Buck Converter Design

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Goals

To design a buck converter which can take in a large range of input voltages and output a controllable current to a battery. It should be robust to different input voltages and different battery models.

I chose some made-up parameters which I will replace once I am ready to design the real PCB. For example, I gave the battery a nominal voltage of 12V with a 0.1Ω internal resistance. This will be updated once I do some testing / read the datasheet on the batteries. Also, I did not worry about choosing resistor and capacitor values which exist in real life. This simulation is just to prove the concept that I can create a buck converter with a working control loop.

In [ ]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib.ticker as mtick
from UliEngineering.EngineerIO import format_value
from si_prefix import si_format
import plecs_helper as helper
%matplotlib
%matplotlib inline

# Imports and setup
from pint import UnitRegistry

# pandas display using scientific notation
# pd.set_option('display.float_format', lambda x: f'{x:.3e}')

# use pint
units = UnitRegistry()
units.default_format = "~P.2f"

def to_db(gain):
    return np.log10(np.abs(gain)) * 20

def to_decibels(gain):
    return to_db(gain)

def from_db(db):
    return 10 ** (db / 20)
```

```
Using matplotlib backend: TkAgg
```

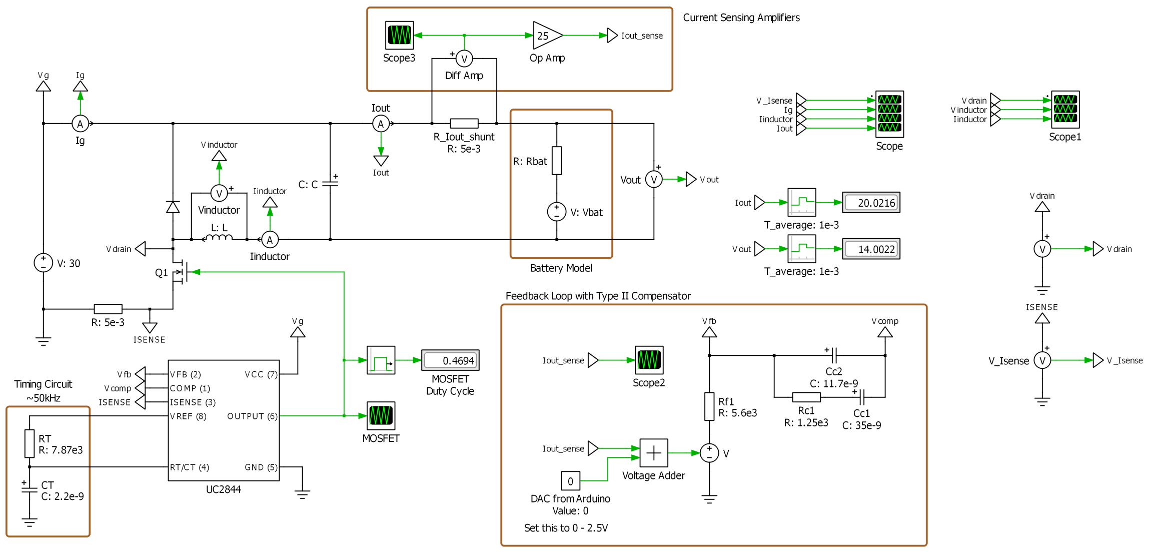
Schematic

The buck converter follows the one we developed in class. The new features are a battery model in place of a load resistor and a current-sensing control loop instead of an output voltage control loop.

The battery model consists of a 12V nominal voltage source in series with a 0.1Ω resistor. This is based on what Beat told me to do. I might be able to replace it with a more accurate battery model, but I do not know if I need to. At least, I will perform measurements or read the datasheet to find the parameters of the real battery I will be using.

The current control loop consists of a shunt resistor in series with the battery straddled by a differential amplifier. The diff amp acts like a voltmeter since it finds the voltage drop across the resistor. I want a nominal 20A charging rate, so I chose a shunt resistance value of 5mΩ. That means a 20A current will create a 0.1V drop across the shunt resistor. The diff amp's value is fed into an op-amp configured as a non-inverting amplifier with a gain of 25. Thus the 0.1V drop will become a 2.5V signal. This is the signal that is fed into the UC2844 and the compensator.

There is one more step, which is a voltage adder connected to an analog pin coming from an Arduino. This voltage is added to the amplified current-sensing signal, such that the output will reach 2.5V at a lower voltage. It turns out that the control loop obeys this voltage adder and works to make the output current equal to $20A \cdot (1 - \frac{V_{control}}{2.5V})$, where $V_{control}$ is the analog voltage created by the Arduino. I am probably going to implement this voltage adder using the same op amp as I will use for the gain.



Define measured constants and nominal values

I aimed for nominally operating very close to boundary condition mode (BCM), but slightly in CCM. Since it is operating with a large amount of current, I thought it was a good balance between energy stored in the inductor and allowing current to flow continuously into the battery (reducing ripple).

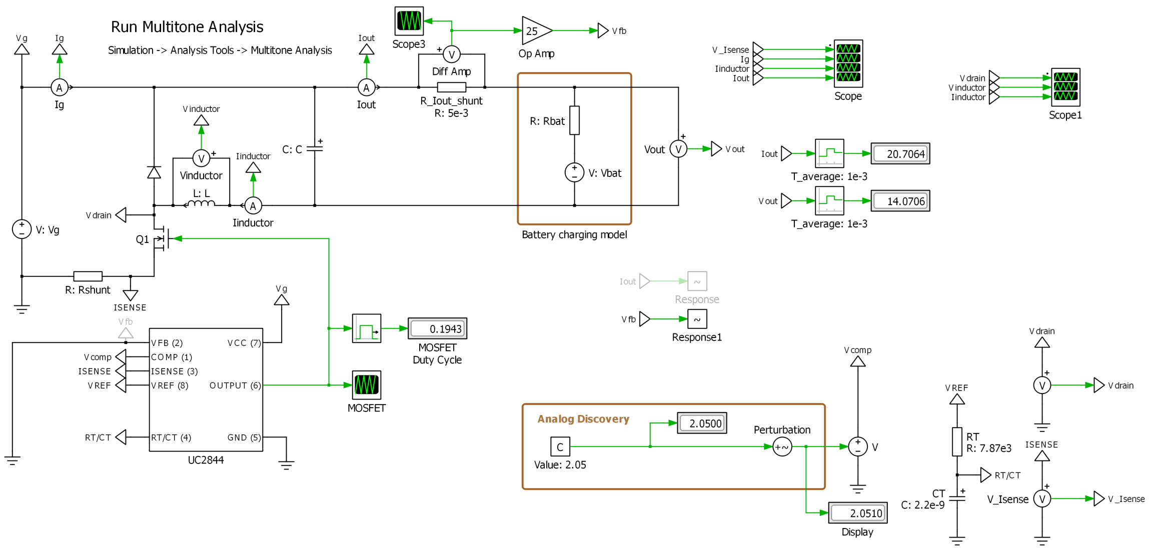
```
In [ ]: Cout = 100 * units.microfarad
        Fs = 50 * units.kilohertz
        Ts = 1 / Fs
        Ibat = 20 * units.amp
        Rbat = 0.1 * units.ohm
        Vg_nominal = 40 * units.volt
        Vbat_nominal = 12 * units.volt + Ibat * Rbat
        Rbat = 0.1 * units.ohm
        Vout_nominal = Vbat_nominal + Ibat * Rbat
        D_nominal = Vout_nominal / Vg_nominal
        Imin = 2 * units.amp
        L = Vout_nominal / (2 * (Ibat - Imin)) * (1 - D_nominal) * Ts

