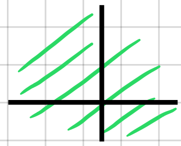
3/4/2025 FUNC VARIAS VARIABLES; CONTINUIDAD; Derivabilidad; Diferenciabilidad. Superficies

⇒ CONJUNTOS DE NIVEL

* Ejercicio 1 → Det dom f, graf dom f analizar tipo de conjunto. Conj nivel f y gráf N_k

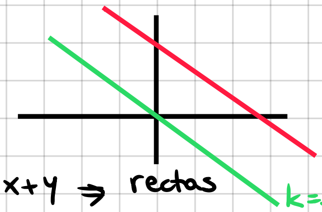
ⓐ $f(x,y) = 3\left(1-\frac{x}{2}-\frac{y}{2}\right)$



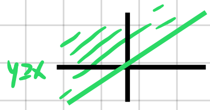
- $\text{Dom } f = \{(x,y) \in \mathbb{R}^2\}$ → conjunto abierto no acotado (todo $\mathbb{R}^2$)
- Conjunto de nivel k

$$N_k = \left\{(x,y) \in \mathbb{R} \;/\; 3\left(1-\frac{x+y}{2}\right) = k,\ k \in \mathbb{R}\right\}$$

$$3\left(1-\frac{x+y}{2}\right) = k \Rightarrow 1-\frac{k}{3} = \frac{x+y}{2} \Rightarrow 2-\frac{2k}{3} = x+y \Rightarrow \text{rectas}$$

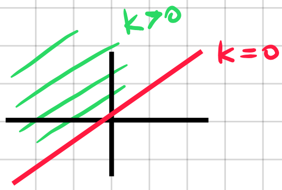


ⓑ $f(x,y) = \sqrt{y-x}$



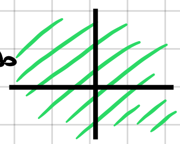
- Hallar dom $\quad y-x \geq 0 \Rightarrow y \geq x \qquad \text{Dom } f = \{(x,y) \in \mathbb{R}^2 / y \geq x\}$ → cerrado, no acotado
  $\nexists B(r, A)$ con $r \in \mathbb{R}$
- Hallar conj Nivel N_k

$$N_k = \{(x,y) \in D \;/\; \sqrt{y-x} = k\}$$

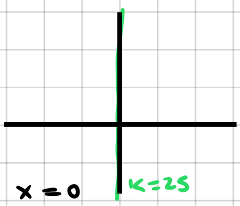


$\sqrt{y-x} = k$
- → $k=0 \Rightarrow \sqrt{y-x} = 0 \Rightarrow \boxed{y=x}$ recta identidad
- → $k>0 \Rightarrow \sqrt{\underbrace{y-x}_{\geq 0}} = k \Rightarrow \boxed{y = k^2+x}$ rectas pendiente $m=1$ ord origen $b=k^2$
- → $k<0 \Rightarrow \nexists\ \sqrt{y-x} = k \quad \phi$

ⓒ $f(x,y) = 25-x^2$



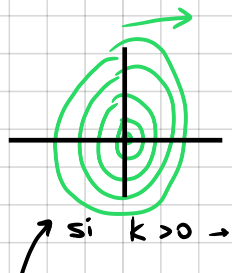
- $\text{Dom}(f) = \{(x,y) \in \mathbb{R}^2\}$
- $\text{Conj Nivel}_k = \{(x,y) \in \mathbb{R}^2 / 25-x^2 = k\}$



si $k=25 \longrightarrow x=0$
si $k<25 \longrightarrow |x| = \sqrt{25-k} \longrightarrow$ rectas verticales $(|x|, y)$ en $\mathbb{R}^2$
$k>25 \rightarrow \phi$

$$25-x^2 = k \Rightarrow \underbrace{x^2}_{\geq 0} = \underbrace{25-k}_{\geq 0} \qquad \boxed{\begin{matrix} 25-k \geq 0 \\ k \leq 25 \end{matrix}}$$

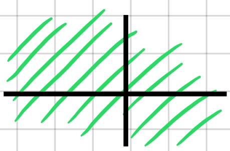
$$N_k = \{(x,y) \in \mathbb{R}^2 / 25-x^2=k,\ k \leq 25\}$$



