Matrix calculator

Matrix A:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{pmatrix}$$

Matrix B:

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

← →

A * B

A + B

A - B

Cells | Clean | + | –

Find the determinant	Find the inverse
Transpose	Find the rank
Multiply by 2	Triangular matrix
Diagonal matrix	Raise to the power of 2
LU-decomposition	Cholesky decomposition

Cells | Clean | + | –

Find the determinant	Find the inverse
Transpose	Find the rank
Multiply by 2	Triangular matrix
Diagonal matrix	Raise to the power of 10
LU-decomposition	Cholesky decomposition

1*A+2*B =

☐ Display decimals

Clean | +

1. Find eigenvalues from the characteristic polynomial :

$$\begin{vmatrix} 0-\lambda & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0-\lambda & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0-\lambda & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0-\lambda & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2}-\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0-\lambda & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2}-\lambda \end{vmatrix} = -\lambda^7+\lambda^6+\frac{3}{4}\times\lambda^5-\frac{5}{8}\times\lambda^4-\frac{1}{4}\times\lambda^3+\frac{3}{32}\times\lambda^2+\frac{1}{32}\times\lambda =$$

$\equiv$

$$\frac{-1}{32}\times\lambda\times\left(32\times\lambda^6-32\times\lambda^5-24\times\lambda^4+20\times\lambda^3+8\times\lambda^2-3\times\lambda-1\right) \underset{(?)}{=} \frac{-1}{32}\times\lambda\times(\lambda-1)\times\left(\lambda+\frac{1}{2}\right)\times\left(\lambda+\frac{1}{2}\right)\times\left(\lambda-\frac{1}{2}\right)\times\left(32\times\lambda^2-16\times\lambda-8\right)$$

$$=\frac{-1}{4}\times\lambda\times(\lambda-1)\times\left(\lambda+\frac{1}{2}\right)\times\left(\lambda+\frac{1}{2}\right)\times\left(\lambda-\frac{1}{2}\right)\times\left(4\times\lambda^2-2\times\lambda-1\right) \underset{(?)}{=}$$

$$4\times\left(\frac{-1}{4}\right)\times\lambda\times(\lambda-1)\times\left(\lambda+\frac{1}{2}\right)\times\left(\lambda+\frac{1}{2}\right)\times\left(\lambda-\frac{1}{2}\right)\times\left(\lambda+\frac{\sqrt{5}-1}{4}\right)\times\left(\lambda-\frac{\sqrt{5}+1}{4}\right)$$

▶ Details (Montante's method (Bareiss algorithm))

▶ Details (Gaussian elimination)

1. $\lambda_1=0$
2. $\lambda_2=1$
3. $\lambda_3=\frac{-1}{2}$
4. $\lambda_4=\frac{1}{2}$
5. $\lambda_5=\frac{-\sqrt{5}+1}{4}$
6. $\lambda_6=\frac{\sqrt{5}+1}{4}$

2. For every λ we find its own vector(s):
   1. $\lambda_1=0$

$$A-\lambda_1 I=\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$\equiv$

$Av = \lambda v$ *

$(A - \lambda I)\, v = 0$

So we have a homogeneous system of linear equations, we solve it by Gaussian Elimination:

$$\left(\begin{array}{ccccccc|c} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}\right) \underset{R_3 \leftrightarrow R_1}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}\right) \times(2) \underset{R_1/\left(\frac{1}{2}\right) \to R_1}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}\right)$$

$$\equiv \quad \equiv \quad \equiv$$

$$\underset{R_3 \leftrightarrow R_2}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}\right) \times(2) \underset{R_2/\left(\frac{1}{2}\right) \to R_2}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}\right) \times(2) \underset{R_3/\left(\frac{1}{2}\right) \to R_3}{\overset{?}{\sim}}$$

$$\equiv \quad \equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}\right) \times\left(\frac{-1}{2}\right) \underset{R_5 - \left(\frac{1}{2}\right) \times R_3 \to R_5}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}\right) \times(2) \underset{R_4/\left(\frac{1}{2}\right) \to R_4}{\overset{?}{\sim}}$$

$$\equiv \quad \equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}\right) \times\left(\frac{-1}{2}\right) \underset{R_5-\left(\frac{1}{2}\right)\times R_4\rightarrow R_5}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}\right) \times(-1) \underset{R_5/(-1)\rightarrow R_5}{\overset{?}{\sim}}$$

$$\equiv \qquad \equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}\right) \times\left(\frac{-1}{2}\right) \underset{R_7-\left(\frac{1}{2}\right)\times R_5\rightarrow R_7}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \end{array}\right) \times\left(\frac{-1}{2}\right) \underset{R_7-\left(\frac{1}{2}\right)\times R_6\rightarrow R_7}{\overset{?}{\sim}}$$

$$\equiv \qquad \equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times(-1) \underset{R_4-1\times R_5\rightarrow R_4}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times(-1) \underset{R_3-1\times R_5\rightarrow R_3}{\overset{?}{\sim}}$$

$$\equiv \qquad \equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times(-1) \underset{R_1-1\times R_5\rightarrow R_1}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\equiv \qquad \equiv$$

$$\left\{\begin{array}{lcl} x_1 & = & 0 \\ x_2 + x_3 & = & 0 \\ x_4 & = & 0 \\ x_5 & = & 0 \\ x_6 & = & 0 \\ x_7 & = & 0 \end{array}\right. \quad (1)$$

$$\equiv$$

- Find the variable x_7 from the equation 6 of the system (1):

$x_7=0$

- Find the variable x_6 from the equation 5 of the system (1):

  $x_6=0$
- Find the variable x_5 from the equation 4 of the system (1):

  $x_5=0$
- Find the variable x_4 from the equation 3 of the system (1):

  $x_4=0$
- Find the variable x_2 from the equation 2 of the system (1):

  $x_2=-x_3$
- Find the variable x_1 from the equation 1 of the system (1):

  $x_1=0$

Answer:

- $x_1=0$
- $x_2=-x_3$
- $x_3=x_3$
- $x_4=0$
- $x_5=0$
- $x_6=0$
- $x_7=0$

General Solution $: X=\begin{pmatrix} 0 \\ -x_3 \\ x_3 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$
$\equiv$

The solution set: $\{x_3\times\begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}\}$
$\equiv$

Let $x_3 = 1$, $\boldsymbol{v_1} = \begin{pmatrix} \mathbf{0} \\ \mathbf{-1} \\ \mathbf{1} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}$

$\equiv$

2. $\lambda_2 = 1$

$$A - \lambda_2 I = \begin{pmatrix} -1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -1 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} \end{pmatrix}$$

