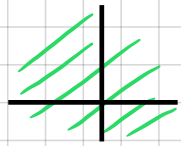
3/4/2025 FUNC VARIAS VARIABLES ; CONTINUIDAD ; Derivabilidad ; Diferenciabilidad . Superficies

⇒ CONJUNTOS DE NIVEL

* Ejercicio 1 → Det dom f , graf dom f analizar tipo de conjunto, Conj nivel f y gráf N_k

ⓐ $f(x,y) = 3\left(1 - \frac{x}{2} - \frac{y}{2}\right)$

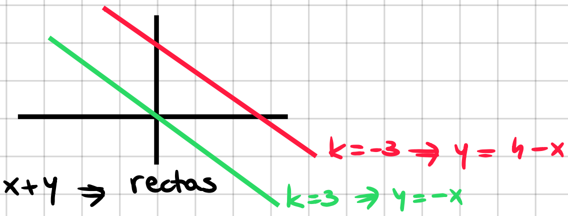


• $\text{Dom} f = \{(x,y) \in \mathbb{R}^2\}$ → conjunto abierto no acotado (todo $\mathbb{R}^2$)

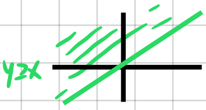
• Conjunto de nivel k

$$N_k = \left\{(x,y) \in \mathbb{R} \,/\, 3\left(1 - \frac{x+y}{2}\right) = k,\ k \in \mathbb{R}\right\}$$

$$3\left(1 - \frac{x+y}{2}\right) = k \Rightarrow 1 - \frac{k}{3} = \frac{x+y}{2} \Rightarrow 2 - \frac{2k}{3} = x + y \Rightarrow \text{rectas}$$



ⓑ $f(x,y) = \sqrt{y - x}$



• Hallar dom $\quad y - x \geq 0 \Rightarrow y \geq x \qquad \text{Dom} f = \{(x,y) \in \mathbb{R}^2 / y \geq x\}$ → cerrado , no acotado

$\nexists B(r, A)$ con $r \in \mathbb{R}$

• Hallar conj Nivel N_k

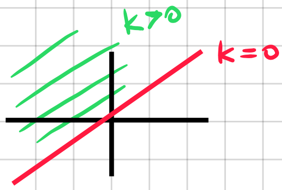
$$N_k = \{(x,y) \in D \,/\, \sqrt{y-x} = k\}$$

$\sqrt{y-x} = k$

→ $k = 0 \Rightarrow \sqrt{y-x} = 0 \Rightarrow \boxed{y = x}$ recta identidad

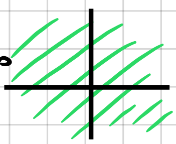
→ $k > 0 \Rightarrow \underbrace{\sqrt{y-x}}_{\geq 0} = k \Rightarrow \boxed{y = k^2 + x}$ rectas pendiente $m=1$ ord origen $b = k^2$

→ $k < 0 \Rightarrow \nexists \sqrt{y-x} = k \quad \phi$



ⓒ $f(x,y) = 25 - x^2$

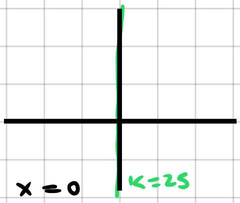
conj abierto no acotado



• $\text{Dom}(f) = \{(x,y) \in \mathbb{R}^2\}$

• $\text{Conj Nivel}_k = \{(x,y) \in \mathbb{R}^2 / 25 - x^2 = k\}$

$$25 - x^2 = k \Rightarrow \underbrace{x^2}_{\geq 0} = \underbrace{25 - k}_{\geq 0} \qquad \boxed{\begin{matrix} 25 - k \geq 0 \\ k \leq 25 \end{matrix}}$$

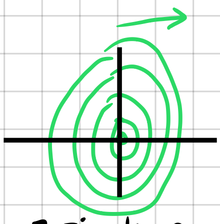


si $k = 25 \longrightarrow x = 0$

si $k < 25 \longrightarrow |x| = \sqrt{25 - k}$ → rectas verticales $(|x|, y)$ en $\mathbb{R}^2$

