Advanced Algorithm Analysis and Design

Assignment 2

Due: Nov. 11, 2024

1. Write down the main steps of proving the NP-Completeness of a problem.

2. Given a graph, a dominating set is a subset of vertices such that any vertex not in this set is adjacent to at least one vertex in this set. The dominating set problem is to check whether a given graph has a dominating set of size at most k.
   2.1 Prove that the dominating set problem is in NP.
   2.2 Prove that the dominating set problem is NP-hard.

3. Prove that: if we can check whether a graph has a clique (a complete graph) of size k in polynomial time then we can also find a clique of size k in polynomial time.

4. A graph is called a 2-plex if each vertex in the graph is not adjacent to at most one other vertex. Prove that it is NP-complete to check whether an input graph has a sub graph of at least k vertices that is a 2-plex.

5. In the multiway cut problem, we are given a undirected graph G=(V,E) and some special vertices in V (called terminals). The problem asks us to delete the minimum number of edges from the graph such that no pair of terminals is connected. Please give a 2-approximation algorithm for this problem.