Color edge preserving smoothing II

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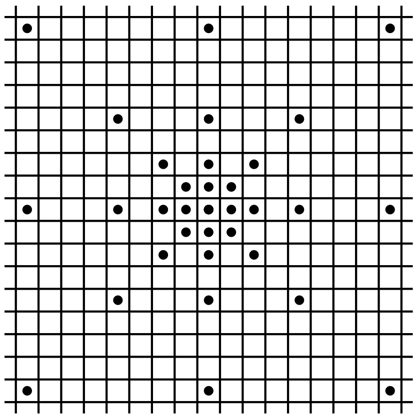
1 Diffusion with control of the directional differences

A diffusion process without any constraints will blur details in images and finally the process converge to one single color. Constraints on diffusion over edges is a much used method for edge preserving smoothing. This is called non-isotropic diffusion. In most methods, the first step is to detect the edges of the image. The presented method uses a constraint that will preserve edges and shade gradients directly without detecting the edges.

This idea is similar to the one presented in the previous paper of Alsam and Rivertz [?,?] from 2011 and 2013. We use a diffusion factor 1/6 horisontally and vertically and a fiffusion factor 1/12 diagonally. The diffusion process is iterative. In the first step, the diffusion is between neighbour pixels, then between second neighbours. The diffusion in the n'th iteration is between 2^{n-1}'th neighbours. We use reflection on the image borders.



We represent an RGB-color image as a function $\mathbf{I}(i,j)$ with values (R,G,B,C) in an affine hyperplane of $\mathbb{R}^4$, where C is a positive constant biger than the maximum noise in the dark regions of the image. Lets consider a 3×3 sub grid centered at (i,j) with distance d between the pixels. Denote the pixel values by

$$\begin{array}{lll} \mathbf{I}_3^d = (\mathbf{I}(i-d,j-d),1) & \mathbf{I}_2^d = \mathbf{I}(i,j-d) & \mathbf{I}_1^d = \mathbf{I}(i+d,j-d) \\ \mathbf{I}_4^d = \mathbf{I}(i-d,j) & \mathbf{I}_8 = \mathbf{I}(i,j). & \mathbf{I}_0^d = \mathbf{I}(i+d,j) \\ \mathbf{I}_5^d = \mathbf{I}(i-d,j+d) & \mathbf{I}_6^d = \mathbf{I}(i,j+d) & \mathbf{I}_7^d = \mathbf{I}(i+d,j+d) \end{array} \quad (1)$$

We decompose each of the fictive pixel values $\mathbf{I}_0^d, \ldots \mathbf{I}_7^d$ as: $\mathbf{I}_i^d = \tilde{\mathbf{I}}_i^d + \mathbf{z}_i^d$, where $\mathbf{z}_i^d \perp \mathbf{I}_8$ and $\tilde{\mathbf{I}}_i^d$ is parallel to $\mathbf{I}_8$. We controll the diffusion of $\tilde{\mathbf{I}}_i^d$ and $\mathbf{z}_i^d$ with $i =$

$0, 1, \ldots, 7$ onto the center pixel $\mathbf{I}_8$. A temporary center pixel value is computed $\mathbf{I}_8' = s\tilde{\mathbf{I}}_i^d + (1-s)\mathbf{I}_8$ where $s \in (0,1)$. The diffusion $\mathbf{I}_8 \to \mathbf{I}_8'$ along the direction i is allowed if

$$\frac{\left\|\tilde{\mathbf{I}}_j^1 - \mathbf{I}_8'\right\|^2 - \left\|\tilde{\mathbf{I}}_j^1 - \mathbf{I}_8\right\|^2}{s} \le \alpha \tag{2}$$

for all $j = 0, 1, \ldots, 7$, where $0 \le \alpha$. This condition can be rewritten as

$$s\left\|\tilde{\mathbf{P}}_i^d\right\|^2 - 2\tilde{\mathbf{P}}_i^d \cdot \tilde{\mathbf{P}}_j^1 \le \alpha \tag{3}$$

where $\tilde{\mathbf{P}}_i^k = \tilde{\mathbf{I}}_i^k - \mathbf{I}_8$, $k = 1, d$. The diffusion $\mathbf{I}_8 \mapsto s\mathbf{z}_i^d + (1-s)\mathbf{I}_8$ for $i = 0, 1, \ldots, 7$ is admissible if

$$\left\|\mathrm{Proj}_{\mathbf{z}_i^d}\, \mathbf{z}_j^1 - s\mathbf{z}_i^d\right\|^2 - \left\|\mathrm{Proj}_{\mathbf{z}_i^d}\, \mathbf{z}_j^1\right\|^2 \le s\beta \tag{4}$$

for all $j = 0, 1, \ldots, 7$, where $s > 0$ and $0 \le \beta$. This condition can be rewritten as

$$s\left\|\mathbf{z}_i^d\right\|^2 - 2\mathbf{z}_i^d \cdot \mathbf{z}_j^1 \le \beta \tag{5}$$

Let A_1 and A_2 denote the set of indices where (3) and (5) are satisfied respectively. The new center pixel value will be

$$\mathbf{I}_8 + s\sum_{i\in A_1} \tilde{\mathbf{P}}_i^d + s\sum_{i\in A_2} \mathbf{z}_i^d.$$