Lasso for the L0.5 or L1/2 norm:-

$$\hat{\theta}_j = \arg\min_{\theta} \left\{ \underbrace{\frac{1}{2} \sum_{i=1}^{N} \left(y_i - \sum_{k=1}^{p} x_{ik} \theta_k \right)^2}_{O} + \lambda \sum_{k=1}^{p} \sqrt{|\theta_k|} \right\}$$

$\lambda \geq 0 \rightarrow$ tuning parameter

For optimization, we take the derivative.

So;

$$\frac{\partial O}{\partial \theta_j} = - \sum_{i=1}^{N} x_{ij} \left(y_i - x_{ij}\theta_j - \sum_{k \neq j}^{p} x_{ij} \theta_j \right) + \frac{\lambda}{2} \frac{\text{sign}(\theta_k)}{\sqrt{|\theta_k|}}$$

$$= - \sum_{i=1}^{N} x_{ij} \left(y_i - \sum_{k \neq j}^{p} x_{ij} \theta_j \right) - \sum_{i=1}^{N} x_{ij}^2 \theta_j + \frac{\lambda}{2} \frac{\text{sign}(\theta_k)}{\sqrt{|\theta_k|}}$$

Now; $\sum_{i=1}^{N} x_{ij}^2 = N-1 \rightarrow$ since our data is standardised.

$$= - \sum_{i=1}^{N} x_{ij} \left(y_i - \sum_{k \neq j}^{p} x_{ij} \theta_j \right) + (N-1)\theta_j + \frac{\lambda}{2} \frac{\text{sign}(\theta_k)}{\sqrt{|\theta_k|}}$$

Now, we set the derivative to 0 ie, $\frac{\partial O}{\partial \theta_j} = 0$.

So;

$$\underbrace{\sum_{i=1}^{N} x_{ij} \left(y_i - \sum_{k \neq j}^{p} x_{ij} \theta_j \right)}_{T} = (N-1)\theta_j + \frac{\lambda}{2} \frac{\text{sign}(\theta_k)}{\sqrt{|\theta_k|}}$$

when $\theta_j \geq 0$; $\quad \frac{T - \frac{\lambda}{2\sqrt{|\theta_k|}}}{N-1}$ $\quad$ So; $T \geq \frac{\lambda}{2\sqrt{|\theta_k|}}$

Eq when, $\theta_j < 0$; $\dfrac{T + \frac{\lambda}{2\sqrt{|\theta_k|}}}{N-1}$ so ; $T < \dfrac{\lambda}{2\sqrt{|\theta_k|}}$

Now, as we can infer, we have 2 properties :-

1) $T \geq \dfrac{\lambda}{2\sqrt{|\theta_k|}}$

2) $\theta_j \geq 0$ if $T \geq 0$ and $\theta_j \leq 0$ if $T \leq 0$.

Now:

$$\theta_j = \frac{\text{sign}(T)\left(|T| - \frac{\lambda}{2\sqrt{|\theta_k|}}\right)}{N-1}$$

(T → $|T|$, B → $\frac{\lambda}{2\sqrt{|\theta_k|}}$; $N-1$ → A.)

$$\theta_j = S(T, B)/N-1.$$

$$= \frac{S\left(\sum_{i=1}^{N} x_{ij}\left(y_i - \sum_{k \neq j}^{p} x_{ij}\theta_j\right), \frac{\lambda}{2\sqrt{|\theta_k|}}\right)}{N-1.}$$

So ; $A = N-1$; $B = \dfrac{\lambda}{2\sqrt{|\theta_k|}}$ //.