$\equiv$

$Av = \lambda v$ *

$(A - \lambda I)v = 0$

So we have a homogeneous system of linear equations, we solve it by Gaussian Elimination:

$$
\left(\begin{array}{ccccccc|c}
-1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0\\
0 & -1 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0\\
\frac{1}{2} & 0 & -1 & 0 & 0 & \frac{1}{2} & 0 & 0\\
0 & 0 & 0 & -1 & \frac{1}{2} & \frac{1}{2} & 0 & 0\\
0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0
\end{array}\right)
\begin{array}{l}
\times(-1)\\ \\ \\ \\ \\ \\ \\
\end{array}
\underset{R_1/(-1)\to R_1}{\overset{?}{\sim}}
\left(\begin{array}{ccccccc|c}
1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 & 0 & 0 & 0\\
0 & -1 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0\\
\frac{1}{2} & 0 & -1 & 0 & 0 & \frac{1}{2} & 0 & 0\\
0 & 0 & 0 & -1 & \frac{1}{2} & \frac{1}{2} & 0 & 0\\
0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0
\end{array}\right)
\begin{array}{l}
\\ \times\left(\frac{-1}{2}\right)\\ \\ \\ \\ \\ \\
\end{array}
$$

$$\equiv \qquad\qquad \equiv$$

$$
\underset{R_3-\left(\frac{1}{2}\right)\times R_1\to R_3}{\overset{?}{\sim}}
\left(\begin{array}{ccccccc|c}
1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 & 0 & 0 & 0\\
0 & -1 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0\\
0 & \frac{1}{4} & \frac{-3}{4} & 0 & 0 & \frac{1}{2} & 0 & 0\\
0 & 0 & 0 & -1 & \frac{1}{2} & \frac{1}{2} & 0 & 0\\
0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0
\end{array}\right)
\begin{array}{l}
\\ \times(-1)\\ \\ \\ \\ \\ \\
\end{array}
\underset{R_2/(-1)\to R_2}{\overset{?}{\sim}}
$$

$$\equiv$$

$$
\left(\begin{array}{ccccccc|c}
1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & \frac{-1}{2} & 0 & \frac{-1}{2} & 0 & 0\\
0 & \frac{1}{4} & \frac{-3}{4} & 0 & 0 & \frac{1}{2} & 0 & 0\\
0 & 0 & 0 & -1 & \frac{1}{2} & \frac{1}{2} & 0 & 0\\
0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0
\end{array}\right)
\begin{array}{l}
\\ \times\left(\frac{-1}{4}\right)\\ \\ \\ \\ \\ \\
\end{array}
\underset{R_3-\left(\frac{1}{4}\right)\times R_2\to R_3}{\overset{?}{\sim}}
\left(\begin{array}{ccccccc|c}
1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & \frac{-1}{2} & 0 & \frac{-1}{2} & 0 & 0\\
0 & 0 & \frac{-3}{4} & \frac{1}{8} & 0 & \frac{5}{8} & 0 & 0\\
0 & 0 & 0 & -1 & \frac{1}{2} & \frac{1}{2} & 0 & 0\\
0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0
\end{array}\right)
\begin{array}{l}
\\ \\ \times\left(\frac{-4}{3}\right)\\ \\ \\ \\ \\
\end{array}
$$

$$\equiv \qquad\qquad \equiv$$

$$\overset{?}{\underset{R_3/\left(\frac{-3}{4}\right)\to R_3}{\sim}}\left(\begin{array}{ccccccc|c} 1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{-1}{2} & 0 & \frac{-1}{2} & 0 & 0 \\ 0 & 0 & 1 & \frac{-1}{6} & 0 & \frac{-5}{6} & 0 & 0 \\ 0 & 0 & 0 & -1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0 \end{array}\right) \times(-1) \quad \overset{?}{\underset{R_4/(-1)\to R_4}{\sim}}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{-1}{2} & 0 & \frac{-1}{2} & 0 & 0 \\ 0 & 0 & 1 & \frac{-1}{6} & 0 & \frac{-5}{6} & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0 \end{array}\right) \times\left(\frac{-1}{2}\right) \quad \overset{?}{\underset{R_5-\left(\frac{1}{2}\right)\times R_4\to R_5}{\sim}} \left(\begin{array}{ccccccc|c} 1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{-1}{2} & 0 & \frac{-1}{2} & 0 & 0 \\ 0 & 0 & 1 & \frac{-1}{6} & 0 & \frac{-5}{6} & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0 \end{array}\right) \times(-4)$$

$$\equiv \qquad \equiv$$

$$\overset{?}{\underset{R_5/\left(\frac{-1}{4}\right)\to R_5}{\sim}}\left(\begin{array}{ccccccc|c} 1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{-1}{2} & 0 & \frac{-1}{2} & 0 & 0 \\ 0 & 0 & 1 & \frac{-1}{6} & 0 & \frac{-5}{6} & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0 \end{array}\right) \times(-1) \quad \overset{?}{\underset{R_6/(-1)\to R_6}{\sim}}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{-1}{2} & 0 & \frac{-1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{-1}{6} & 0 & \frac{-5}{6} & 0 & 0 \\
0 & 0 & 0 & 1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{1} & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0
\end{array}\right) \times\left(\frac{-1}{2}\right) \quad \underset{R_7-\left(\frac{1}{2}\right)\times R_6\to R_7}{\overset{?}{\sim}} \quad \left(\begin{array}{ccccccc|c}
1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{-1}{2} & 0 & \frac{-1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{-1}{6} & 0 & \frac{-5}{6} & 0 & 0 \\
0 & 0 & 0 & 1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{1} & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right) \times(1)$$

$$\equiv \quad \equiv$$

$$\underset{R_5-(-1)\times R_6\to R_5}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c}
1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{-1}{2} & 0 & \frac{-1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{-1}{6} & 0 & \frac{-5}{6} & 0 & 0 \\
0 & 0 & 0 & 1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{1} & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right) \times\left(\frac{1}{2}\right) \quad \underset{R_4-\left(\frac{-1}{2}\right)\times R_6\to R_4}{\overset{?}{\sim}}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{-1}{2} & 0 & \frac{-1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{-1}{6} & 0 & \frac{-5}{6} & 0 & 0 \\
0 & 0 & 0 & 1 & \frac{-1}{2} & 0 & \frac{-1}{2} & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{1} & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right) \times\left(\frac{5}{6}\right) \quad \underset{R_3-\left(\frac{-5}{6}\right)\times R_6\to R_3}{\overset{?}{\sim}} \quad \left(\begin{array}{ccccccc|c}
1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{-1}{2} & 0 & \frac{-1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{-1}{6} & 0 & 0 & \frac{-5}{6} & 0 \\
0 & 0 & 0 & 1 & \frac{-1}{2} & 0 & \frac{-1}{2} & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{1} & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right) \times\left(\frac{1}{2}\right)$$

