Resolução de Problemas do Livro

Mecânica Clássica (Taylor, J. R.)

por

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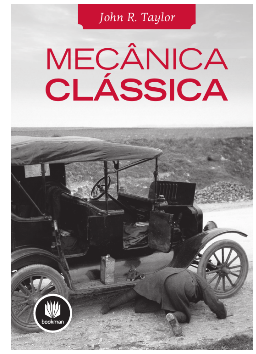

Referência

TAYLOR, J. R.. **Mecânica Clássica**. Porto Alegre, Bookman, 2013.

Capítulo 1: Leis de Newton do Movimento

PROBLEMAS

Seção 1.2 Espaço e Tempo

1.1 Dados dois vetores $\mathbf{v} = \hat{\mathbf{x}} + \hat{\mathbf{y}}$ e $\mathbf{c} = \hat{\mathbf{x}} + \hat{\mathbf{z}}$, determine $\mathbf{b}+\mathbf{c}$, $5\mathbf{b}+2\mathbf{c}$, $\mathbf{b}\cdot\mathbf{c}$ e $\mathbf{b}\times\mathbf{c}$.

Solução:

$$
\begin{aligned}
\mathbf{b}+\mathbf{c} &= (\hat{\mathbf{x}}+\hat{\mathbf{y}}) + (\hat{\mathbf{x}}+\hat{\mathbf{z}}) = \hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{x}}+\hat{\mathbf{z}} = 2\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}} \\
5\mathbf{b}+2\mathbf{c} &= 5(\hat{\mathbf{x}}+\hat{\mathbf{y}}) + 2(\hat{\mathbf{x}}+\hat{\mathbf{z}}) = 5\hat{\mathbf{x}}+5\hat{\mathbf{y}}+2\hat{\mathbf{x}}+2\hat{\mathbf{z}} = 7\hat{\mathbf{x}}+5\hat{\mathbf{y}}+2\hat{\mathbf{z}} \\
\mathbf{b}\cdot\mathbf{c} &= (\hat{\mathbf{x}}+\hat{\mathbf{y}})\cdot(\hat{\mathbf{x}}+\hat{\mathbf{z}}) = \hat{\mathbf{x}}\cdot\hat{\mathbf{x}}+\hat{\mathbf{x}}\cdot\hat{\mathbf{z}}+\hat{\mathbf{y}}\cdot\hat{\mathbf{x}}+\hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = 1+0+0+0 = 1 \\
\mathbf{b}\times\mathbf{c} &= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \hat{\mathbf{x}}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - \hat{\mathbf{y}}\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} + \hat{\mathbf{z}}\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \\
&= \hat{\mathbf{x}}(1\cdot1-0\cdot0) - \hat{\mathbf{y}}(1\cdot1-1\cdot0) + \hat{\mathbf{z}}(1\cdot0-1\cdot1) = \hat{\mathbf{x}}-\hat{\mathbf{y}}-\hat{\mathbf{z}}
\end{aligned}
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1.2 Dois vetores são dados como $\mathbf{b} = (1,2,3)$ e $\mathbf{c} = (3,2,1)$. (Lembre-se de que essas declarações são uma forma compacta de fornacer as componentes dos vetores.) Determine $\mathbf{b}+\mathbf{c}$, $5\mathbf{b}+2\mathbf{c}$, $\mathbf{b}\cdot\mathbf{c}$ e $\mathbf{b}\times\mathbf{c}$.

Solução:

$$
\begin{aligned}
\mathbf{b}+\mathbf{c} &= (1+3, 2+2, 3+1) = (4,4,4) = 2\hat{\mathbf{x}}+4\hat{\mathbf{y}}+4\hat{\mathbf{z}} \\
5\mathbf{b}+2\mathbf{c} &= (5\cdot1+2\cdot3, 5\cdot2+2\cdot2, 5\cdot3+2\cdot1) = (5+6, 10+4, 15+2) = (11,14,17) = 11\hat{\mathbf{x}}+14\hat{\mathbf{y}}+17\hat{\mathbf{z}} \\
\mathbf{b}\cdot\mathbf{c} &= 1\cdot3+2\cdot2+3\cdot1 = 3+4+3 = 10 \\
\mathbf{b}\times\mathbf{c} &= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = \hat{\mathbf{x}}\begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} - \hat{\mathbf{y}}\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} + \hat{\mathbf{z}}\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} \\
&= \hat{\mathbf{x}}(2\cdot1-2\cdot3) - \hat{\mathbf{y}}(1\cdot1-3\cdot3) + \hat{\mathbf{z}}(1\cdot2-3\cdot2) = -4\hat{\mathbf{x}}+8\hat{\mathbf{y}}-4\hat{\mathbf{z}}
\end{aligned}
$$

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