Verifying Bell's Theorem Using a CHSH Experiment:

Measuring Entangled Photon Polarizations

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Physics 111B Oral, November 2021

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- 1964: Bell shows quantum mechanics (QM) incompatible with local hidden variable theories (HVTs) via Bell's inequality, experiments testing HVTs are physically realizable
- 1969: CHSH proposes explicit polarization experiment to test HVTs and violate Bell's inequality

Motivation

Is QM an exact theory or is there some more fundamental theory that reduces to QM in the limit that some small parameter goes to 0? This question hasn't been answered yet, but there are many theories.

Violating Bell's inequality shows that any fundamental theory which extends QM or incorporates it cannot be a HVT.

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Bell's Theorem

Start with QM system and hidden variable (hv) information contained in parameter λ. Probability becomes:

$$P(a) = \int \mathrm{d}\lambda \rho(\lambda) A(a, \lambda)$$

Needs to pass three tests

1. Statistical nature of QM measurements must be explained by hv's
2. HVT must preserve locality
3. For an HVT, the expectation value must be equivalent to the QM expectation value

$$P(a) = \int \mathrm{d}\lambda \rho(\lambda) A(a, \lambda) = \langle \Psi | \hat{\mathcal{A}} | \Psi \rangle$$

Bell's Theorem cont.

Bell's Theorem

No HVT can reproduce the predictions of QM

1. Bell showed that any HVT can pass (1) without any issue.
2. However, in showing that hv's explain the randomness in QM measurements, the theory no longer becomes local and so any HVT fails (2).
3. Bell showed using an analytic proof that the expectation values of QM and any HVT do not coincide so any HVT fails (3). This is expressed in Bell's inequality.

CHSH Inequality

The equivalent CHSH ineqality bounds polarization correlations at different polarization angle values.

Probability of observing polarizations p_i and p_j in rotated basis is

$$P_{p_i,p_j}(\alpha, \beta) = \left| \langle p_i|_\alpha \langle p_j|_\beta |\Psi\rangle \right|^2 \tag{1}$$

where $|\Psi\rangle$ is some entangled state and $\langle p_i|_\alpha = \langle p_i| R_\alpha$ where R_α is the standard 2 dimensional rotation matrix through an angle α.

The observable

$$E(\alpha, \beta) = P_{VV}(\alpha, \beta) + P_{HH}(\alpha, \beta) - P_{VH}(\alpha, \beta) - P_{HV}(\alpha, \beta) \tag{2}$$

is a measure of the correlation between polarizations.

CHSH Inequality cont.

The observable S is defined as

$$S(\alpha, \beta, \alpha', \beta') = E(\alpha, \beta) - E(\alpha, \beta') + E(\alpha', \beta) + E(\alpha', \beta') \quad (3)$$

No clear interpretation of S, but CHSH showed

$$S^{(\mathrm{QM})} \leq 2\sqrt{2} \quad (4)$$

and

$$S^{(\mathrm{HVT})} \leq 2 \quad (5)$$

So, observations that yield $S > 2$ violate Bell's inequality and show an HVT cannot reproduce QM predictions.

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Overview of Apparatus

Main Idea

Entangle two photons to get $|\Psi\rangle = \frac{1}{\sqrt{2}}(|H\rangle_s |H\rangle_i + |V\rangle_s |V\rangle_i)$ and measure their polarization correlations for various angles

1. Polarize photons at some angle θ_l using a half-wave plate
2. Apply a phase shift ϕ_l to counteract impending phase shift from downconversion
3. Downconvert 405nm photons to 810nm photon pairs using non-linear crystals
4. Use half-wave plates to adjust measurement basis and beam splitters to collapse wave functions

Creating the Down-converted State

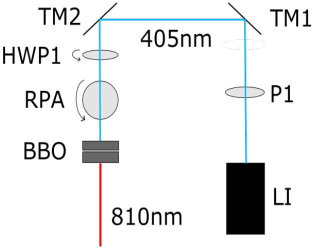


Figure: LI: Laser and Isolator, P1: Polarizer, TM: Tuning Mirrors, HWP1: Half-wave plate, RPA: Relative phase adjuster, BBO: Non-linear crystal

Start by sending $|\psi\rangle$ through HWP1 yielding

$$|\psi\rangle = \cos\theta_l\, |V\rangle_p + \sin\theta_l\, |H\rangle_p \qquad (6)$$

then the RPA to get

$$|\psi\rangle = \cos\theta_l\, |V\rangle_p + e^{i\phi_l} \sin\theta_l\, |H\rangle_p \qquad (7)$$

The BBOs downconvert the 405nm wave to an 810nm entangled photon pair

BBOs and Down-conversion

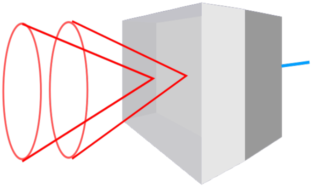


Figure: 405nm beam enters both BBOs with optical axes rotated 90 degrees from each other. Downconverted entangled photons exit.