        print(f"D = {np.round(D_nominal, 2)}")
        print(f"L = {L.to_compact(units.henry)}")
        print(f"Vout = {Vout_nominal.to_compact(units.volt)}")
```

```
D = 0.40
L = 5.33 µH
Vout = 16.00 V
```

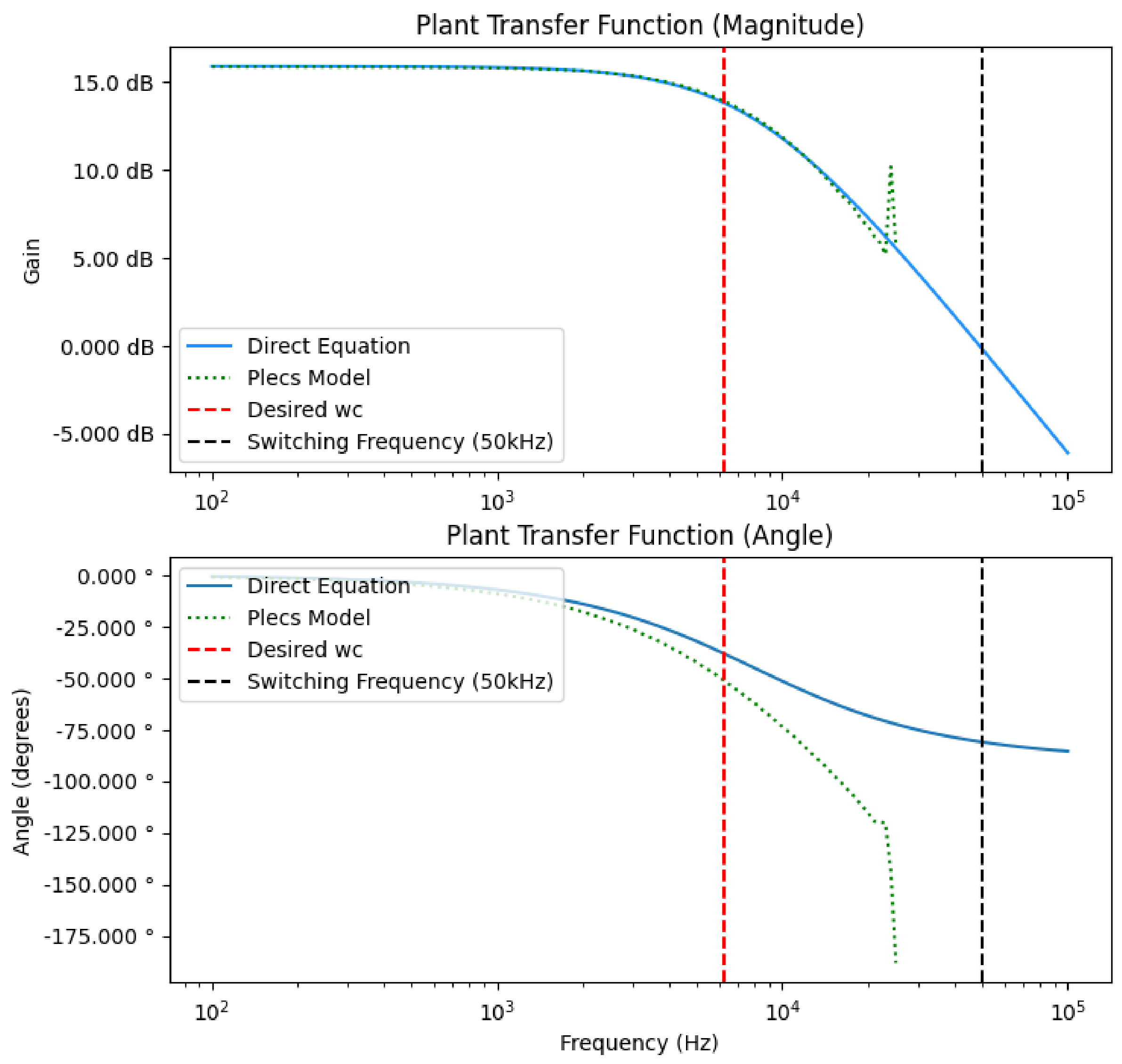
Plant Frequency Analysis

Ok, so here's where I took a shortcut. Before doing any frequency analysis or compensator design in class, we went through a whole derivation of the large-signal and small-signal models for the buck-boost converter and came up with a theoretical prediction of what the plant transfer function would look like. I tried to comprehend the latter half of Chapter 12 in *Fundamentals of Power Electronics*, but I got lost in all the math and could not reproduce this derivation for the buck converter. I found Table 12.3 in that textbook which listed the simple model transfer function as being $\frac{\hat{v}}{\hat{i}_c} = \frac{R}{1+sRC}$, where $\frac{\hat{v}}{\hat{i}_c} \overset{?}{=} G_p$ is presumably the transfer function I am looking for and R is the load resistance and C is the load capacitance. Simple enough! I hope it's right.

With that in hand, I went into PLECS and made a bode plot of the plant transfer function just like we did with the physical hardware in Lab 9. Below, I plotted the simulated results of the multitone analysis. Apart from a relatively large constant offset, it matches the textbook equation pretty well.



