Advanced Algorithm Analysis and Design

Assignment 1

Due: Oct. 31, 2024

1. Arrange the following functions in ascending asymptotic order of growth rate:

(a) $f_1(n) = n^{2.024} + 2024^{100}n^2$, $f_2(n) = 2024^{\log n + \log\log n}$, $f_3(n) = \sqrt{n^{3.5}}$,

$f_4(n) = 2^{2n}$, $f_5(n) = 3^n$;

(b) $f_1(n) = n^{\log^2 n}$, $f_2(n) = 2^{\log n + \log\log n}$, $f_3(n) = \log^n \log^2 n$, $f_4(n) = n^{\sqrt{n}\log n}$.

2. Solve the following recurrences:

1、 $f(n) = 12f(\frac{n}{8}) + O(n\log n).$

2、 $f(n) = 3f(n-3) + O(n).$

3、 $f(n) = 3f(\frac{n}{2}) + O(n^2).$

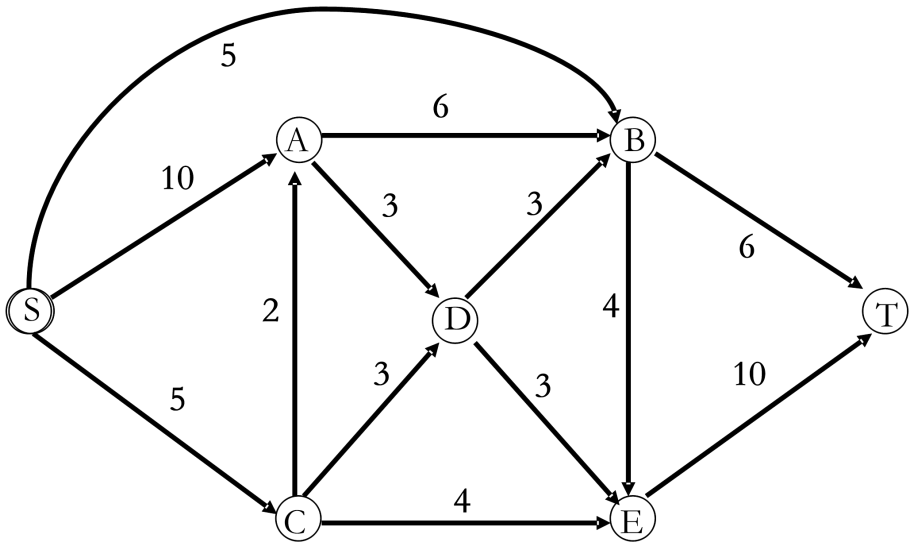
3. Given currency denominations: 1,5,10,25,100, devise a method to pay amount x to customer using fewest number of coins. How about the case that the currency denominations are 1, 5, 7, 35, 70?

4. Please using dynamic programming to solve the following knapsack problem. We are given 7 items and a knapsack. Each item i has weight of $w_i > 0$ kilograms and value of $v_i > 0$ dollars (given in table 1). The capacity of the knapsack is 14 kilograms. Then how to fill the knapsack to maximize the total value?

Items	Weight	Value
1	3	2
2	4	3
3	3	4
4	2	2
5	7	6
6	6	4
7	6	5

Table 1

5. Compute a maximum flow from S to T in the following graph.



6. You have a box of identical eggs and you need to find out the strength of these eggs. The strength of an egg is measured by an integer i from 1 to 9, which corresponds to a height h_i (for any $1 <= i < 9$, we have $h_i < h_{i+1}$) that the egg will not be broken when it is dropped at or below that height. The test is done by dropping an egg at a height h_i. If it is not broken, then pick up the egg and drop it again from a new height $h_j > h_i$; otherwise use a new egg and drop it at a lower height. Repeat the process until the strength of the eggs is determined. Design a strategy that uses as few drops as possible, under the condition that you can break at most two eggs, to determine the strength of the eggs.
   How about under the condition that you can break at most three eggs?