

# Math 258A Challenge #1

Ian Gallagher

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## Problem 9: Geometry Challenge via Non-linear Optimization Models

Your challenge is packing  $m$  spheres in a box of minimal area. The spheres have a given radius  $r_i$ , and the problem is to determine the precise location of the centers  $x_i$ . The constraints in this problem are that the spheres should not overlap, and should be contained in a square of center 0 and half-size  $R$ . The objective is to minimize the area of the containing box.

- (a) Show that two spheres of radius  $r_1, r_2$  and centers  $x_1, x_2$  respectively do not intersect if and only if  $\|x_1 - x_2\|_2$  exceeds a certain number, which you will determine.
- (b) Formulate the sphere packing problem as an optimization model. Is the formulation you have found convex optimization?
- (c) Write (in SCIP, Python, MATLAB, or any other environment) code to solve the packing problem of five and six circular disks of the same radius inside a square of half-size  $R$ . What is the optimal size if the disks have radius 1?
- (d) Do some drawings using MATLAB of the packings you have discovered. Is the solution unique?

**Solution**

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