```
In [ ]: df_plant = pd.read_csv("plecs_data/plant_bode.csv")
        df_plant.set_axis(["w", "CH1", "CH2"], axis = "columns", inplace = True)

        expected_wc = 50e3 / 8 # to get it into Hz
        print(f"Lead element center voltage (ω_c): {si_format(expected_wc, precision = 2)}H
        w = np.logspace(2, 5, 100) # * units.hertz

        R_Iout_shunt = 5e-3 # * units.ohm
        Rload = 0.1 # * units.ohm
        Cload = 200e-6 # * units.farad
        mystery_factor = 50 # for a total of * 62.5 = 5^4 / 10 = 5^3 / 2
        theoretical_plant_bode = Rload / (1 + (2 * np.pi * 1j * w) * Rload * Cload) / Rload

        fig, (ax1, ax2) = plt.subplots(nrows = 2, ncols = 1, sharex = False, sharey = False
        helper.axes_labels("", "Hz", "Gain", "dB", title = "Plant Transfer Function (Magnit
        ax1.semilogx(w, to_decibels(theoretical_plant_bode), linestyle = "solid", color = "
        ax1.semilogx(df_plant["w"], df_plant["CH1"], linestyle = "dotted", color = "green",
        ax1.axvline(x = expected_wc, linestyle = "dashed", color = "red", label = "Desired 
        ax1.axvline(x = 50e3, linestyle = "dashed", color = "black", label = "Switching Fre
        ax1.legend(loc = "lower left")

        helper.axes_labels("Frequency (Hz)", "Hz", "Angle (degrees)", "°", title = "Plant T
        ax2.semilogx(w, np.angle(theoretical_plant_bode, deg = True), linestyle = "solid", 
        ax2.semilogx(df_plant["w"], df_plant["CH2"], linestyle = "dotted", color = "green",
        ax2.axvline(x = expected_wc, linestyle = "dashed", color = "red", label = "Desired 
        ax2.axvline(x = 50e3, linestyle = "dashed", color = "black", label = "Switching Fre
        ax2.legend(loc = "upper left")

        Lead element center voltage (ω_c): 6.25 kHz

Out[ ]: <matplotlib.legend.Legend at 0x1f226a04c10>
```



Ok, so the PLECS model matches the theory pretty well. You will notice that the magnitude plot matches almost exactly, but I said they were off by a significant factor. What I've shown in the graph above is after multiplying the theoretical curve by a magical factor of 50 to make them line up.

```
    mystery_factor = 50
    theoretical_plant_bode = Rload / (1 + (2 * np.pi * 1j * w) * Rload
    * Cload) / Rload * R_Iout_shunt * 25 * mystery_factor
```

More clarification on the above line of code: `Rload / (1 + (2 * np.pi * 1j * w) * Rload * Cload)` This is the equation given in the textbook as $\frac{R}{1+sRC}$, with some terms to convert from real-valued `w` in Hertz to complex-valued s in radians per second. But this equation is designed to capture the transfer function from I_{peak} to V_{out}, and I want from V_{COMP} to V_{FB}. To convert V_{out} into V_{FB}, I use `/ Rload` to convert output voltage into to current (assuming the equation was designed for a regular load resistor, not a battery model), `* R_Iout_shunt` to convert the current into the voltage seen by the diff amp, and `* 25` for the amplifier. There is still a mystery factor of `* mystery_factor`, which I assume originates from the conversion of V_{COMP} through the innards of the UC2844 and into I_{peak}.

There is obviously some second order behaviour going on above a couple of kHz, especially for the angle, but I'm hoping I can just ignore that.

Finding the compensator values

Using code copied from Lab 11

Design parameters are a gain cross-over frequency at 6.25kHz, gain factor of -13dB to make the gain crossover occur at that frequency (obtained from the simulated transfer function above), and phase boost of 30 degrees. I used a Type II compensator consisting of an integrating pole and a lead element like we used in class for the buck-boost converter.

In [ ]:
```
class Compensator:
    def __init__(self, Rf1_unitless):
        self.Rf1 = Rf1_unitless * units.ohm
        self.find_values()

    def __repr__(self) -> str:
        return f"K: {self.K},\t Gc0: {self.Gc0.to_compact()},\t Rf1: {self.Rf1.to_c

    def find_values(self):
        self.wc = 50 * units.kilohertz * units.revolutions / 8

        self.angle_boost = 30 * units.degree
        self.K = np.tan((self.angle_boost / 2 + 45 * units.degree))
        self.Kp = 1 / from_db(13) # from_db(32)
        self.wz = self.wc / self.K
        self.wp = self.wc * self.K
        self.Gc0 = self.Kp * self.wc * np.sqrt((1 + self.K ** (-2)) / (1 + self.K *
        self.Cc1 = 1 / (self.Gc0 * self.Rf1)
        self.Rc1 = 1 / (self.wz * self.Cc1)
        self.Cc2 = 1 / (self.wp * self.Rc1)

for R in [2, 2.2, 2.7, 3.3, 3.9, 4.3, 4.7, 5.1, 5.6, 6.2, 6.8, 7.5, 8.2, 10]: # val
    R_to_highlight = 5.6
    if R == R_to_highlight:
        print() # highlight the chosen value
    compensator = Compensator(R * 1e3)
    print(compensator)
    if R == R_to_highlight:
        print()
```

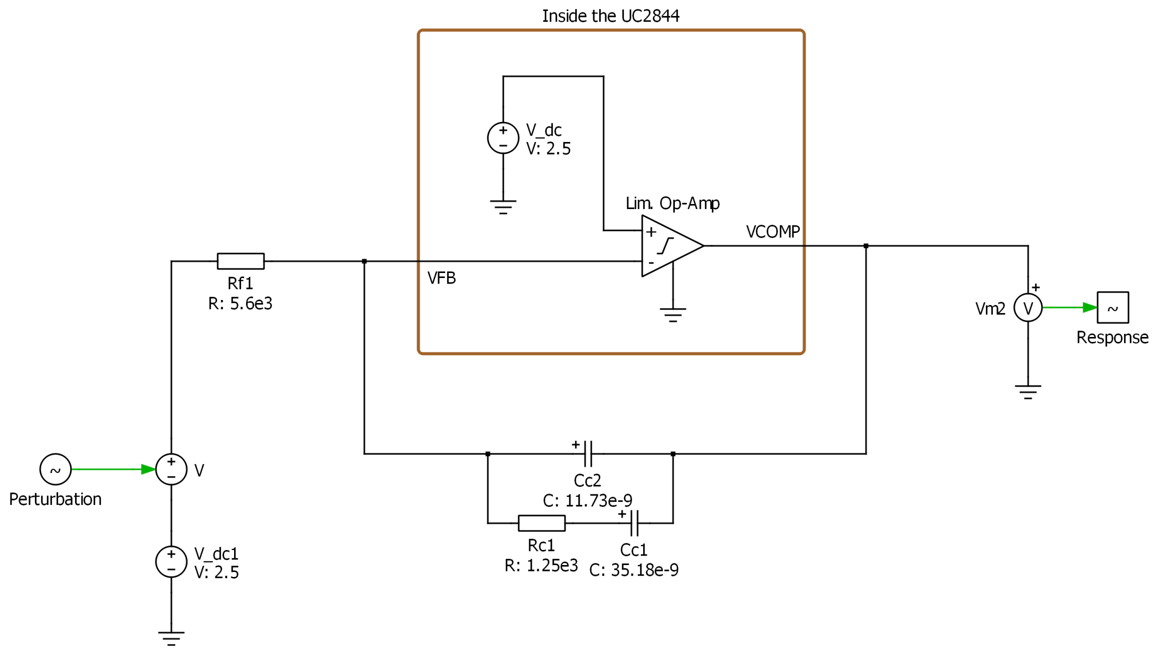
```
K: 1.73,         Gc0: 807.83 Hz·turn,    Rf1: 2.00 kΩ,   Rc1: 447.74 Ω,  Cc1: 98.5
1 nF,    Cc2: 32.84 nF
K: 1.73,         Gc0: 807.83 Hz·turn,    Rf1: 2.20 kΩ,   Rc1: 492.52 Ω,  Cc1: 89.5
5 nF,    Cc2: 29.85 nF
K: 1.73,         Gc0: 807.83 Hz·turn,    Rf1: 2.70 kΩ,   Rc1: 604.45 Ω,  Cc1: 72.9
7 nF,    Cc2: 24.32 nF
K: 1.73,         Gc0: 807.83 Hz·turn,    Rf1: 3.30 kΩ,   Rc1: 738.78 Ω,  Cc1: 59.7
0 nF,    Cc2: 19.90 nF
K: 1.73,         Gc0: 807.83 Hz·turn,    Rf1: 3.90 kΩ,   Rc1: 873.10 Ω,  Cc1: 50.5
2 nF,    Cc2: 16.84 nF
K: 1.73,         Gc0: 807.83 Hz·turn,    Rf1: 4.30 kΩ,   Rc1: 962.65 Ω,  Cc1: 45.8
2 nF,    Cc2: 15.27 nF
K: 1.73,         Gc0: 807.83 Hz·turn,    Rf1: 4.70 kΩ,   Rc1: 1.05 kΩ,   Cc1: 41.9
2 nF,    Cc2: 13.97 nF
K: 1.73,         Gc0: 807.83 Hz·turn,    Rf1: 5.10 kΩ,   Rc1: 1.14 kΩ,   Cc1: 38.6
3 nF,    Cc2: 12.88 nF