$$\equiv \quad \equiv$$

$$\underset{R_2-\left(\frac{-1}{2}\right)\times R_6\to R_2}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c}
1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{-1}{2} & 0 & 0 & \frac{-1}{2} & 0 \\
0 & 0 & 1 & \frac{-1}{6} & 0 & 0 & \frac{-5}{6} & 0 \\
0 & 0 & 0 & 1 & \frac{-1}{2} & 0 & \frac{-1}{2} & 0 \\
0 & 0 & 0 & 0 & \mathbf{1} & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right) \times\left(\frac{1}{2}\right) \quad \underset{R_4-\left(\frac{-1}{2}\right)\times R_5\to R_4}{\overset{?}{\sim}}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{-1}{2} & 0 & 0 & \frac{-1}{2} & 0 \\ 0 & 0 & 1 & \frac{-1}{6} & 0 & 0 & \frac{-5}{6} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times\left(\frac{1}{6}\right) \underset{R_3-\left(\frac{-1}{6}\right)\times R_4\to R_3}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{-1}{2} & 0 & 0 & \frac{-1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times\left(\frac{1}{2}\right)$$

$$\equiv \qquad \equiv$$

$$\underset{R_2-\left(\frac{-1}{2}\right)\times R_4\to R_2}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times\left(\frac{1}{2}\right) \underset{R_1-\left(\frac{-1}{2}\right)\times R_3\to R_1}{\overset{?}{\sim}}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & \frac{-1}{2} & 0 & 0 & 0 & 0 & \frac{-1}{2} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times\left(\frac{1}{2}\right) \underset{R_1-\left(\frac{-1}{2}\right)\times R_2\to R_1}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\equiv \qquad \equiv$$

$$\left\{\begin{array}{lllllllll} x_1 & & & & & & -x_7 & = & 0 \\ & x_2 & & & & & -x_7 & = & 0 \\ & & x_3 & & & & -x_7 & = & 0 \\ & & & x_4 & & & -x_7 & = & 0 \\ & & & & x_5 & & -x_7 & = & 0 \\ & & & & & x_6 & -x_7 & = & 0 \end{array}\right. \quad (1)$$

$$\equiv$$

- Find the variable x_6 from the equation 6 of the system (1):
  $x_6=x_7$
- Find the variable x_5 from the equation 5 of the system (1):
  $x_5=x_7$
- Find the variable x_4 from the equation 4 of the system (1):
  $x_4=x_7$
- Find the variable x_3 from the equation 3 of the system (1):

$x_3 = x_7$

- Find the variable x_2 from the equation 2 of the system (1):

  $x_2 = x_7$
- Find the variable x_1 from the equation 1 of the system (1):

  $x_1 = x_7$

Answer:

- $x_1 = x_7$
- $x_2 = x_7$
- $x_3 = x_7$
- $x_4 = x_7$
- $x_5 = x_7$
- $x_6 = x_7$
- $x_7 = x_7$

General Solution $: X = \begin{pmatrix} x_7 \\ x_7 \\ x_7 \\ x_7 \\ x_7 \\ x_7 \\ x_7 \end{pmatrix}$

$\equiv$

The solution set: $\{ x_7 \times \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \}$

$\equiv$

Let $x_7 = 1$, $\boldsymbol{v_2} = \begin{pmatrix} \mathbf{1} \\ \mathbf{1} \\ \mathbf{1} \\ \mathbf{1} \\ \mathbf{1} \\ \mathbf{1} \\ \mathbf{1} \end{pmatrix}$

$\equiv$

3. $\lambda_3 = \frac{-1}{2}$

$$A-\lambda_3 I=\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 1 \end{pmatrix}$$

$$\equiv$$

$Av=\lambda v$ *

$(A-\lambda I)v=0$

So we have a homogeneous system of linear equations, we solve it by Gaussian Elimination:

$$\left(\begin{array}{ccccccc|c} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 1 & 0 \end{array}\right) \times(2) \quad \underset{R_1/\left(\frac{1}{2}\right)\to R_1}{\overset{?}{\sim}} \quad \left(\begin{array}{ccccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 1 & 0 \end{array}\right) \times\left(\frac{-1}{2}\right) \quad \underset{R_3-\left(\frac{1}{2}\right)\times R_1\to R_3}{\overset{?}{\sim}}$$

$$\equiv \qquad \equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{-1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 1 & 0 \end{array}\right) \times(2) \quad \underset{R_2/\left(\frac{1}{2}\right)\to R_2}{\overset{?}{\sim}} \quad \left(\begin{array}{ccccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & \frac{-1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 1 & 0 \end{array}\right) \times\left(\frac{1}{2}\right) \quad \underset{R_3-\left(\frac{-1}{2}\right)\times R_2\to R_3}{\overset{?}{\sim}}$$

