1. Find a symmetric matrix $\mathbf{H}$, vector $\vec{b}$, and constant c so that

$$q(x_1, x_2, x_3) = 4x_1^2 + 4x_1x_2 + 2x_1x_3 + x_2^2 + 6x_2x_3 + 3x_3^2 - 2x_1 + x_2 - x_3 + 5 = \frac{1}{2}\vec{x}^T\mathbf{H}\vec{x} + \vec{x}^T\vec{b} + c.$$

2. Suppose that you have a finite-dimensional vector space V whose elements are functions of x (e.g. polynomials or trig functions), and you have a basis for that space $\vec{v}_1, \ldots, \vec{v}_m$. (These 'vectors' are functions.) Assume that you have an inner product on V $\langle \cdot, \cdot \rangle$. You have some function f such that $\langle \vec{v}_i, f \rangle$ exists for all i. You want to find $\vec{v} \in V$ that minimizes $\|f - \vec{v}\|^2 = \langle f - \vec{v}, f - \vec{v} \rangle$. Explain why a solution always exists and give a formula for the solution.

3. Consider the problem of optimizing a quadratic function

$$q(\vec{x}) = \frac{1}{2}\vec{x}^T\mathbf{K}\vec{x} + \vec{x}^T\vec{b} + c$$

subject to a linear equality constraint

$$\vec{d}^T\vec{x} = e \;\text{ i.e. }\; g(\vec{x}) = \vec{d}^T\vec{x} - e = 0$$

where we can assume that $\mathbf{K}$ is symmetric without loss of generality, and also assume $\vec{d} \neq \vec{0}$.

   (a) Apply the method of Lagrange multipliers and write the optimality condition as a linear system of equations for $\vec{x}$ and λ.

   (b) The coefficient matrix in (a) should be symmetric. Explain why it cannot be positive definite even if $\mathbf{K}$ is positive definite.

   (c) Assuming that $\mathbf{K}$ is positive definite, use block Gaussian elimination to find an explicit expression for the solution $\vec{x}$.