K: 1.73,         Gc0: 807.83 Hz·turn,    Rf1: 5.60 kΩ,   Rc1: 1.25 kΩ,   Cc1: 35.1
8 nF,    Cc2: 11.73 nF

K: 1.73,         Gc0: 807.83 Hz·turn,    Rf1: 6.20 kΩ,   Rc1: 1.39 kΩ,   Cc1: 31.7
8 nF,    Cc2: 10.59 nF
K: 1.73,         Gc0: 807.83 Hz·turn,    Rf1: 6.80 kΩ,   Rc1: 1.52 kΩ,   Cc1: 28.9
7 nF,    Cc2: 9.66 nF
K: 1.73,         Gc0: 807.83 Hz·turn,    Rf1: 7.50 kΩ,   Rc1: 1.68 kΩ,   Cc1: 26.2
7 nF,    Cc2: 8.76 nF
K: 1.73,         Gc0: 807.83 Hz·turn,    Rf1: 8.20 kΩ,   Rc1: 1.84 kΩ,   Cc1: 24.0
3 nF,    Cc2: 8.01 nF
K: 1.73,         Gc0: 807.83 Hz·turn,    Rf1: 10.00 kΩ,  Rc1: 2.24 kΩ,   Cc1: 19.7
0 nF,    Cc2: 6.57 nF
```

Compensator Transfer Function

The values I chose for the compensator are shown in the schematic below. These will change when I update the model parameters for my final design, but the behavior of the compensator should remain the same.

Run Multitone Analysis

Simulation -> Analysis Tools -> Multitone Analysis



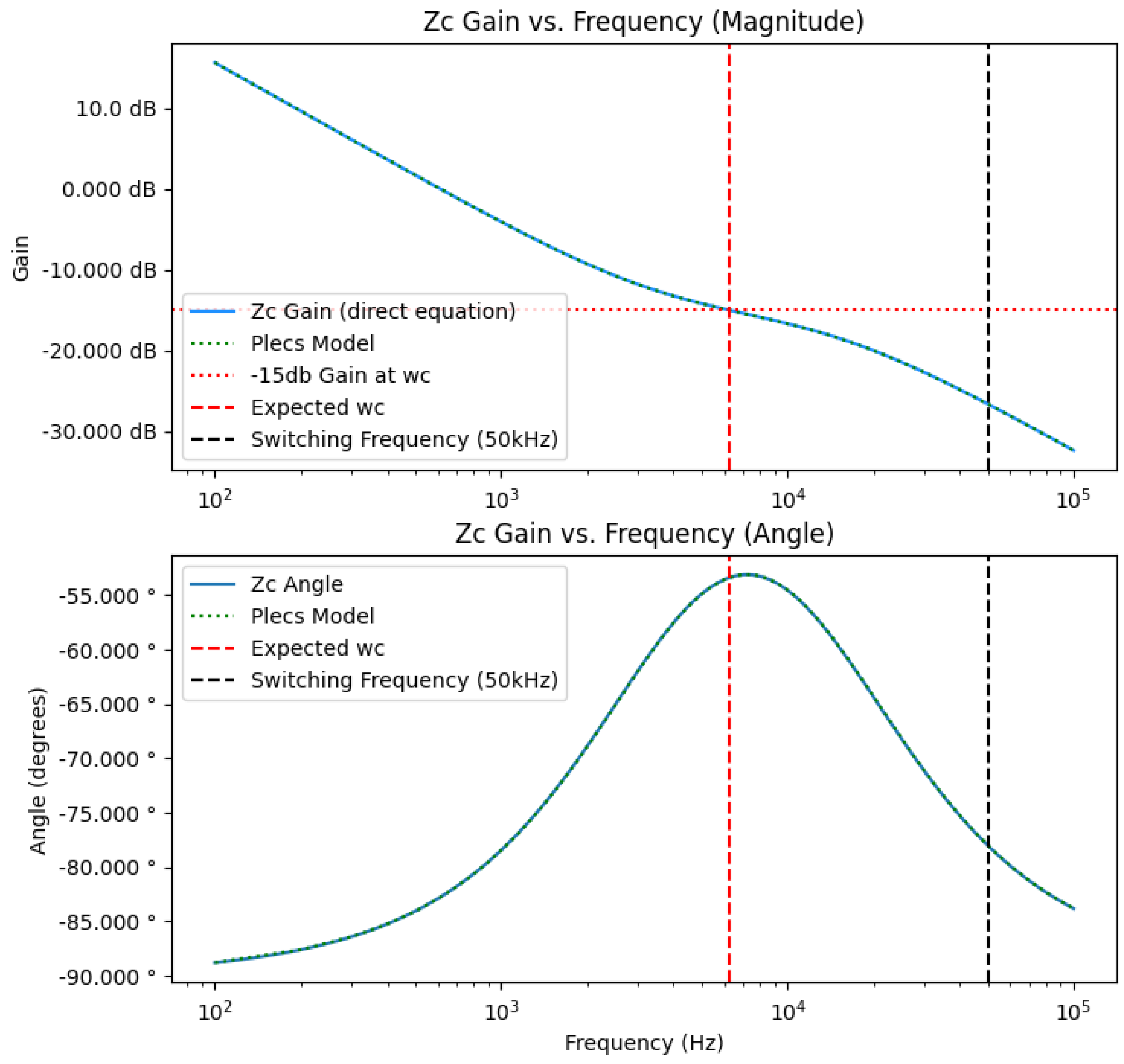
I ran the frequency analysis in PLECS to determine the transfer function, as well as writing down the equation based on the components. They match exactly.