$$\equiv \qquad \equiv$$

$$\left(\begin{array}{ccccccc|c} 1&1&1&0&0&0&0&0\\ 0&1&0&1&0&1&0&0\\ 0&0&0&\frac{1}{2}&0&1&0&0\\ 0&0&0&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&0\\ 0&0&0&\frac{1}{2}&1&0&0&0\\ 0&0&0&0&0&\frac{1}{2}&1&0\\ 0&0&0&0&0&\frac{1}{2}&1&0 \end{array}\right)\times(2) \quad \overset{?}{\sim} \; R_3/\left(\frac{1}{2}\right)\to R_3$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1&1&1&0&0&0&0&0\\ 0&1&0&1&0&1&0&0\\ 0&0&0&1&0&2&0&0\\ 0&0&0&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&0\\ 0&0&0&\frac{1}{2}&1&0&0&0\\ 0&0&0&0&0&\frac{1}{2}&1&0\\ 0&0&0&0&0&\frac{1}{2}&1&0 \end{array}\right)\times\left(\frac{-1}{2}\right) \quad \overset{?}{\sim} \; R_4-\left(\frac{1}{2}\right)\times R_3\to R_4$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1&1&1&0&0&0&0&0\\ 0&1&0&1&0&1&0&0\\ 0&0&0&1&0&2&0&0\\ 0&0&0&0&\frac{1}{2}&\frac{-1}{2}&0&0\\ 0&0&0&\frac{1}{2}&1&0&0&0\\ 0&0&0&0&0&\frac{1}{2}&1&0\\ 0&0&0&0&0&\frac{1}{2}&1&0 \end{array}\right)\times\left(\frac{-1}{2}\right) \quad \overset{?}{\sim} \; R_5-\left(\frac{1}{2}\right)\times R_3\to R_5$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1&1&1&0&0&0&0&0\\ 0&1&0&1&0&1&0&0\\ 0&0&0&1&0&2&0&0\\ 0&0&0&0&\frac{1}{2}&\frac{-1}{2}&0&0\\ 0&0&0&0&1&-1&0&0\\ 0&0&0&0&0&\frac{1}{2}&1&0\\ 0&0&0&0&0&\frac{1}{2}&1&0 \end{array}\right)\times(2) \quad \overset{?}{\sim} \; R_4/\left(\frac{1}{2}\right)\to R_4$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1&1&1&0&0&0&0&0\\ 0&1&0&1&0&1&0&0\\ 0&0&0&1&0&2&0&0\\ 0&0&0&0&1&-1&0&0\\ 0&0&0&0&1&-1&0&0\\ 0&0&0&0&0&\frac{1}{2}&1&0\\ 0&0&0&0&0&\frac{1}{2}&1&0 \end{array}\right)\times(-1) \quad \overset{?}{\sim} \; R_5-1\times R_4\to R_5$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1&1&1&0&0&0&0&0\\ 0&1&0&1&0&1&0&0\\ 0&0&0&1&0&2&0&0\\ 0&0&0&0&1&-1&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&\frac{1}{2}&1&0\\ 0&0&0&0&0&\frac{1}{2}&1&0 \end{array}\right) \quad \overset{?}{\sim} \; R_6\leftrightarrow R_5$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1&1&1&0&0&0&0&0\\ 0&1&0&1&0&1&0&0\\ 0&0&0&1&0&2&0&0\\ 0&0&0&0&1&-1&0&0\\ 0&0&0&0&0&\frac{1}{2}&1&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&\frac{1}{2}&1&0 \end{array}\right)\times(2) \quad \overset{?}{\sim} \; R_5/\left(\frac{1}{2}\right)\to R_5$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1&1&1&0&0&0&0&0\\ 0&1&0&1&0&1&0&0\\ 0&0&0&1&0&2&0&0\\ 0&0&0&0&1&-1&0&0\\ 0&0&0&0&0&1&2&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&\frac{1}{2}&1&0 \end{array}\right)\times\left(\frac{-1}{2}\right) \quad \overset{?}{\sim} \; R_7-\left(\frac{1}{2}\right)\times R_5\to R_7$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times(1) \underset{R_4-(-1)\times R_5\rightarrow R_4}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times(-2) \underset{R_3-2\times R_5\rightarrow R_3}{\overset{?}{\sim}}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times(-1) \underset{R_2-1\times R_5\rightarrow R_2}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times(-1) \underset{R_2-1\times R_3\rightarrow R_2}{\overset{?}{\sim}}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times(-1) \underset{R_1-1\times R_2\rightarrow R_1}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\equiv$$

$$\begin{cases} x_1 + x_3 - 2\times x_7 = 0 \\ x_2 + 2\times x_7 = 0 \\ x_4 - 4\times x_7 = 0 \\ x_5 + 2\times x_7 = 0 \\ x_6 + 2\times x_7 = 0 \end{cases} \quad (1)$$

$$\equiv$$

- Find the variable x_6 from the equation 5 of the system (1):
  $x_6 = -2\times x_7$
- Find the variable x_5 from the equation 4 of the system (1):
  $x_5 = -2\times x_7$
- Find the variable x_4 from the equation 3 of the system (1):
  $x_4 = 4\times x_7$
- Find the variable x_2 from the equation 2 of the system (1):
  $x_2 = -2\times x_7$
- Find the variable x_1 from the equation 1 of the system (1):
  $x_1 = -x_3 + 2\times x_7$

Answer:

- $x_1 = -x_3 + 2 \times x_7$
- $x_2 = -2 \times x_7$
- $x_3 = x_3$
- $x_4 = 4 \times x_7$
- $x_5 = -2 \times x_7$
- $x_6 = -2 \times x_7$
- $x_7 = x_7$

General Solution $: X = \begin{pmatrix} -x_3 + 2 \times x_7 \\ -2 \times x_7 \\ x_3 \\ 4 \times x_7 \\ -2 \times x_7 \\ -2 \times x_7 \\ x_7 \end{pmatrix}$
$\equiv$

The solution set: $\{x_3 \times \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_7 \times \begin{pmatrix} 2 \\ -2 \\ 0 \\ 4 \\ -2 \\ -2 \\ 1 \end{pmatrix}\}$
$\equiv$ $\equiv$

Let $x_3 = 1, x_7 = 0$, $\boldsymbol{v_3} = \begin{pmatrix} \mathbf{-1} \\ \mathbf{0} \\ \mathbf{1} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}$; Let $x_3 = 0, x_7 = 1$, $\boldsymbol{v_4} = \begin{pmatrix} \mathbf{2} \\ \mathbf{-2} \\ \mathbf{0} \\ \mathbf{4} \\ \mathbf{-2} \\ \mathbf{-2} \\ \mathbf{1} \end{pmatrix}$
$\equiv$ $\equiv$

4. $\lambda_4 = \frac{1}{2}$

$$A-\lambda_4 I=\begin{pmatrix} \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{-1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{2} & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$$

$$\equiv$$

$Av=\lambda v$ *

$(A-\lambda I)v=0$

So we have a homogeneous system of linear equations, we solve it by Gaussian Elimination:

$$\left(\begin{array}{ccccccc|c} \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{-1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{array}\right) \times(-2) \quad \underset{R_1/\left(\frac{-1}{2}\right)\to R_1}{\overset{?}{\sim}} \quad \left(\begin{array}{ccccccc|c} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{-1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{array}\right) \times\left(\frac{-1}{2}\right)$$

$$\equiv \qquad\qquad \equiv$$

$$\underset{R_3-\left(\frac{1}{2}\right)\times R_1\to R_3}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{array}\right) \times(-2) \quad \underset{R_2/\left(\frac{-1}{2}\right)\to R_2}{\overset{?}{\sim}}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{array}\right) \times\left(\frac{-1}{2}\right) \quad \overset{?}{\sim} \quad R_3-\left(\frac{1}{2}\right)\times R_2 \to R_3 \quad \left(\begin{array}{ccccccc|c} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{array}\right) \times(2) \quad \overset{?}{\sim} \quad R_3/\left(\frac{1}{2}\right) \to R_3$$

$$\equiv \qquad \equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{array}\right) \times\left(\frac{1}{2}\right) \quad \overset{?}{\sim} \quad R_4-\left(\frac{-1}{2}\right)\times R_3 \to R_4 \quad \left(\begin{array}{ccccccc|c} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{3}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{array}\right) \times\left(\frac{-1}{2}\right)$$

$$\equiv \qquad \equiv$$

$$\overset{?}{\sim} \quad R_5-\left(\frac{1}{2}\right)\times R_3 \to R_5 \quad \left(\begin{array}{ccccccc|c} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{3}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{array}\right) \times(2) \quad \overset{?}{\sim} \quad R_4/\left(\frac{1}{2}\right) \to R_4 \quad \left(\begin{array}{ccccccc|c} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{array}\right) \times(-1)$$

