# On the intricacies of running it twice in No-Limit Texas Hold 'em

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**ABSTRACT**

In poker games, sometimes you're all-in before the river and you'll be asked if you want to run it twice. If you agree the remaining streets will be dealt twice with half the pot going to the winner of each completed board. Note that the dealer does not complete the board once, shuffle the cards back in and complete a second board again. No, the cards are dealt *without replacement*. Turns out, however that the expected winnings are the same either way. So why do people do it? Because it reduces the variance. In some sense, that means there's less luck involved. That said, in the limit, if you play an infinite number of hands of poker, it doesn't change your earnings.

## Proofs

Let's start by defining a reward $R(X_1)$ over a random variable $X_1$ that represents either a win or a loss on the first board. Let $M$ be the total amount in the pot. Furthermore, we let $w_1$ represent the event that $X_1 = win$ and $l_1$ represent the event that $X_1 = loss$. If we run it once, the expected winnings are:

$$\mathbb{E}[R(X_1)] = MP(w_1)$$

Extending this to two boards, we use $R(X_1, X_2)$ to be the reward given for the joint variables $X_1, X_2$. Running it twice we get:

$$\begin{aligned}
\mathbb{E}[R(X_1, X_2)] &= MP(w_1, w_2) + \frac{1}{2}MP(l_1, w_2) + \frac{1}{2}MP(w_1, l_2) + 0P(l_1, l_2) \\
&= MP(w_1, w_2) + MP(w_1, l_2) \\
&= MP(w_1)P(w_2|w_1) + MP(w_1)P(l_2|w_1) \\
&= MP(w_1)[P(w_2|w_1) + P(l_2|w_1)] \\
&= MP(w_1)[1] \\
&= MP(w_1)
\end{aligned}$$

Now let's show that $Var[R(X_1, X_2)] < Var[R(X_1)]$. In other words, we want to show that running it twice yields lower variance than running it once. First, we write out the variance for the running it once case. Recall that $Var[X] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$.

$$\begin{aligned}
Var[R(X_1)] &= \mathbb{E}[R(X_1)^2] - \mathbb{E}[R(X_1)]^2 \\
&= M^2P(w_1) - M^2P(w_1)^2 \\
&= M^2P(w_1)\left[1 - P(w_1)\right]
\end{aligned}$$

Now let's do the running it twice case.

$$\begin{aligned}
Var[R(X_1, X_2)] &= \mathbb{E}[R(X_1, X_2)^2] - \mathbb{E}[R(X_1, X_2)]^2 \\
&= M^2P(w_1, w_2) + \frac{1}{4}M^2P(w_1, l_2) + \frac{1}{4}M^2P(l_1, w_2) + 0P(l_1, l_2) - M^2P(w_1)^2 \\
&= M^2P(w_1, w_2) + \frac{1}{2}M^2P(w_1, l_2) - M^2P(w_1)^2 \\
&= M^2P(w_1)\left[P(w_2|w_1, ) + \frac{1}{2}P(l_2|w_1) - P(w_1)\right] \\
&\leq M^2P(w_1)\left[1 - P(w_1)\right] = Var[R(X_1)]
\end{aligned}$$