```
In [ ]: df_compensator = pd.read_csv("plecs_data/compensator_bode.csv")
        df_compensator.set_axis(["w", "CH1", "CH2"], axis = "columns", inplace = True)

        # chosen values
        compensator = Compensator(5.6 * 1e3)
        Rf1 = compensator.Rf1.to(units.ohm).magnitude
        Rc1 = compensator.Rc1.to(units.ohm).magnitude
        Cc1 = compensator.Cc1.to(units.farad).magnitude
        Cc2 = compensator.Cc2.to(units.farad).magnitude
        expected_wc = 1 / (Rc1 * np.sqrt(Cc1 * Cc2)) / 2 / np.pi # to get it into Hz
        print(f"Lead element center voltage (w_c): {si_format(expected_wc, precision = 2)}H

        w = np.logspace(2, 5, 100) # * units.hertz
        def get_Zc(my_w):
            my_Zc = 1 / (1 / (Rc1 + 1 / (2 * np.pi * 1j * my_w * Cc1)) + 1 / (1 / (2 * np.p
            return my_Zc
        Zc = get_Zc(w) # direct equation

        fig, (ax1, ax2) = plt.subplots(nrows = 2, ncols = 1, sharex = False, sharey = False
        helper.axes_labels("", "Hz", "Gain", "dB", title = "Zc Gain vs. Frequency (Magnitud
        ax1.semilogx(w, to_decibels(Zc), linestyle = "solid", color = "dodgerblue", label =
        ax1.semilogx(df_compensator["w"], df_compensator["CH1"], linestyle = "dotted", colo
        ax1.axhline(y = -15, linestyle = "dotted", color = "red", label = "-15db Gain at wc
        ax1.axvline(x = expected_wc, linestyle = "dashed", color = "red", label = "Expected
        ax1.axvline(x = 50e3, linestyle = "dashed", color = "black", label = "Switching Fre
        ax1.legend(loc = "lower left")
```

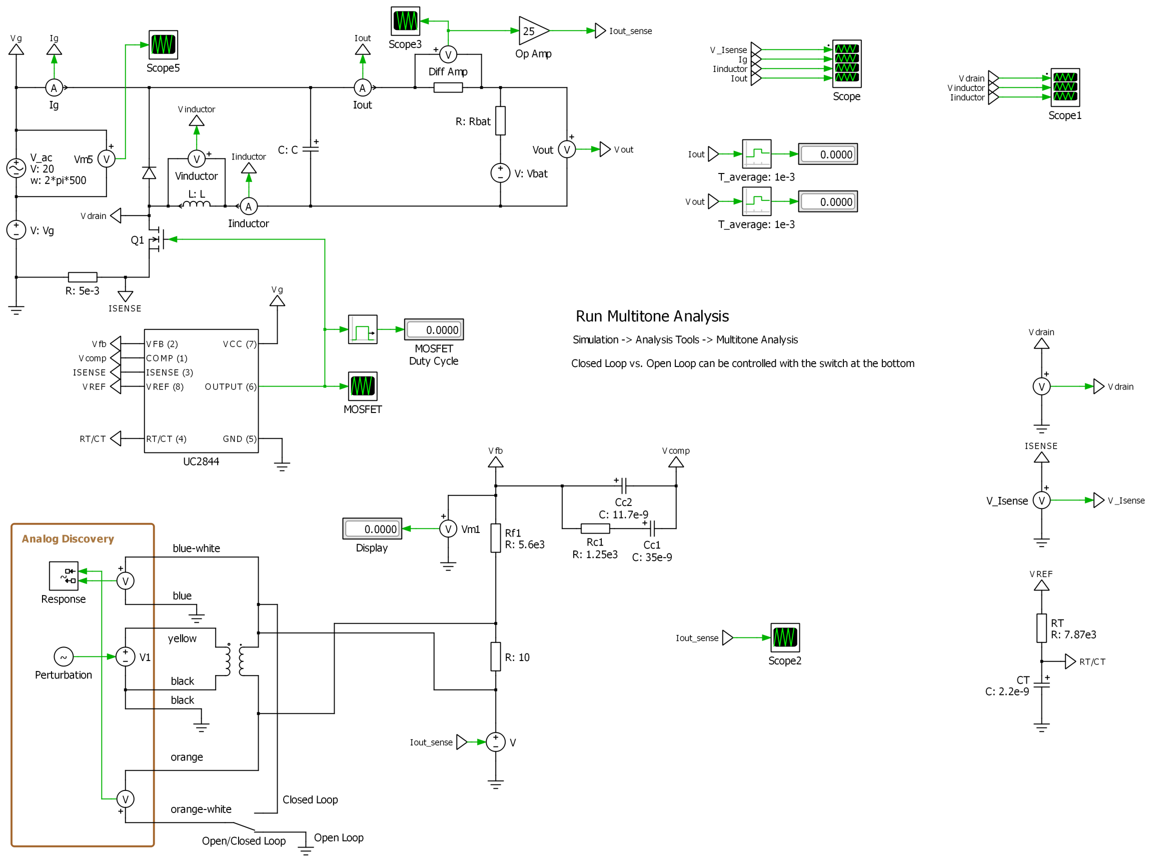
```
helper.axes_labels("Frequency (Hz)", "Hz", "Angle (degrees)", "°", title = "Zc Gain
ax2.semilogx(w, np.angle(Zc, deg = True), linestyle = "solid", label = "Zc Angle")
ax2.semilogx(df_compensator["w"], df_compensator["CH2"] + 180, linestyle = "dotted"
ax2.axvline(x = expected_wc, linestyle = "dashed", color = "red", label = "Expected
ax2.axvline(x = 50e3, linestyle = "dashed", color = "black", label = "Switching Fre
ax2.legend(loc = "upper left")
```

