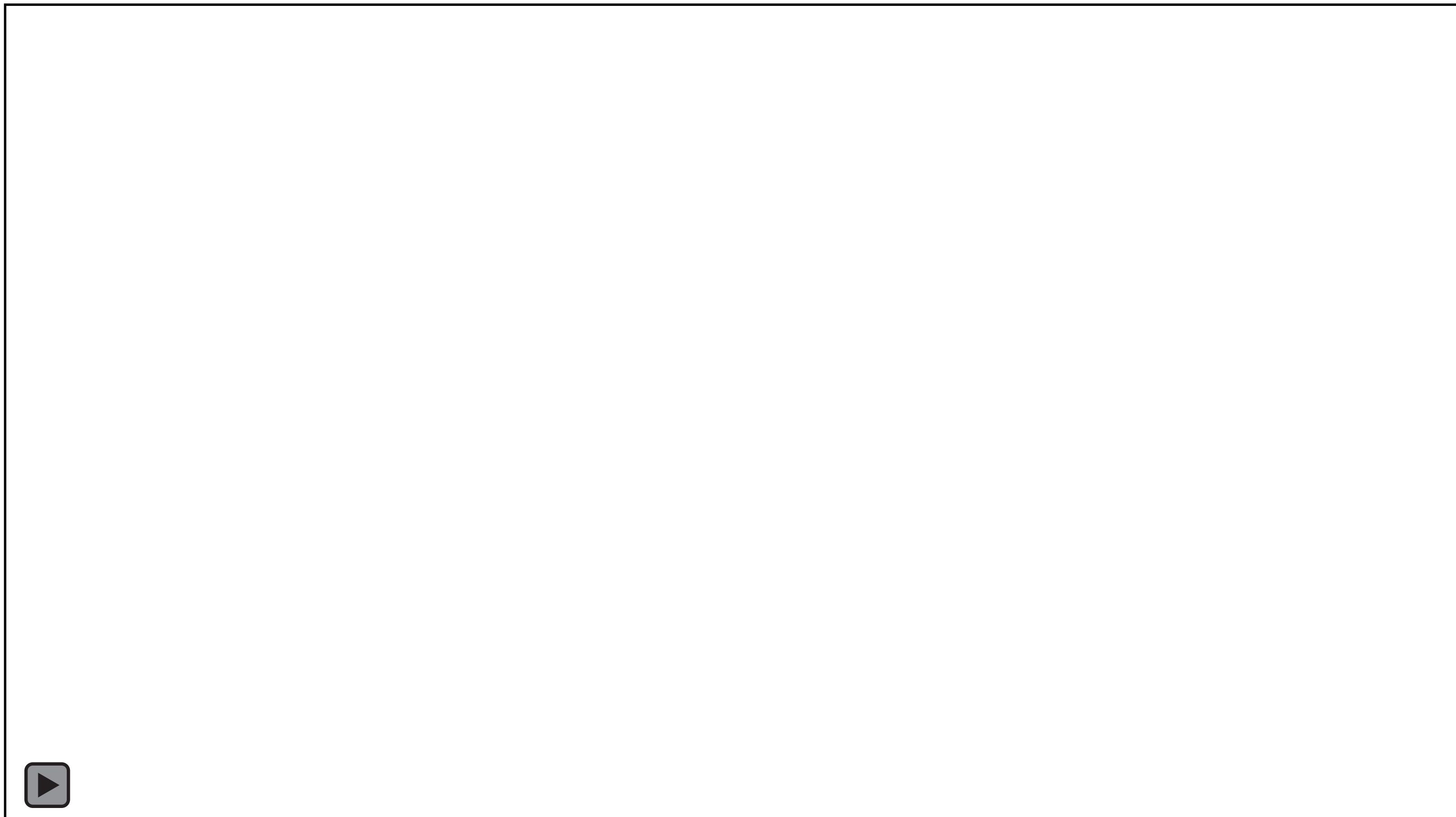
From the book: Assumes axis going into the table (incorrect)

Inverse Kinematics Equations

$$\phi_L = \frac{\dot{X}_R - \frac{\dot{\theta} d}{2}}{r}$$

$$\phi_R = \frac{\dot{X}_R + \frac{\dot{\theta} d}{2}}{r}$$

Where do we go from here?



Summary for IK Lab:

- Calculate suitable velocities that drive the robot toward your goal

$$\begin{aligned} \dot{x} &= p_1\rho \\ \dot{\theta} &= p_2\alpha + p_3\eta \end{aligned}$$

- Calculate the necessary wheel-speed

$$\begin{aligned} \dot{\phi}_l &= (2\dot{x}_R - \dot{\theta}d)/2r \\ \dot{\phi}_r &= (2\dot{x}_R + \dot{\theta}d)/2r \end{aligned}$$

Problems

- How to deal with obstacles?
- How to find short(est) paths?