ⓓ $f(x,y) = 9x^2+4y^2$

- $\text{Dom}(f): \{(x,y) \in \mathbb{R}^2\}$
- Conjuntos de nivel $N_k = \{(x,y) \in \mathbb{R}^2 / 9x^2+4y^2 = k\}$

si $k>0 \rightarrow$ elipse
si $k=0 \rightarrow \phi$
si $k<0 \rightarrow \phi$



$9x^2+4y^2 = k$

$$\frac{9x^2}{k} + \frac{4y^2}{k} = 1 \Rightarrow \underbrace{\frac{x^2}{\left(\frac{k}{9}\right)} + \frac{y^2}{\left(\frac{k}{4}\right)}}_{\boxed{k>0}} = 1 \rightarrow \text{elipses} \quad \left.\begin{matrix} a = \sqrt{\frac{k}{9}} \\ b = \sqrt{\frac{k}{4}} \end{matrix}\right\} \rightarrow \boxed{k \geq 0}$$

(e) $f(x,y) = \sqrt{x^2+2y^2}$

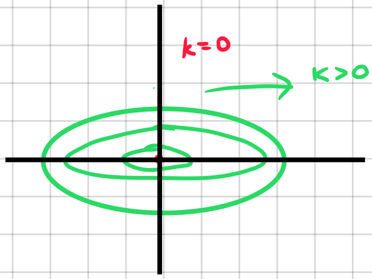
• $\text{Dom}(f) = \{(x,y) \in \mathbb{R}^2\}$

$\underbrace{x^2+2y^2}_{\text{siempre positivo}} \geq 0$

• Conjunt Nivel $N_k = \{(x,y) \in \mathbb{R}^2 \,/\, \sqrt{x^2+2y^2} = k\}$

$\sqrt{x^2+2y^2} = k \longrightarrow |x^2+2y^2| = k^2 \longrightarrow \underbrace{x^2+2y^2 = k^2}_{\text{elipse}}$

$\longrightarrow \dfrac{x^2}{(k^2)} + \dfrac{y^2}{\left(\frac{k^2}{2}\right)} = 1$, donde $k > 0$



$$\begin{cases} \text{si } k>0 \longrightarrow \dfrac{x^2}{k^2} + \dfrac{y^2}{\frac{k^2}{2}} = 1 \longrightarrow \text{elipses} \\ \text{si } k=0 \longrightarrow \sqrt{x^2+2y^2} = k^2 \longrightarrow (0,0) \\ \text{si } k<0 \longrightarrow \phi \end{cases}$$

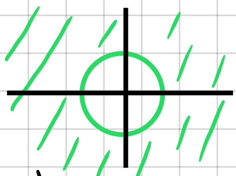
(f) $f(x,y) = (x^2+y^2-16)^{-\frac{1}{2}} = \dfrac{1}{\sqrt{(x^2+y^2-16)}}$

$\text{Dom}(f) \rightarrow \{(x,y) \in \mathbb{R}^2 \,/\, x^2+y^2 > 16\}$

$\downarrow$

$x^2+y^2-16 > 0$

$x^2+y^2 > 16$



conjunto abierto, no acotado

• $N_k = \{(x,y) \in D \,/\, \dfrac{1}{\sqrt{x^2+y^2-16}} = k\}$

$\dfrac{1}{\sqrt{x^2+y^2-16}} = k$

$\underbrace{k}_{\neq 0} \underbrace{\sqrt{x^2+y^2-16}}_{>0 \text{ En Dom}(f)} = 1 \longrightarrow x^2+y^2-16 = \dfrac{1}{k}$

$\longrightarrow \underbrace{x^2+y^2}_{>0} = \underbrace{\dfrac{1}{k} + 16}_{>0}$