```
Lead element center voltage (ω_c): 6.25 kHz
```

```
Out[ ]: <matplotlib.legend.Legend at 0x1f226b29d00>
```



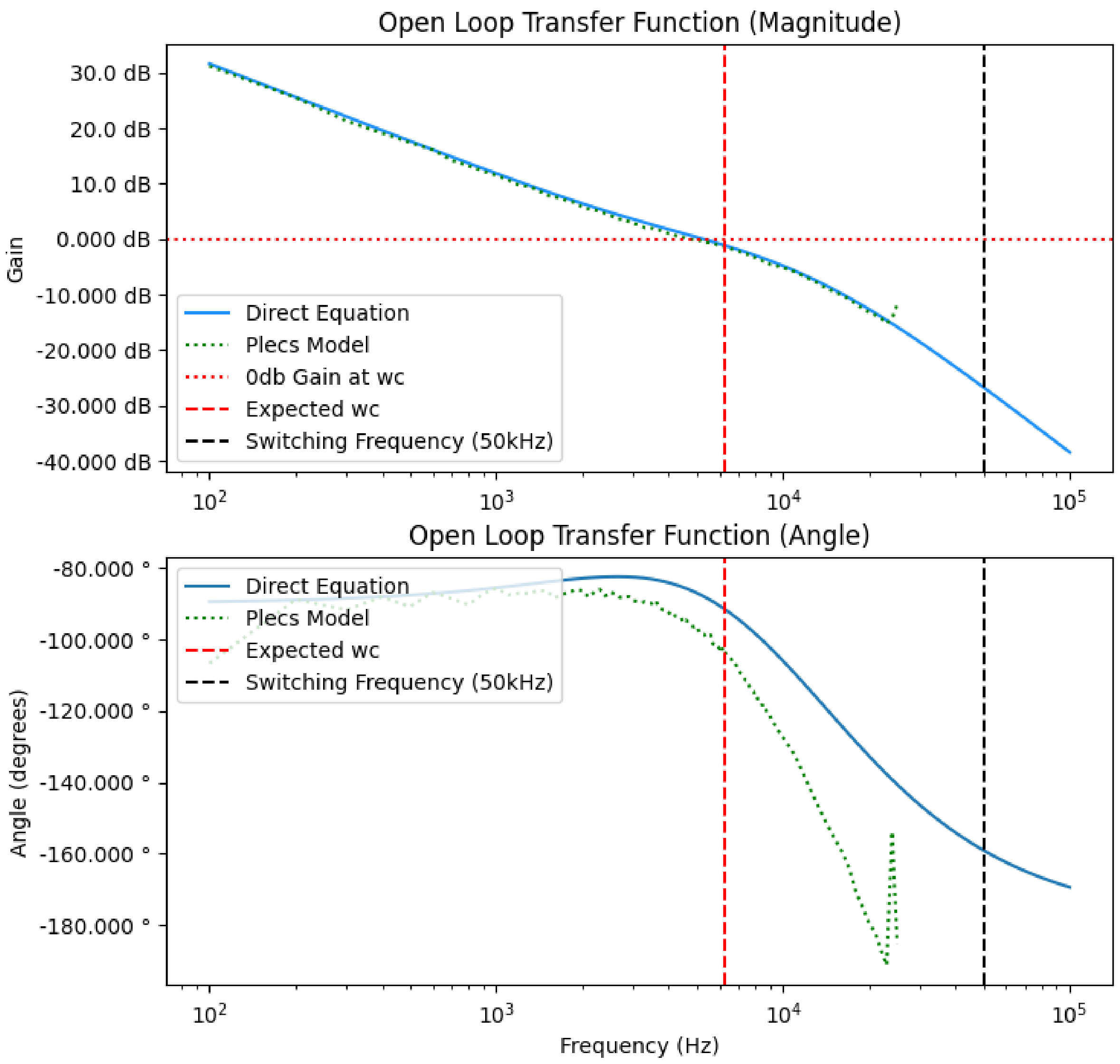
Open-Loop Transfer Function



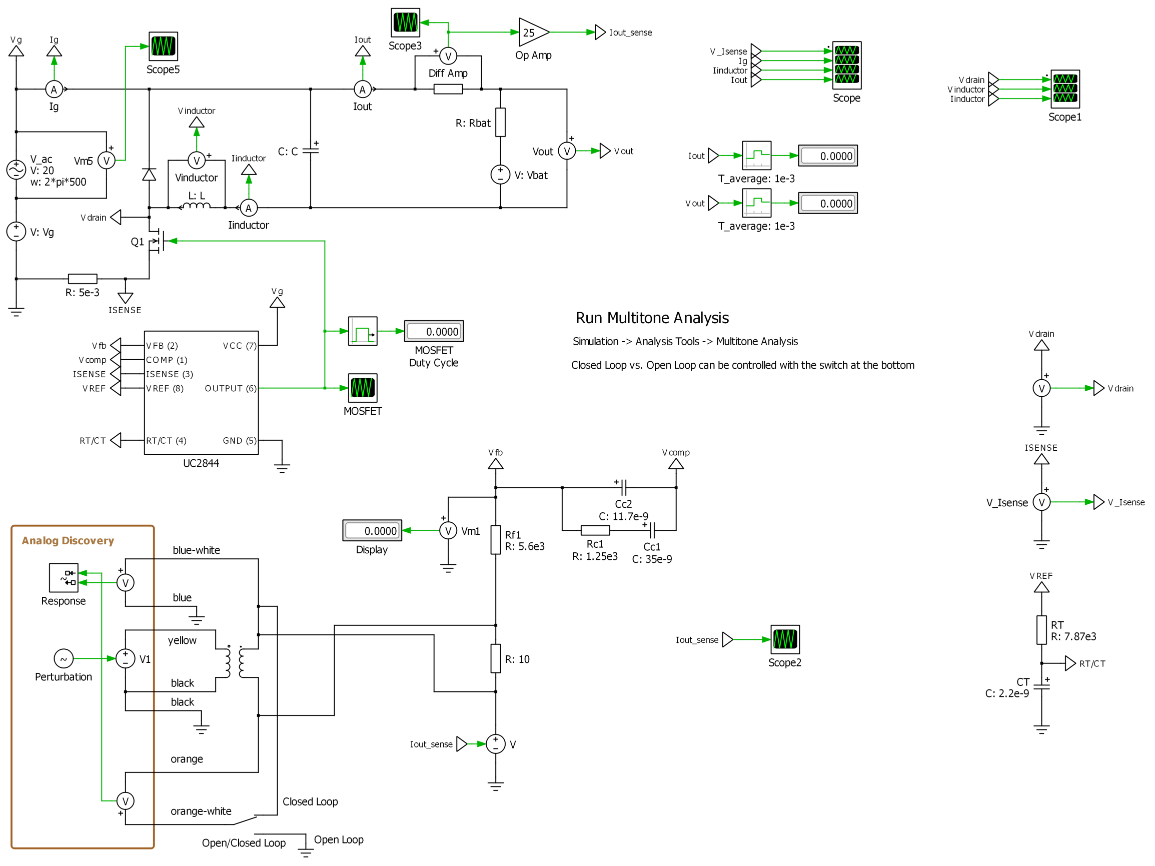
I implemented the compensator in PLECS and ran open-loop and closed-loop frequency analyses, copying the Analog Discovery (AD2) model from Lab 10. Lo and behold, it does the things we want, including infinite open-loop gain at DC, a crossover frequency where we wanted it, and an excellent phase margin between 70 and 90 degrees, depending on if you're looking at the PLECS results or the theoretical equation.

In [ ]:
```
df_open_loop = pd.read_csv("plecs_data/open_loop_bode.csv")
df_open_loop.set_axis(["w", "CH1", "CH2"], axis = "columns", inplace = True)