$$\equiv \qquad \equiv$$

$$\overset{?}{\sim} \quad R_5/(-1) \to R_5 \quad \left(\begin{array}{ccccccc|c} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{array}\right) \times\left(\frac{1}{2}\right) \quad R_6-\left(\frac{-1}{2}\right)\times R_5 \to R_6 \quad \overset{?}{\sim}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{array}\right) \times\left(\frac{-1}{2}\right) \underset{R_7-\left(\frac{1}{2}\right)\times R_5\to R_7}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times(-3)$$

$$\equiv$$

$$\underset{R_4-3\times R_5\to R_4}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times(-2) \underset{R_3-2\times R_5\to R_3}{\overset{?}{\sim}}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times(1) \underset{R_2-(-1)\times R_5\to R_2}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times(1)$$

$$\equiv$$

$$\underset{R_2-(-1)\times R_3\to R_2}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times(1) \underset{R_1-(-1)\times R_2\to R_1}{\overset{?}{\sim}} \left(\begin{array}{ccccccc|c} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\equiv$$

$$\left\{\begin{array}{l} x_1 - x_3 = 0 \\ x_2 = 0 \\ x_4 = 0 \\ x_5 = 0 \\ x_6 = 0 \\ x_7 = 0 \end{array}\right. \quad (1)$$

$$\equiv$$

- Find the variable x_7 from the equation 6 of the system (1):
  $x_7=0$
- Find the variable x_6 from the equation 5 of the system (1):

$x_6=0$

- Find the variable x_5 from the equation 4 of the system (1):

  $x_5=0$
- Find the variable x_4 from the equation 3 of the system (1):

  $x_4=0$
- Find the variable x_2 from the equation 2 of the system (1):

  $x_2=0$
- Find the variable x_1 from the equation 1 of the system (1):

  $x_1=x_3$

Answer:

- $x_1=x_3$
- $x_2=0$
- $x_3=x_3$
- $x_4=0$
- $x_5=0$
- $x_6=0$
- $x_7=0$

General Solution $: X=\begin{pmatrix} x_3 \\ 0 \\ x_3 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$\equiv$

The solution set: $\{x_3\times\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}\}$

$\equiv$

Let $x_3=1$, $\boldsymbol{v_5}=\begin{pmatrix} \mathbf{1} \\ \mathbf{0} \\ \mathbf{1} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}$

$\equiv$

5. $\lambda_5 = \frac{-\sqrt{5}+1}{4}$

$$A - \lambda_5 I = \begin{pmatrix} \frac{\sqrt{5}-1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{5}-1}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{\sqrt{5}-1}{4} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{5}-1}{4} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{5}+1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{5}-1}{4} & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{5}+1}{4} \end{pmatrix}$$

$\equiv$

$Av = \lambda v$ *

$(A - \lambda I)v = 0$

So we have a homogeneous system of linear equations, we solve it by Gaussian Elimination:

$$\left(\begin{array}{ccccccc|c} \frac{\sqrt{5}-1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{5}-1}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{\sqrt{5}-1}{4} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{5}-1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{5}+1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{5}-1}{4} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{5}+1}{4} & 0 \end{array}\right) \times(\sqrt{5}+1)$$

$$\overset{?}{\sim} \quad R_1 / \left(\frac{\sqrt{5}-1}{4}\right) \rightarrow R_1$$

$\equiv$

$$\left(\begin{array}{ccccccc|c} 1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{5}-1}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{\sqrt{5}-1}{4} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{5}-1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{5}+1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{5}-1}{4} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{5}+1}{4} & 0 \end{array}\right) \times\left(\frac{-1}{2}\right)$$

$$\overset{?}{\sim} \quad R_3 - \left(\frac{1}{2}\right) \times R_1 \rightarrow R_3$$

$\equiv$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\sqrt{5}-1}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\
0 & \frac{-\sqrt{5}-1}{4} & \frac{-1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{\sqrt{5}-1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{5}+1}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\sqrt{5}-1}{4} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{5}+1}{4} & 0
\end{array}\right) \times\left(\sqrt{5}+1\right) \qquad \overset{?}{\underset{R_2/\left(\frac{\sqrt{5}-1}{4}\right)\to R_2}{\sim}}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{\sqrt{5}+1}{2} & 0 & \frac{\sqrt{5}+1}{2} & 0 & 0 \\
0 & \frac{-\sqrt{5}-1}{4} & \frac{-1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{\sqrt{5}-1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{5}+1}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\sqrt{5}-1}{4} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{5}+1}{4} & 0
\end{array}\right) \times\left(\frac{\sqrt{5}+1}{4}\right) \qquad \overset{?}{\underset{R_3-\left(\frac{-\sqrt{5}-1}{4}\right)\times R_2\to R_3}{\sim}}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{\sqrt{5}+1}{2} & 0 & \frac{\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & \frac{-1}{2} & \frac{\sqrt{5}+3}{4} & 0 & \frac{\sqrt{5}+5}{4} & 0 & 0 \\
0 & 0 & 0 & \frac{\sqrt{5}-1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{5}+1}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\sqrt{5}-1}{4} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{5}+1}{4} & 0
\end{array}\right) \times\left(-2\right) \qquad \overset{?}{\underset{R_3/\left(\frac{-1}{2}\right)\to R_3}{\sim}}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{\sqrt{5}+1}{2} & 0 & \frac{\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{-\sqrt{5}-3}{2} & 0 & \frac{-\sqrt{5}-5}{2} & 0 & 0 \\
0 & 0 & 0 & \mathbf{\frac{\sqrt{5}-1}{4}} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{5}+1}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\sqrt{5}-1}{4} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{5}+1}{4} & 0
\end{array}\right) \times\left(\sqrt{5}+1\right) \quad \overset{?}{\sim} \quad R_4/\left(\frac{\sqrt{5}-1}{4}\right) \rightarrow R_4$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{\sqrt{5}+1}{2} & 0 & \frac{\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{-\sqrt{5}-3}{2} & 0 & \frac{-\sqrt{5}-5}{2} & 0 & 0 \\
0 & 0 & 0 & \mathbf{1} & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{5}+1}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\sqrt{5}-1}{4} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{5}+1}{4} & 0
\end{array}\right) \times\left(\frac{-1}{2}\right) \quad R_5-\left(\frac{1}{2}\right)\overset{?}{\sim}\times R_4 \rightarrow R_5$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{\sqrt{5}+1}{2} & 0 & \frac{\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{-\sqrt{5}-3}{2} & 0 & \frac{-\sqrt{5}-5}{2} & 0 & 0 \\
0 & 0 & 0 & 1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{\frac{-\sqrt{5}-1}{4}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\sqrt{5}-1}{4} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{5}+1}{4} & 0
\end{array}\right) \times\left(-\sqrt{5}+1\right) \quad \overset{?}{\sim} \quad R_5/\left(\frac{-\sqrt{5}-1}{4}\right) \rightarrow R_5$$