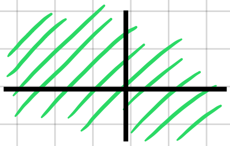
$k > 25 \rightarrow \phi$

$$N_k = \{(x,y) \in \mathbb{R}^2 / 25 - x^2 = k,\ k \leq 25\}$$



ⓓ $f(x,y) = 9x^2 + 4y^2$

• $\text{Dom}(f) : \{(x,y) \in \mathbb{R}^2\}$



conj abierto no acotado

• Conjuntos de nivel $N_k = \{(x,y) \in \mathbb{R}^2 / 9x^2 + 4y^2 = k\}$

si $k > 0$ → elipse

si $k = 0 \rightarrow \phi$

si $k < 0 \rightarrow \phi$

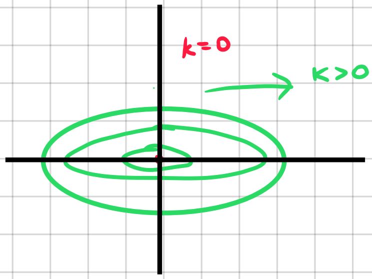
$$9x^2 + 4y^2 = k$$

$$\frac{9x^2}{k} + \frac{4y^2}{k} = 1 \Rightarrow \underbrace{\frac{x^2}{\left(\frac{k}{9}\right)} + \frac{y^2}{\left(\frac{k}{4}\right)}}_{\boxed{k > 0}} = 1 \rightarrow \text{elipses} \quad \left.\begin{matrix} a = \sqrt{\frac{k}{9}} \\ b = \sqrt{\frac{k}{4}} \end{matrix}\right\} \rightarrow \boxed{k \geq 0}$$

(e) $f(x,y) = \sqrt{x^2+2y^2}$

• $\text{Dom}(f) = \{(x,y) \in \mathbb{R}^2\}$

$\underbrace{x^2+2y^2}_{\text{siempre positivo}} \geq 0$



• Conjunt Nivel $N_k = \{(x,y) \in \mathbb{R}^2 / \sqrt{x^2+2y^2} = k\}$

$\sqrt{x^2+2y^2} = k \longrightarrow |x^2+2y^2| = k^2 \longrightarrow \underbrace{x^2+2y^2 = k^2}_{\text{elipse}}$

$\longrightarrow \dfrac{x^2}{(k^2)} + \dfrac{y^2}{\left(\frac{k^2}{2}\right)} = 1$, donde $k > 0$

$$\begin{cases} \text{si } k > 0 \longrightarrow \dfrac{x^2}{k^2} + \dfrac{y^2}{\frac{k^2}{2}} = 1 \longrightarrow \text{elipses} \\ \text{si } k = 0 \longrightarrow \sqrt{x^2+2y^2} = k^2 \longrightarrow (0,0) \\ \text{si } k < 0 \longrightarrow \phi \end{cases}$$

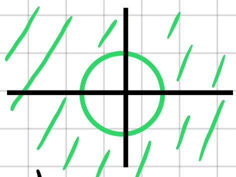
(f) $f(x,y) = (x^2+y^2-16)^{-\frac{1}{2}} = \dfrac{1}{\sqrt{(x^2+y^2-16)}}$

$\text{Dom}(f) \rightarrow \{(x,y) \in \mathbb{R}^2 / x^2+y^2 > 16\}$

$\downarrow$

$x^2+y^2-16 > 0$

$x^2+y^2 > 16$



conjunto abierto, no acotado

• $N_k = \{(x,y) \in D / \dfrac{1}{\sqrt{x^2+y^2-16}} = k\}$

$\dfrac{1}{\sqrt{x^2+y^2-16}} = k$

$\underbrace{k}_{\neq 0} \underbrace{\sqrt{x^2+y^2-16}}_{>0 \text{ En Dom}(f)} = 1 \longrightarrow x^2+y^2-16 = \dfrac{1}{k}$

$\longrightarrow \underbrace{x^2+y^2}_{>0} = \underbrace{\dfrac{1}{k}+16}_{>0}$

• Niveles disponibles (k)

si $k = 0 \longrightarrow \phi$

si $k > \frac{-1}{16} \wedge k \neq 0 \rightarrow$ circunferencias

si $k \leq \frac{-1}{16} \longrightarrow \phi$

Hallar k para $\frac{1}{k} + 16 \geq 0$

$\frac{1}{k} + 16 > 0$ $\quad -16k < 1$

$\frac{1}{k} > -16$ $\quad k > \frac{1}{-16}$

(g) $f(x,y) = e^{-x^2-y^2} = e^{-(x^2+y^2)} = \dfrac{1}{e^{x^2+y^2}}$

$\text{Dom}(f) = \{(x,y) \in \mathbb{R}^2\}$

$\downarrow$

$\underbrace{e^{x^2+y^2}}_{\geq 0}$, $x^2+y^2 \rightarrow \geq 0$

función exponencial nunca negativo

• Hallar conjunto Nivel : $\{(x,y) \in \mathbb{R}^2 / e^{-x^2-y^2} = k\}$

$\dfrac{1}{e^{x^2+y^2}} = k \longrightarrow e^{x^2+y^2} = \dfrac{1}{k}$

$\underbrace{x^2+y^2 = \ln\left(\frac{1}{k}\right)}_{\text{circunferencias}}$, $\underbrace{k \neq 0 \wedge \frac{1}{k} > 0}_{k>0}$