theoretical_open_loop = theoretical_plant_bode * Zc

fig, (ax1, ax2) = plt.subplots(nrows = 2, ncols = 1, sharex = False, sharey = False
helper.axes_labels("", "Hz", "Gain", "dB", title = "Open Loop Transfer Function (Ma
ax1.semilogx(w, to_decibels(theoretical_open_loop), linestyle = "solid", color = "d
ax1.semilogx(df_open_loop["w"], df_open_loop["CH1"], linestyle = "dotted", color =
ax1.axhline(y = 0, linestyle = "dotted", color = "red", label = "0db Gain at wc")
ax1.axvline(x = expected_wc, linestyle = "dashed", color = "red", label = "Expected
ax1.axvline(x = 50e3, linestyle = "dashed", color = "black", label = "Switching Fre
ax1.legend(loc = "lower left")

helper.axes_labels("Frequency (Hz)", "Hz", "Angle (degrees)", "°", title = "Open Lo
ax2.semilogx(w, np.angle(theoretical_open_loop, deg = True), linestyle = "solid", l
ax2.semilogx(df_open_loop["w"], df_open_loop["CH2"], linestyle = "dotted", color =
ax2.axvline(x = expected_wc, linestyle = "dashed", color = "red", label = "Expected
ax2.axvline(x = 50e3, linestyle = "dashed", color = "black", label = "Switching Fre
ax2.legend(loc = "upper left")
```

```
Out[ ]: <matplotlib.legend.Legend at 0x1f226f61a30>
```

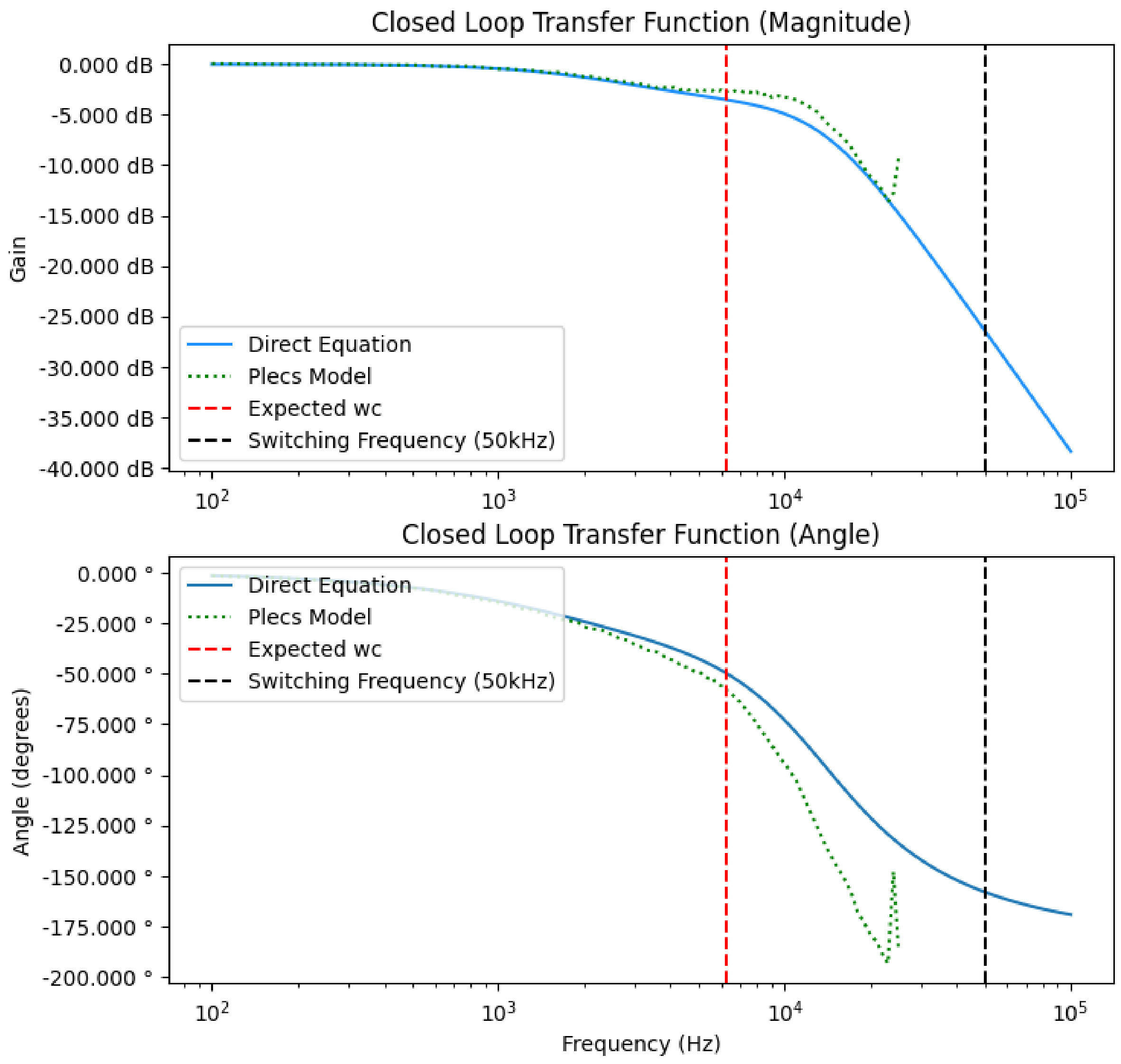


Closed-Loop Transfer Function



Run Multitone Analysis
Simulation -> Analysis Tools -> Multitone Analysis
Closed Loop vs. Open Loop can be controlled with the switch at the bottom
Analog Discovery
Response
Perturbation
UC2844
MOSFET Duty Cycle
Open/Closed Loop
Closed Loop
Open Loop

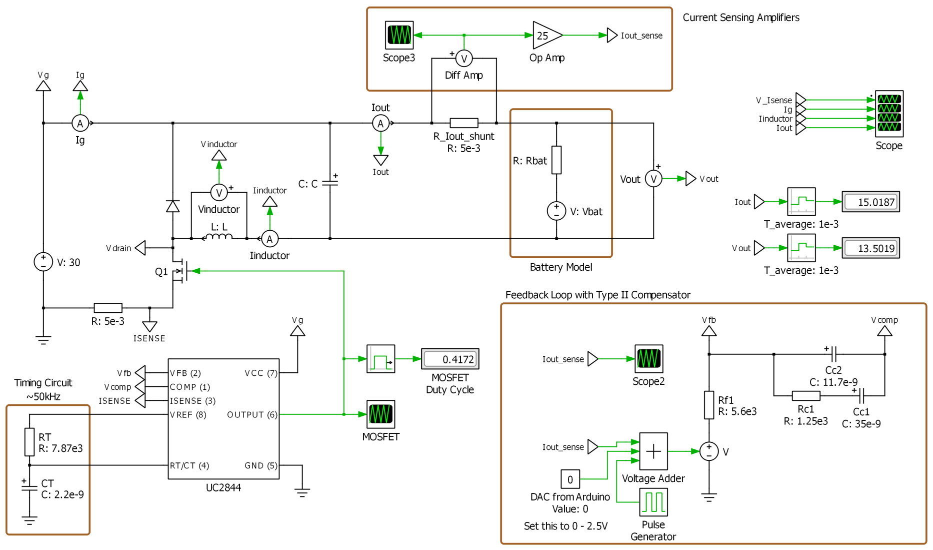
```
In [ ]: df_closed_loop = pd.read_csv("plecs_data/closed_loop_bode.csv")
        df_closed_loop.set_axis(["w", "CH1", "CH2"], axis = "columns", inplace = True)