$$\equiv$$

$$
\left(\begin{array}{ccccccc|c}
1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{\sqrt{5}+1}{2} & 0 & \frac{\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{-\sqrt{5}-3}{2} & 0 & \frac{-\sqrt{5}-5}{2} & 0 & 0 \\
0 & 0 & 0 & 1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\sqrt{5}-1}{4} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{5}+1}{4} & 0
\end{array}\right)
\times\left(\frac{-\sqrt{5}+1}{4}\right)
\quad
\overset{?}{\sim}\; R_6-\left(\frac{\sqrt{5}-1}{4}\right)\times R_5 \to R_6
$$

$$\equiv$$

$$
\left(\begin{array}{ccccccc|c}
1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{\sqrt{5}+1}{2} & 0 & \frac{\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{-\sqrt{5}-3}{2} & 0 & \frac{-\sqrt{5}-5}{2} & 0 & 0 \\
0 & 0 & 0 & 1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{5}+1}{4} & 0
\end{array}\right)
\times\left(\frac{-1}{2}\right)
\quad
\overset{?}{\sim}\; R_7-\left(\frac{1}{2}\right)\times R_5 \to R_7
$$

$$\equiv$$

$$
\left(\begin{array}{ccccccc|c}
1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{\sqrt{5}+1}{2} & 0 & \frac{\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{-\sqrt{5}-3}{2} & 0 & \frac{-\sqrt{5}-5}{2} & 0 & 0 \\
0 & 0 & 0 & 1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{5}+1}{4} & 0
\end{array}\right)
\times\left(\frac{-\sqrt{5}-1}{4}\right)
\quad
\overset{?}{\sim}\; R_7-\left(\frac{\sqrt{5}+1}{4}\right)\times R_6 \to R_7
$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{\sqrt{5}+1}{2} & 0 & \frac{\sqrt{5}+1}{2} & 0 & 0 \\ 0 & 0 & 1 & \frac{-\sqrt{5}-3}{2} & 0 & \frac{-\sqrt{5}-5}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times\left(\frac{-\sqrt{5}-1}{2}\right) \quad R_4-\left(\frac{\sqrt{5}+1}{2}\right)\times R_5 \to R_4 \quad \overset{?}{\sim}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{\sqrt{5}+1}{2} & 0 & \frac{\sqrt{5}+1}{2} & 0 & 0 \\ 0 & 0 & 1 & \frac{-\sqrt{5}-3}{2} & 0 & \frac{-\sqrt{5}-5}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times\left(\frac{\sqrt{5}+5}{2}\right) \quad R_3-\left(\frac{-\sqrt{5}-5}{2}\right)\times R_5 \to R_3 \quad \overset{?}{\sim}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{\sqrt{5}+1}{2} & 0 & \frac{\sqrt{5}+1}{2} & 0 & 0 \\ 0 & 0 & 1 & \frac{-\sqrt{5}-3}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times\left(\frac{-\sqrt{5}-1}{2}\right) \quad R_2-\left(\frac{\sqrt{5}+1}{2}\right)\times R_5 \to R_2 \quad \overset{?}{\sim}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{-\sqrt{5}-3}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times\left(\frac{\sqrt{5}+3}{2}\right) \quad R_3-\left(\frac{-\sqrt{5}-3}{2}\right)\times R_4 \to R_3 \quad \overset{?}{\sim}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \sqrt{5}+2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times\left(\frac{-\sqrt{5}-1}{2}\right) \quad \underset{R_2-\left(\frac{\sqrt{5}+1}{2}\right)\times R_4\to R_2}{\overset{?}{\sim}}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & \frac{\sqrt{5}+1}{2} & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{-\sqrt{5}-3}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \sqrt{5}+2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times\left(\frac{-\sqrt{5}-1}{2}\right) \quad \underset{R_1-\left(\frac{\sqrt{5}+1}{2}\right)\times R_3\to R_1}{\overset{?}{\sim}}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & \frac{\sqrt{5}+1}{2} & 0 & 0 & \frac{-3\times\sqrt{5}-7}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{-\sqrt{5}-3}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \sqrt{5}+2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times\left(\frac{-\sqrt{5}-1}{2}\right) \quad \underset{R_1-\left(\frac{\sqrt{5}+1}{2}\right)\times R_2\to R_1}{\overset{?}{\sim}} \quad \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & \frac{-\sqrt{5}-3}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{-\sqrt{5}-3}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \sqrt{5}+2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{\sqrt{5}+1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\equiv \qquad \equiv$$

$$\begin{cases} x_1 - \frac{\sqrt{5}+3}{2}\times x_5 = 0 \\ x_2 - \frac{\sqrt{5}+3}{2}\times x_5 = 0 \\ x_3 + \left(\sqrt{5}+2\right)\times x_5 = 0 \\ x_4 + \frac{\sqrt{5}+1}{2}\times x_5 = 0 \\ x_6 = 0 \\ x_7 = 0 \end{cases} \quad (1)$$

$$\equiv$$

- Find the variable x_7 from the equation 6 of the system (1):
  $x_7=0$
- Find the variable x_6 from the equation 5 of the system (1):

$x_6=0$

- Find the variable x_4 from the equation 4 of the system (1):
  $x_4=\frac{-\sqrt{5}-1}{2}\times x_5$
- Find the variable x_3 from the equation 3 of the system (1):
  $x_3=\left(-\sqrt{5}-2\right)\times x_5$
- Find the variable x_2 from the equation 2 of the system (1):
  $x_2=\frac{\sqrt{5}+3}{2}\times x_5$
- Find the variable x_1 from the equation 1 of the system (1):
  $x_1=\frac{\sqrt{5}+3}{2}\times x_5$

Answer:

- $x_1=\frac{\sqrt{5}+3}{2}\times x_5$
- $x_2=\frac{\sqrt{5}+3}{2}\times x_5$
- $x_3=\left(-\sqrt{5}-2\right)\times x_5$
- $x_4=\frac{-\sqrt{5}-1}{2}\times x_5$
- $x_5=x_5$
- $x_6=0$
- $x_7=0$

General Solution $: X=\begin{pmatrix} \frac{\sqrt{5}+3}{2}\times x_5 \\ \frac{\sqrt{5}+3}{2}\times x_5 \\ \left(-\sqrt{5}-2\right)\times x_5 \\ \frac{-\sqrt{5}-1}{2}\times x_5 \\ x_5 \\ 0 \\ 0 \end{pmatrix}$
$\equiv$

The solution set: $\{x_5\times\begin{pmatrix} \frac{\sqrt{5}+3}{2} \\ \frac{\sqrt{5}+3}{2} \\ -\sqrt{5}-2 \\ \frac{-\sqrt{5}-1}{2} \\ 1 \\ 0 \\ 0 \end{pmatrix}\}$
$\equiv$

