Assigning Boy Scouts to Patrols

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MATH 480, Community Project

Problem Description

- Scoutmaster Gene Bruce of Troop 407 in Kent, Washington must assign new boy scouts into their patrols. In these patrols, the scouts engage in numerous activities and form close bonds.

- The number of incoming scouts are increasing requiring multiple patrols to better organize activities and to form a small community between scouts. The younger scouts are temperamental and awkward when dealing with adversity, ruining their experience and causing many to quit.

- **Goal: Assign 13 boy scouts of appropriate size (of 6-8 scouts) to two patrols such that we maximize the retention rate of the scouts by avoiding severe conflicts and maximizing positive relationships between scouts.**

Simplifications

- Instead of considering the scouts' complex personality and their interactions with other scouts, we simplified by only asking each scout which other scouts they liked or disliked.

- Quality of the a patrol can be determined by the quality of the relationships between the unique pairs of the group.
  i.e. the whole is equal to the sum of its parts.

Acknowledgments

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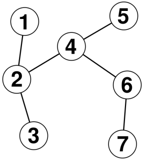
A Mathematical Interpretation - Let's Use Graphs!

Definition

A **graph** consists of a group of objects called vertices where some of the vertices are linked together. When two vertices are linked, we say that there is an **edge** between them.

Implied by the name, graphs have a very visual interpretation, with the following being a typical example of what one would look like:



The numbered circles represent vertices and the lines between them represent edges. We can model a wide variety of real-world things with graphs, namely any collection of objects where clear links can be drawn between pairs that share a relationship

A Mathematical Interpretation cont.

To give a mathematical structure to the patrol problem, we will use what is called a weighted directed graph:

Definition

A graph is **directed** if edges only represent one way relationships from one vertex to another

Definition

A graph is **weighted** if a numerical value can be assigned to each edge

With these definitions, we can define a graph where each scout is a vertex, and a directed edge from scout A to scout B is defined with weight 1 if scout A likes scout B, 0 if scout A is indifferent to B, and -1 if scout A dislikes scout B.

A Mathematical Interpretation cont.

We have our graph, but how can we use it to divide the scouts into patrols? We can split up the vertices into groups that don't overlap, where this splitting is called a **partition**.

Each group would correspond to a potential patrol.

We can imagine that there are many different possible partitions for any graph, the question is about which is "best".

If we can define some numerical measure of a partition that is best, we can try to **optimize** that measure of goodness, which manifests as minimizing or maximizing some function dependent on the partition, where we choose to maximize or minimize depending on the measure

The Objective Functions

1. **MinCut**. This is the traditional metric of a cut on a weighted graph and is the sum of the weights of the edges we would "cut" if we were to sever the edges between distinct patrols. More formally, for some partition I, J $I \neq J$, the MinCut is defined as:

$$MinCut(I, J) := \sum_{e=(i,j), i \in I, j \in J} c(e)$$

   We want this to be as low as possible.

2. **FriendCut**. This is the sum of the positive, or friendly edge weights within each patrol. That is, if $I_+ = \{e = (i,j) | i, j \in I, c(e) = 1\}$ is the the set of edges in patrol I with positive weight and $J_+ = \{e = (i,j) | i, j \in J, c(e) = 1\}$ is the set of edges in patrol J with positive weight, then (assuming all positive weights are 1):

$$FriendCut(I, J) := |I_+ \cup J_+|$$

   We want this to be as high as possible.

3. **EnemyCut**. This is similar to FriendCut, except we now measure the number of negative edges going from one scout to another within the same patrol. Let $I_- = \{e = (i,j) | i, j \in I, c(e) = -1\}$ be the the set of edges in patrol I with negative weight and $J_- = \{e = (i,j) | i, j \in J, c(e) = -1\}$ be the set of edges in patrol J with negative weight, then

$$EnemyCut(I, J) := |I_- \cup J_-|$$

   We want this to be as low as possible.

4. **AwkwardCut (+i)**. Same in all respects to MinCut, except for an additional penalty of i if there are two scouts, Scout A and Scout B, in the same patrol such that Scout A likes Scout B and Scout B dislikes Scout A.

$$AwkwardCut(I,J,i) := \left[\sum_{e=(k,j),k\in I,j\in J} c(e)\right] + i * |\{e_+ = (k,j), e_- = (j,k) : e_+, e_- \in I, c(e_+) = 1, c(e_-) = -1\}|$$

5. **HybridCut**. A convex combination of MinCut and FriendCut, each with equal weight (that is, half the MinCut loss plus half the FriendCut loss).

$$HybridCut(I,J) := \frac{1}{2}MinCut(I,J) + \frac{1}{2}FriendCut(I,J)$$

Our Solution

MinCut		FriendCut		EnemyCut	
Patrol 1	Patrol 2	Patrol 1	Patrol 2	Patrol 1	Patrol 2
Brandon	Christian	Brandon	Daniel	Brandon	Christian
Cameron	Daniel	Cameron	Darwin	Cameron	Jake
Colby	Jake	Christian	Jake	Colby	Jordan
Darwin	Jordan	Colby	Jordan	Daniel	Nathan
Evan	Nathan	Evan	Nathan	Darwin	Patrick
Tommy	Patrick	Tommy	Patrick	Evan	Timmy
	Timmy		Timmy	Tommy	

AwkwardCut (+1)		AwkwardCut (+2)		HybridCut	
Patrol 1	Patrol 2	Patrol 1	Patrol 2	Patrol 1	Patrol 2
Brandon	Christian	Brandon	Cameron	Brandon	Christian
Cameron	Daniel	Christian	Evan	Cameron	Daniel
Colby	Jake	Daniel	Evan	Colby	Darwin
Darwin	Jordan	Darwin	Nathan	Evan	Jake
Evan	Nathan	Jake	Timmy	Timmy	Jordan
Tommy	Patrick	Jordan	Tommy	Tommy	Nathan
	Timmy	Patrick			Patrick

Table: The partitions arrived at by minimizing each objective function

How Good Are These Solutions?

Method	MinCut	FriendCut	EnemyCut
MinCut	0	17	0
FriendCut	1	18	1
EnemyCut	2	15	0
AwkwardCut (+1)	0	17	0
AwkwardCut (+2)	1	18	2
HybridCut	0	18	1

Figure: The scores according to the three primary metrics.

Blocks

Block Title

You can also highlight sections of your presentation in a block, with it's own title

Theorem

There are separate environments for theorems, examples, definitions and proofs.

Example

Here is an example of an example block.

Summary

- The first main message of your talk in one or two lines.
- The second main message of your talk in one or two lines.
- Perhaps a third message, but not more than that.

- Outlook
  - Something you haven't solved.
  - Something else you haven't solved.

For Further Reading I

A. Author.
Handbook of Everything.
Some Press, 1990.

S. Someone.
On this and that.
Journal of This and That, 2(1):50–100, 2000.