Lasso for L 0.5 or L1/2 norm:-

$$heta_j = rg \min_{\Theta} \left\{ rac{1}{2} \sum_{i=1}^N \left(Y_i - \sum_{k=1}^p x_{ik} heta_k
ight)^2 + \lambda \sum_{k=1}^p \sqrt{| heta_k|}
ight\}$$
 $\lambda \geq 0 \qquad ext{$->$ tuning parameter}$

For optimisation, we have to take the derivative.

So,

$$egin{aligned} rac{\partial O}{\partial heta_i} &= -\sum_{i=1}^N x_{ij} \left(Y_i - x_{ij} \Theta_j - \sum_{k
eq j}^p x_{ij} \Theta_j
ight) + rac{\lambda}{2} rac{sign\left(heta_k
ight)}{\sqrt{| heta_k|}} \ &= rac{\sum_{i=1}^N x_{ij} \left(Y_i - \sum_{k
eq j}^p x_{ij} \Theta_j
ight)}{-\sum_{i=1}^N x_{ij}^2 heta_j + rac{\lambda}{2}} rac{\overline{sign}\left(heta_k
ight)}{\sqrt{| heta_k|}} \ &= Now; \ &= \sum_{i=1}^N x_{ij} rac{1}{2} \left(Y_i - \sum_{k
eq j}^p x_{ij} \Theta_j
ight) + \left(N - 1
ight) heta_j + rac{\lambda}{2} rac{sign\left(heta_k
ight)}{\sqrt{| heta_k|}} \end{aligned}$$

Now, we set the derivative to 0 i.e. $\dfrac{\partial O}{\partial heta_j} = 0$

 S_0 ;

$$\sum_{i=1}^{N} x_{ij} \left(Y_i - \sum_{k=j}^{p} x_{ij} \Theta j
ight) = (N-1) \, \Theta j + rac{\lambda}{2} rac{sign \left(heta_k
ight)}{\sqrt{| heta_k|}}$$

When
$$\theta_j \geq 0$$

$$\frac{T - \frac{\lambda}{2\sqrt{|\theta_k|}}}{N-1}$$
 $S_0, \quad T \geq \frac{\lambda}{2\sqrt{|\theta_k|}}$

And when
$$heta_j < 0$$
 ; $T + rac{\lambda}{2\sqrt{| heta_k|}}$ $S_0; \quad T < rac{\lambda}{2\sqrt{| heta_k|}}$

Now, as we can infer, we have 2 properties;

$$1) T \ge \frac{\lambda}{2\sqrt{|\theta_k|}}$$

2)
$$\theta_j \geq 0$$
 if $T \geq 0$ $\theta_j \leq 0$ if $T \leq 0$

$$heta j = rac{sign\left(au
ight)\left(\left|T
ight| - rac{\lambda}{2\sqrt{\left| heta_k
ight|}}
ight)}{N-1}$$

$$S_0; \qquad A = N-1 \qquad \qquad ; \qquad \qquad B = rac{\lambda}{2\sqrt{| heta_k|}}$$