Let $x_5 = 1$, $\boldsymbol{v_6} = \begin{pmatrix} \frac{\sqrt{5}+3}{2} \\ \frac{\sqrt{5}+3}{2} \\ -\sqrt{5}-2 \\ \frac{-\sqrt{5}-1}{2} \\ 1 \\ 0 \\ 0 \end{pmatrix}$

≡

6. $\lambda_6 = \frac{\sqrt{5}+1}{4}$

$$A - \lambda_6 I = \begin{pmatrix} \frac{-\sqrt{5}-1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{-\sqrt{5}-1}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{-\sqrt{5}-1}{4} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{-\sqrt{5}-1}{4} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{-\sqrt{5}+1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-\sqrt{5}-1}{4} & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-\sqrt{5}+1}{4} \end{pmatrix}$$

≡

$Av = \lambda v$ *

$(A - \lambda I)v = 0$

So we have a homogeneous system of linear equations, we solve it by Gaussian Elimination:

$$\left(\begin{array}{ccccccc|c}
\frac{-\sqrt{5}-1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-\sqrt{5}-1}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\
\frac{1}{2} & 0 & \frac{-\sqrt{5}-1}{4} & 0 & 0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{-\sqrt{5}-1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & \frac{-\sqrt{5}+1}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{-\sqrt{5}-1}{4} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-\sqrt{5}+1}{4} & 0
\end{array}\right) \times\left(-\sqrt{5}+1\right)$$

$$\stackrel{?}{\sim} \quad R_1/\left(\frac{-\sqrt{5}-1}{4}\right) \rightarrow R_1$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-\sqrt{5}-1}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\
\frac{1}{2} & 0 & \frac{-\sqrt{5}-1}{4} & 0 & 0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{-\sqrt{5}-1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & \frac{-\sqrt{5}+1}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{-\sqrt{5}-1}{4} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-\sqrt{5}+1}{4} & 0
\end{array}\right) \times\left(\frac{-1}{2}\right)$$

$$\stackrel{?}{\sim} \quad R_3-\left(\frac{1}{2}\right)\times R_1 \rightarrow R_3$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-\sqrt{5}-1}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\
0 & \frac{\sqrt{5}-1}{4} & \frac{-1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{-\sqrt{5}-1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & \frac{-\sqrt{5}+1}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{-\sqrt{5}-1}{4} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-\sqrt{5}+1}{4} & 0
\end{array}\right) \times\left(-\sqrt{5}+1\right)$$

$$\stackrel{?}{\sim} \quad R_2/\left(\frac{-\sqrt{5}-1}{4}\right) \rightarrow R_2$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{-\sqrt{5}+1}{2} & 0 & \frac{-\sqrt{5}+1}{2} & 0 & 0 \\
0 & \frac{\sqrt{5}-1}{4} & \frac{-1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{-\sqrt{5}-1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & \frac{-\sqrt{5}+1}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{-\sqrt{5}-1}{4} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-\sqrt{5}+1}{4} & 0
\end{array}\right) \times\left(\frac{-\sqrt{5}+1}{4}\right) \quad \stackrel{?}{\sim} \quad R_3-\left(\frac{\sqrt{5}-1}{4}\right)\times R_2 \to R_3$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{-\sqrt{5}+1}{2} & 0 & \frac{-\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & \frac{-1}{2} & \frac{-\sqrt{5}+3}{4} & 0 & \frac{-\sqrt{5}+5}{4} & 0 & 0 \\
0 & 0 & 0 & \frac{-\sqrt{5}-1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & \frac{-\sqrt{5}+1}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{-\sqrt{5}-1}{4} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-\sqrt{5}+1}{4} & 0
\end{array}\right) \times(-2) \quad \stackrel{?}{\sim} \quad R_3/\left(\frac{-1}{2}\right) \to R_3$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{-\sqrt{5}+1}{2} & 0 & \frac{-\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{\sqrt{5}-3}{2} & 0 & \frac{\sqrt{5}-5}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{-\sqrt{5}-1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & \frac{-\sqrt{5}+1}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{-\sqrt{5}-1}{4} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-\sqrt{5}+1}{4} & 0
\end{array}\right) \times(-\sqrt{5}+1) \quad \stackrel{?}{\sim} \quad R_4/\left(\frac{-\sqrt{5}-1}{4}\right) \to R_4$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{-\sqrt{5}+1}{2} & 0 & \frac{-\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{\sqrt{5}-3}{2} & 0 & \frac{\sqrt{5}-5}{2} & 0 & 0 \\
0 & 0 & 0 & 1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & \frac{-\sqrt{5}+1}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{-\sqrt{5}-1}{4} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-\sqrt{5}+1}{4} & 0
\end{array}\right) \times\left(\frac{-1}{2}\right) \quad R_5-\left(\frac{1}{2}\right)\times R_4 \to R_5 \quad \overset{?}{\sim}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{-\sqrt{5}+1}{2} & 0 & \frac{-\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{\sqrt{5}-3}{2} & 0 & \frac{\sqrt{5}-5}{2} & 0 & 0 \\
0 & 0 & 0 & 1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\sqrt{5}-1}{4} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{-\sqrt{5}-1}{4} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-\sqrt{5}+1}{4} & 0
\end{array}\right) \times\left(\sqrt{5}+1\right) \quad R_5/\left(\frac{\sqrt{5}-1}{4}\right) \to R_5 \quad \overset{?}{\sim}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{-\sqrt{5}+1}{2} & 0 & \frac{-\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{\sqrt{5}-3}{2} & 0 & \frac{\sqrt{5}-5}{2} & 0 & 0 \\
0 & 0 & 0 & 1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{-\sqrt{5}-1}{4} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-\sqrt{5}+1}{4} & 0
\end{array}\right) \times\left(\frac{\sqrt{5}+1}{4}\right) \quad R_6-\left(\frac{-\sqrt{5}-1}{4}\right)\times R_5 \to R_6 \quad \overset{?}{\sim}$$

$$\equiv$$

$$
\left(\begin{array}{ccccccc|c}
1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{-\sqrt{5}+1}{2} & 0 & \frac{-\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{\sqrt{5}-3}{2} & 0 & \frac{\sqrt{5}-5}{2} & 0 & 0 \\
0 & 0 & 0 & 1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{-\sqrt{5}+1}{4} & 0
\end{array}\right)
\times\left(\frac{-1}{2}\right)
\quad \underset{R_7-\left(\frac{1}{2}\right)\times R_5\to R_7}{\overset{?}{\sim}}
$$