Hallar k para $\frac{1}{k} + 16 \geq 0$

$\frac{1}{k} + 16 > 0 \qquad -16k < 1$

$\frac{1}{k} > -16 \qquad k > \frac{1}{-16}$

• Niveles disponibles (k)

si $k = 0 \longrightarrow \phi$

si $k > \frac{-1}{16} \wedge k \neq 0 \rightarrow$ circunferencias

si $k \leq \frac{-1}{16} \longrightarrow \phi$

(g) $f(x,y) = e^{-x^2-y^2} = e^{-(x^2+y^2)} = \dfrac{1}{e^{x^2+y^2}}$

$\text{Dom}(f) = \{(x,y) \in \mathbb{R}^2\}$

$\downarrow$

$\underbrace{e^{\overbrace{x^2+y^2}^{\geq 0}}}_{\geq 0}$

función exponencial nunca negativo

• Hallar conjunto Nivel : $\{(x,y) \in \mathbb{R}^2 \,/\, e^{-x^2-y^2} = k\}$

$\dfrac{1}{e^{x^2+y^2}} = k \longrightarrow e^{x^2+y^2} = \dfrac{1}{k}$

$\underbrace{x^2+y^2 = \ln\left(\frac{1}{k}\right)}_{\text{circunferencias}}$, $\underbrace{k \neq 0 \wedge \frac{1}{k} > 0}_{k>0}$

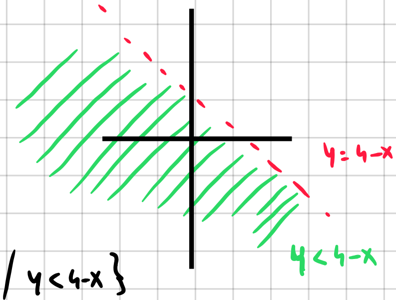
$$\begin{cases} \text{si } k = 1 \longrightarrow x^2+y^2 = 0 \longrightarrow (0,0) \\ \text{si } k>0 \wedge k \neq 1 \longrightarrow x^2+y^2 = \ln\left(\frac{1}{k}\right) \longrightarrow \text{circunferencias radio } \sqrt{\ln\left(\frac{1}{k}\right)} \\ \text{si } k<0 \longrightarrow \phi \end{cases}$$

(h) $f(x,y,z) = \ln(4-x-y)$

- Hallar dominio

$4-x-y > 0$

$4-x > y$



$Dom(f) = \{(x,y,z) \in \mathbb{R}^3 / y < 4-x\}$

→ conj abierto → no incluye frontera $4-x=y$

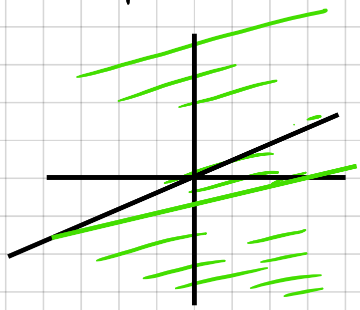
→ no acotado

- Hallar conjunto nivel $N_k = \{(x,y,z) \in \mathbb{R}^3 / \ln(4-x-y) = k\}$

$\ln(4-x-y) = k$

$4-x-y = e^k$

$\underbrace{4-e^k-x = y}_{\text{plano en } \mathbb{R}^3}$



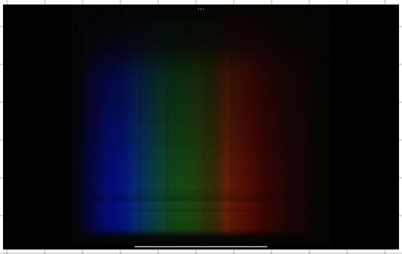
(I) $f(x,y,z) = x+y+2z$

$Dom(f) = \mathbb{R}^3$

- $N_k = \{(x,y,z) \in \mathbb{R}^3 / \underbrace{x+y+2z = k}_{\text{plano}}\}$

(J) $f(x,y,z) = e^{x^2+y^2-z^2}$

$Dom(f) = \{\underbrace{(x,y,z) \in \mathbb{R}^3}\}$
función exponencial continua en $\mathbb{R}^3$



- $N_k = \{(x,y,z) \in \mathbb{R}^3 / e^{x^2+y^2-z} = k\}$

$e^{x^2+y^2+2z} = k$

$x^2+y^2+2z = \underbrace{\ln(k)}_{\in \mathbb{R}}$ si $k \leq 0 \longrightarrow N_k = \emptyset$

$\frac{x^2+y^2-\ln(k)}{-2} = z \Rightarrow \underbrace{z = \frac{-x^2}{2} - \frac{y^2}{2} + \ln(k)}_{(?)}$