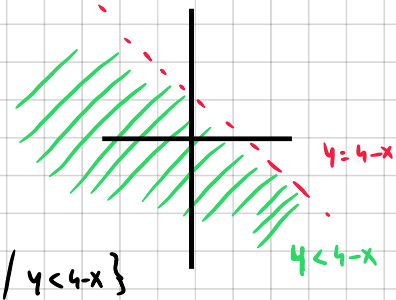
$$\begin{cases} \text{si } k = 1 \longrightarrow x^2+y^2 = 0 \longrightarrow (0,0) \\ \text{si } k > 0 \wedge k \neq 1 \longrightarrow x^2+y^2 = \ln\left(\frac{1}{k}\right) \longrightarrow \text{circunferencias radio } \sqrt{\ln\left(\frac{1}{k}\right)} \\ \text{si } k < 0 \longrightarrow \phi \end{cases}$$

(h) $f(x,y,z) = \ln(4-x-y)$

- Hallar dominio

$4-x-y > 0$

$4-x > y$



$\text{Dom}(f) = \{(x,y,z) \in \mathbb{R}^3 / \ y < 4-x\}$

↳ conj abierto → no incluye frontera $4-x=y$

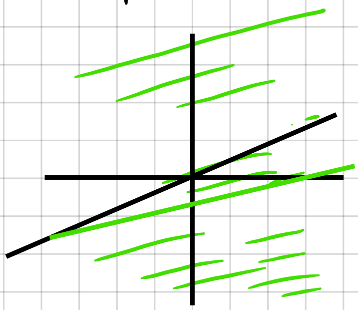
↳ no acotado

- Hallar conjunto nivel $N_k = \{(x,y,z) \in \mathbb{R}^3 / \ \ln(4-x-y) = k\}$

$\ln(4-x-y) = k$

$4-x-y = e^k$

$\underbrace{4-e^k-x = y}_{\text{plano en } \mathbb{R}^3}$



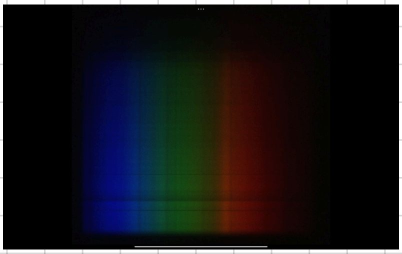
(I) $f(x,y,z) = x+y+2z$

$\text{Dom}(f) = \mathbb{R}^3$

- $N_k = \{(x,y,z) \in \mathbb{R}^3 / \ \underbrace{x+y+2z = k}_{\text{plano}}\}$

(J) $f(x,y,z) = e^{x^2+y^2-z^2}$

$\text{Dom}(f) = \underbrace{\{(x,y,z) \in \mathbb{R}^3\}}_{\text{función exponencial continua en } \mathbb{R}^3}$



- $N_k = \{(x,y,z) \in \mathbb{R}^3 / \ e^{x^2+y^2-z} = k\}$

$e^{x^2+y^2+2z} = k$

$x^2+y^2+2z = \underbrace{\ln(k)}_{\in \mathbb{R}}$ si $k \leq 0 \longrightarrow N_k = \emptyset$

$\dfrac{x^2+y^2-\ln(k)}{-2} = z \Rightarrow \underbrace{z = \dfrac{-x^2}{2} - \dfrac{y^2}{2} + \ln(k)}_{(?)}$

4/4

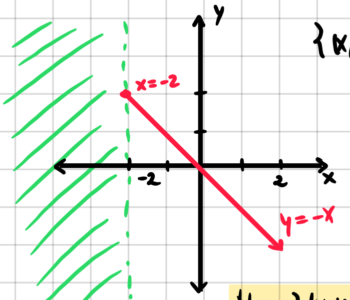
* Ejercicio 2 = Graf, si quedan def, N_0 y N_4

(a) $f(x,y) = \begin{cases} x+y & \text{si } x \geq -2 \\ 0 & \text{si } x < -2 \end{cases}$