$$\equiv$$

$$
\left(\begin{array}{ccccccc|c}
1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{-\sqrt{5}+1}{2} & 0 & \frac{-\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{\sqrt{5}-3}{2} & 0 & \frac{\sqrt{5}-5}{2} & 0 & 0 \\
0 & 0 & 0 & 1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{-\sqrt{5}+1}{4} & 0
\end{array}\right)
\times\left(\frac{\sqrt{5}-1}{4}\right)
\quad \underset{R_7-\left(\frac{-\sqrt{5}+1}{4}\right)\times R_6\to R_7}{\overset{?}{\sim}}
$$

$$\equiv$$

$$
\left(\begin{array}{ccccccc|c}
1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{-\sqrt{5}+1}{2} & 0 & \frac{-\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{\sqrt{5}-3}{2} & 0 & \frac{\sqrt{5}-5}{2} & 0 & 0 \\
0 & 0 & 0 & 1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right)
\times\left(\frac{\sqrt{5}-1}{2}\right)
\quad \underset{R_4-\left(\frac{-\sqrt{5}+1}{2}\right)\times R_5\to R_4}{\overset{?}{\sim}}
$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{-\sqrt{5}+1}{2} & 0 & \frac{-\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{\sqrt{5}-3}{2} & 0 & \frac{\sqrt{5}-5}{2} & 0 & 0 \\
0 & 0 & 0 & 1 & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right) \times\left(\frac{-\sqrt{5}+5}{2}\right) \quad R_3-\left(\frac{\sqrt{5}-5}{2}\right)\times R_5 \to R_3 \overset{?}{\sim}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{-\sqrt{5}+1}{2} & 0 & \frac{-\sqrt{5}+1}{2} & 0 & 0 \\
0 & 0 & 1 & \frac{\sqrt{5}-3}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right) \times\left(\frac{\sqrt{5}-1}{2}\right) \quad R_2-\left(\frac{-\sqrt{5}+1}{2}\right)\times R_5 \to R_2 \overset{?}{\sim}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & \frac{\sqrt{5}-3}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right) \times\left(\frac{-\sqrt{5}+3}{2}\right) \quad R_3-\left(\frac{\sqrt{5}-3}{2}\right)\times R_4 \to R_3 \overset{?}{\sim}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c}
1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & -\sqrt{5}+2 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right) \times\left(\frac{\sqrt{5}-1}{2}\right) \quad R_2-\left(\frac{-\sqrt{5}+1}{2}\right)\times R_4 \to R_2 \overset{?}{\sim}$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & \frac{-\sqrt{5}+1}{2} & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{\sqrt{5}-3}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -\sqrt{5}+2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times\left(\frac{\sqrt{5}-1}{2}\right) \overset{?}{\sim} R_1-\left(\frac{-\sqrt{5}+1}{2}\right)\times R_3 \rightarrow R_1$$

$$\equiv$$

$$\left(\begin{array}{ccccccc|c} 1 & \frac{-\sqrt{5}+1}{2} & 0 & 0 & \frac{3\times\sqrt{5}-7}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{\sqrt{5}-3}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -\sqrt{5}+2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \times\left(\frac{\sqrt{5}-1}{2}\right) \overset{?}{\sim} R_1-\left(\frac{-\sqrt{5}+1}{2}\right)\times R_2 \rightarrow R_1 \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & \frac{\sqrt{5}-3}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{\sqrt{5}-3}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -\sqrt{5}+2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{-\sqrt{5}+1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\equiv \qquad \equiv$$

$$\left\{\begin{array}{rcl} x_1 + \frac{\sqrt{5}-3}{2}\times x_5 & = & 0 \\ x_2 + \frac{\sqrt{5}-3}{2}\times x_5 & = & 0 \\ x_3 - \left(\sqrt{5}-2\right)\times x_5 & = & 0 \\ x_4 - \frac{\sqrt{5}-1}{2}\times x_5 & = & 0 \\ x_6 & = & 0 \\ x_7 & = & 0 \end{array}\right. \qquad (1)$$

$$\equiv$$

- Find the variable x_7 from the equation 6 of the system (1):
  $x_7=0$
- Find the variable x_6 from the equation 5 of the system (1):
  $x_6=0$
- Find the variable x_4 from the equation 4 of the system (1):
  $x_4=\frac{\sqrt{5}-1}{2}\times x_5$
- Find the variable x_3 from the equation 3 of the system (1):
  $x_3=\left(\sqrt{5}-2\right)\times x_5$
- Find the variable x_2 from the equation 2 of the system (1):
  $x_2=\frac{-\sqrt{5}+3}{2}\times x_5$

- Find the variable x_1 from the equation 1 of the system (1):
  $x_1=\frac{-\sqrt{5}+3}{2}\times x_5$

Answer:

- $x_1=\frac{-\sqrt{5}+3}{2}\times x_5$
- $x_2=\frac{-\sqrt{5}+3}{2}\times x_5$
- $x_3=\left(\sqrt{5}-2\right)\times x_5$
- $x_4=\frac{\sqrt{5}-1}{2}\times x_5$
- $x_5=x_5$
- $x_6=0$
- $x_7=0$

General Solution $: X=\begin{pmatrix} \frac{-\sqrt{5}+3}{2}\times x_5 \\ \frac{-\sqrt{5}+3}{2}\times x_5 \\ \left(\sqrt{5}-2\right)\times x_5 \\ \frac{\sqrt{5}-1}{2}\times x_5 \\ x_5 \\ 0 \\ 0 \end{pmatrix}$

$\equiv$

The solution set: $\left\{x_5\times\begin{pmatrix} \frac{-\sqrt{5}+3}{2} \\ \frac{-\sqrt{5}+3}{2} \\ \sqrt{5}-2 \\ \frac{\sqrt{5}-1}{2} \\ 1 \\ 0 \\ 0 \end{pmatrix}\right\}$

$\equiv$

Let $x_5=1$, $\boldsymbol{v_7}=\begin{pmatrix} \mathbf{\frac{-\sqrt{5}+3}{2}} \\ \mathbf{\frac{-\sqrt{5}+3}{2}} \\ \mathbf{\sqrt{5}-2} \\ \mathbf{\frac{\sqrt{5}-1}{2}} \\ \mathbf{1} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}$

$\equiv$

Clean

With help of this calculator you can: find the matrix determinant, the rank, raise the matrix to a power, find the sum and the multiplication of matrices, calculate the inverse matrix. Just type matrix elements and click the button.

- Leave extra cells *empty* to enter non-square matrices.
- You can use decimal (finite and periodic) fractions: `1/3`, `3.14`, `-1.3(56)`, `1.2e-4`; or arithmetic expressions: `2/3+3*(10-4)`, `(1+x)/y^2`, `2^0.5`, `sin(\phi)`.
- Use ↲ Enter, Space, ←, →, ↑, ↓ to navigate between cells.
- Drag-and-drop matrices from the results, or even from/to a text editor.
- To learn more about matrices use Wikipedia.

▶ Examples

▶ Comments

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