        theoretical_closed_loop = theoretical_open_loop / (1 + theoretical_open_loop)

        fig, (ax1, ax2) = plt.subplots(nrows = 2, ncols = 1, sharex = False, sharey = False
        helper.axes_labels("", "Hz", "Gain", "dB", title = "Closed Loop Transfer Function (
        ax1.semilogx(w, to_decibels(theoretical_closed_loop), linestyle = "solid", color =
        ax1.semilogx(df_closed_loop["w"], df_closed_loop["CH1"], linestyle = "dotted", colo
        ax1.axvline(x = expected_wc, linestyle = "dashed", color = "red", label = "Expected
        ax1.axvline(x = 50e3, linestyle = "dashed", color = "black", label = "Switching Fre
        ax1.legend(loc = "lower left")

        helper.axes_labels("Frequency (Hz)", "Hz", "Angle (degrees)", "°", title = "Closed
        ax2.semilogx(w, np.angle(theoretical_closed_loop, deg = True), linestyle = "solid",
        ax2.semilogx(df_closed_loop["w"], df_closed_loop["CH2"], linestyle = "dotted", colo
        ax2.axvline(x = expected_wc, linestyle = "dashed", color = "red", label = "Expected
        ax2.axvline(x = 50e3, linestyle = "dashed", color = "black", label = "Switching Fre
        ax2.legend(loc = "upper left")

Out[ ]: <matplotlib.legend.Legend at 0x1f2285b95b0>
```



Again, the theory and the PLECS model match pretty well. The system has a gain of 1 at DC and strong attenuation well before the switching frequency, exactly as we want.

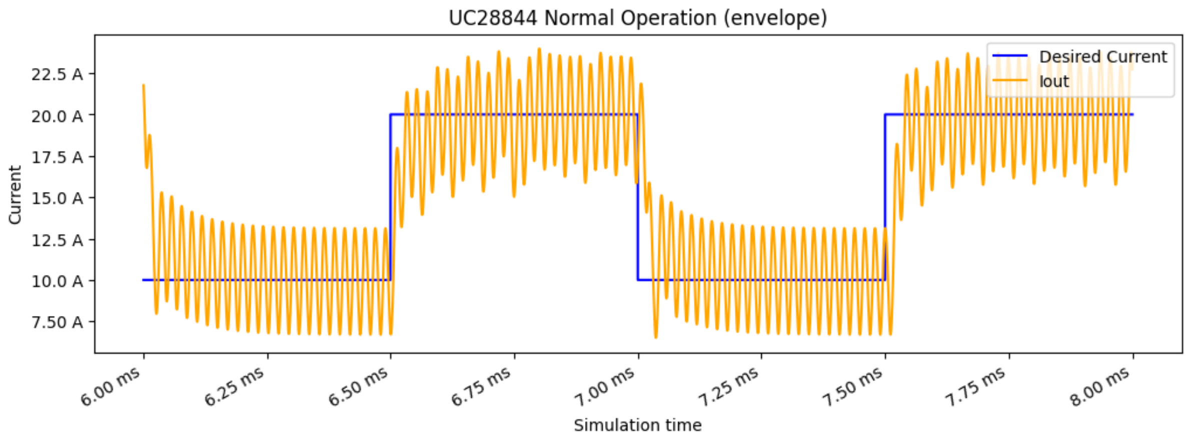
Time Response

Lastly, I implemented a square wave signal to the voltage adder to simulate a change in the desired current.



In [ ]:
```python
df = pd.read_csv("plecs_data/square_wave_response.csv")
df.rename(mapper = helper.strip_labels, axis = "columns", inplace = True)
df["Iexpected"] = 20 * (1 - df["Pulse Generator"] / 2.5)
df_zoom = df.loc[(df["Time / s"] > 6e-3) & (df["Time / s"] < 8e-3)]

fig, (ax1) = plt.subplots(nrows = 1, ncols = 1, sharex = True, sharey = False, figs
# ax2 = ax1.twinx()
fig.autofmt_xdate()
ax1.plot(df_zoom["Time / s"], df_zoom["Iexpected"], color = "blue", label = "Desire
ax1.plot(df_zoom["Time / s"], df_zoom["Iout"], color = "orange", label = "Iout")
helper.axes_labels("Simulation time", "s", "Current", "A", title = "UC28844 Normal
ax1.legend(loc = "upper right")
```

Out[ ]:
```
<matplotlib.legend.Legend at 0x1f225a7feb0>
```



There is significant ripple in the output current, but the average value approaches the set point very quickly. The system appears slightly underdamped, since there is no overshoot, but it still settles within a quarter of a millisecond, which is fast enough for me. The ripple can be significantly reduced by increasing the load capacitance, say from 100uF to 200uF, although this would likely change the plant transfer function and necessitate recalculating the compensator values.

Next Steps

If this design looks good to you, I will start refining the parameter values, such as the input voltage, desired current, and battery model. With the updated parameters, I will re-run the simulation and make sure it behaves as I expect. Then I will probably start speccing physical components and designing a PCB in KiCAD.