- Hallar Dominio $\text{Dom}(f) = \{(x,y) \in \mathbb{R}^2\}$

- Hallar $N_0 \to \{(x,y) \in D / \ f(x,y) = 0\}$

$\{(x,y) \in D / \ x < -2\}$



¿Qué pasa si $x \geq -2$ y $f(x,y) = 0$

$x+y = 0 \Rightarrow y = -x$

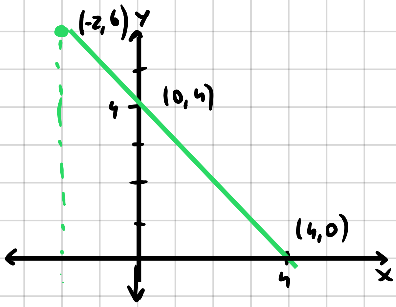
$N_0 = \{(x,y) \in \mathbb{R}^2 / \ x < -2\} \cup \{(x,y) \in D / \ y = -x, \ x \geq -2\}$

- Hallar $N_4 = \{(x,y) \in D / \ f(x,y) = 4\}$

si $x \geq -2$

$x+y = 4 \Rightarrow y = 4-x$ recta

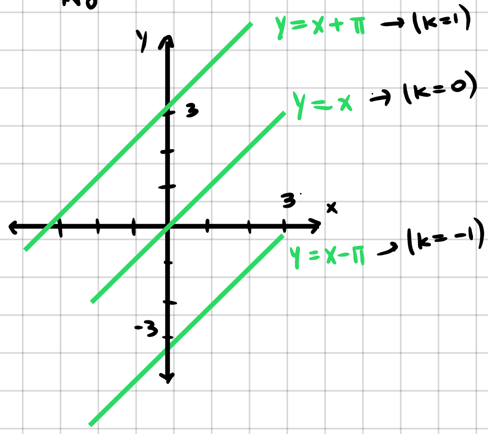
$N_4 = \{(x,y) \in D / \ y = 4-x \ \wedge \ x \geq -2\}$



* (b) $f(x,y) = \text{sen}(y-x) \to$ función periódica $\to f(x,y) \in [0,1]$

- Hallar dominio $\text{Dom} f = \{(x,y) \in \mathbb{R}^2\}$

- Gráfico N_0



- Hallar conjunto Nivel 0

$f(x,y) = \text{sen}(y-x) = 0$

$y - x = \text{arcsen}(0) = \underbrace{\pi k, \; k \in \mathbb{Z}}$ función periódica

$y = x + \pi k \Rightarrow$ rectas en $\mathbb{R}^2$

$N_4 = \phi$

- Hallar $N_4 \to \nexists f(x,y) = 4 \to f(x,y)$ oscila entre $[0,1]$

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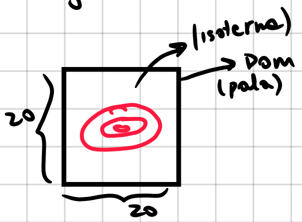
* $D = \{(x,y) \in \mathbb{R}^2 : x \in [-10,10], y \in [-10,10]\} = [-10,10] \times [-10,10] \to$ placa metálica

$T(x,y) = 64 - 4x^2 - 8y^2 \to$ temperatura de punto $(x,y) \in D$

Dibujar "isotermas" $\to$ Conj de puntos con misma temperatura k

- Figura de análisis



- Hallar conjunto de nivel N_k (en general) → Hallar isotermas de Temperatura k

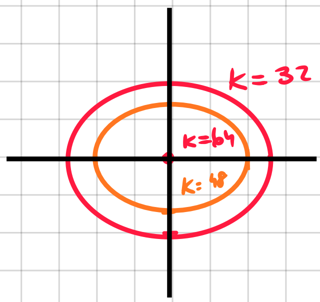
$T(x,y) = 64 - 4x^2 - 8y^2 = k$

$64 - k = 4x^2 + 8y^2 \Rightarrow$ elipses

$1 = \dfrac{4x^2 + 8y^2}{64-k} \Rightarrow 1 = \dfrac{x^2}{\left(\frac{64-k}{4}\right)} + \dfrac{y^2}{\left(\frac{64-k}{8}\right)}$ $\quad 64 - k > 0$ $\quad \boxed{k < 64}$

- Gráfico

$k = 32 \Rightarrow 1 = \dfrac{x^2}{8} + \dfrac{y^2}{4}$ $\quad a = \sqrt{8} \cong 2{,}82$, $b = 2$



