Math 258A Challenge #1

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Problem 9: Geometry Challenge via Non-linear Optimization Models

Your challenge is packing m spheres in a box of minimal area. The spheres have a given radius r_i, and the problem is to determine the precise location of the centers x_i. The constraints in this problem are that the spheres should not overlap, and should be contained in a square of center 0 and half-size R. The objective is to minimize the area of the containing box.

(a) Show that two spheres of radius r_1, r_2 and centers x_1, x_2 respectively do not intersect if and only if $\|x_1 - x_2\|_2$ exceeds a certain number, which you will determine.

(b) Formulate the sphere packing problem as an optimization model. Is the formulation you have found convex optimization?

(c) Write (in SCIP, Python, MATLAB, or any other environment) code to solve the packing problem of five and six circular disks of the same radius inside a square of half-size R. What is the optimal size if the disks have radius 1?

(d) Do some drawings using MATLAB of the packings you have discovered. Is the solution unique?

Solution

(a) The sphere with center x_i and radius r_i is denoted by

$$S(x_i, r_i)^n = \{x \quad | \quad \|x - x_i\|_2 = r_i\}$$

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