$k = 16 \Rightarrow 1 = \dfrac{x^2}{12} + \dfrac{y^2}{6}$

$k = 48 \Rightarrow 1 = \dfrac{x^2}{4} + \dfrac{y^2}{2}$ $\quad a = 2$, $b = \sqrt{2} \cong 1{,}41$

$k = 8 \Rightarrow 1 = \dfrac{x^2}{14} + \dfrac{y^2}{7}$

$k = 64 \Rightarrow 0 = 4x^2 + 8y^2 \to (0,0)$

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* Ejercicio 4 $\quad U(x,y)$ esp potencial electrostático $(x,y) \in D \subset \mathbb{R}^2$

$U(x,y) = k / \sqrt{x^2 + y^2}$ $\quad (x,y) \in \mathbb{R}^2 - \{\bar{0}\}$ siendo $k \in \mathbb{R}_{>0}$

- Hallar líneas equipotenciales (E)

$U(x,y) = \dfrac{k}{\underbrace{\sqrt{x^2+y^2}}_{>0 \; \forall (x,y) \in \mathbb{R}^2}} = E \Rightarrow \left(\dfrac{k}{E}\right)^2 = x^2 + y^2$ → circunferencias en $\mathbb{R}^2$ de radio $r = \dfrac{k}{E}$, y $E \neq 0$

no existe equipotencial 0

LÍMITES Y CONTINUIDAD DE CAMPOS

* Ejercicios : Determinar existencia del lím, fundamentar rta

ⓐ $\lim\limits_{(x,y)\to(1,1)} \overset{\to 1}{x}\overset{\to 1}{y} - y^2 = 0 \Rightarrow \exists \lim\limits_{(x,y)\to(1,1)} (xy - y^2) = 0$ (existe lim y es 0)

ⓑ $\lim\limits_{(x,y)\to(0,4)} x y^{-\frac{1}{2}} = \lim\limits_{(x,y)\to(0,4)} \dfrac{1}{\sqrt{\underbrace{xy}_{\to 0}}} = \infty$ no existe lim

ⓒ $\lim\limits_{(x,y)\to(0,0)} \sqrt{x^2+y^2} \to 0 = 0 \Rightarrow \exists \lim\limits_{(x,y)\to(0,0)} \sqrt{x^2+y^2} = 0$ existe lim y es 0

ⓓ $\lim\limits_{(x,y)\to(0,0)} \dfrac{x \to 0}{x+y \to 0} \Rightarrow$ ind $\frac{0}{0}$ → probar acercamiento (0,0) por curvas

- familia de rectas $(0,0) \to y = mx$

$f(x, mx) = \dfrac{x}{x+mx} = \dfrac{x}{x(1+m)}$

$\Rightarrow \lim\limits_{x\to 0} \dfrac{x}{x(1+m)} = \lim\limits_{x\to 0} \dfrac{1}{1+m} = \dfrac{1}{1+m}$

→ si $m = 1 \to \lim\limits_{x\to 0} f(x, xm) = \frac{1}{2}$

→ si $m = 2 \to \lim\limits_{x\to 0} f(x, mx) = \frac{1}{3}$

→ si limite existe es único $\longrightarrow \nexists \lim\limits_{(x,y)\to(0,0)} \dfrac{x}{x+y}$

ⓔ $\lim\limits_{(x,y)\to(0,2)} \dfrac{\overset{\to 0}{x^2}\,\overset{\to 0}{(y-2)^2}}{x^2 + (y-2)^2 \to 0} = \text{Ind } \frac{0}{0} \Rightarrow \lim\limits_{(x,y)\to(0,2)} \dfrac{x^2 (y-2)^2}{x^2+(y-2)^2} = 0$ (acotado; $(y-2)^2 \to 0$)

⇒ Probar acotada

$x^2 \leq x^2 + \underbrace{(y-2)^2}_{\geq 0} \to \dfrac{x^2}{x^2+(y-2)^2} \leq 1 \to \underbrace{\left| \dfrac{x^2}{x^2+(y-2)^2} \right|}_{>0 \ \forall (x,y) \in \mathbb{R}^2} \leq \dfrac{1}{M}$

ⓕ $\lim\limits_{x\to 0} \left(\dfrac{x}{|x|}, \sqrt{x} \right)$ $\quad f_1 = \begin{cases} \frac{x}{x} & \text{si } x > 0 \\ \frac{-x}{x} & \text{si } x < 0 \end{cases}$