Each BBO downconverts one polarization, ignoring the other:

$$|V\rangle_p \rightarrow \quad |H\rangle_s |H\rangle_i \tag{8}$$

$$|H\rangle_p \rightarrow \quad e^{i\Delta} |V\rangle_s |V\rangle_i \tag{9}$$

where Δ is the phase shift introduced due to imperfect conditions and birefringence. We end up with

$$|\Psi\rangle_{DC} = \cos\theta_l |H\rangle_s |H\rangle_i + e^{i\phi} \sin\theta |V\rangle_s |V\rangle_i \tag{10}$$

Measuring the State

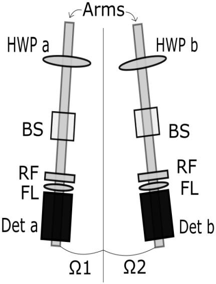


Figure: Arms: adjustment arms to change the angle at which the dc beam hits the optical components, BS: beam splitters, RF: long pass filter, FL: focusing lens, Dets: photon detectors, Ω: arm angles

HWP a (b) polarizes the beam with respect to an angle α (β):

$$\begin{pmatrix} |V\rangle_\alpha \\ |H\rangle_\alpha \end{pmatrix} = R_\alpha \begin{pmatrix} |V\rangle \\ |H\rangle \end{pmatrix} = \begin{pmatrix} \cos\alpha\,|V\rangle - \sin\alpha\,|H\rangle \\ \sin\alpha\,|V\rangle + \cos\alpha\,|H\rangle \end{pmatrix} \tag{11}$$

The BS only allow a certain polarization to pass making the photons distinguishable, while the RF allow red light to filter through

Coincidence Counting

How the detector works:

- A coincidence is an event in which both detectors register a photon hit within a 5ns window

What do we expect?

$$N(\alpha, \beta) = \sum_i \sum_j A\big(P_{p_i,p_j}(\alpha, \beta)\big) + C \tag{12}$$

$$P_{p_i p_j}(\alpha, \beta) = \left| \langle p_i|_\alpha \langle p_j|_\beta |\Psi\rangle_{DC} \right|^2 \tag{13}$$

$$C = N(0^\circ, 90^\circ) \tag{14}$$

$$A = N(0^\circ, 0^\circ) + N(90^\circ, 90^\circ) - 2C \tag{15}$$

Coincidence Counting cont.

Both theories show strong and weak correlations but QM theory predicts a sinusoidally varying change in correlations where as for HVTs it is a linearly changing correlation.

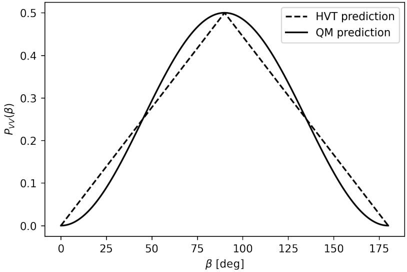
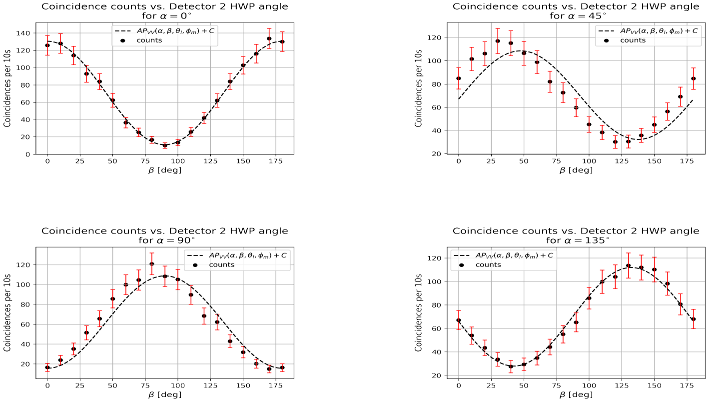


Figure: Probability of observing VV polarization at angle β for $\alpha = 90°$. Note the stationary points for the QM predictions.

Tuning the Bell State



Figures: Polarization correlations between detectors A and B. Error bars are Poissonian errors.

Tuning the Bell State cont.

Using the graphs above and our calculated values for A and C, we can get initial polarization angle and relative phase adjustment angle:

$$\theta_l = \arctan\sqrt{\frac{N(90^\circ, 90^\circ) - C}{N(0^\circ, 0^\circ) - C}} \approx 42.6^\circ \tag{16}$$

$$\phi_m = \arccos\left[\frac{1}{2\sin\theta_l}\left(4\frac{N(45^\circ, 45^\circ)}{A} - 1\right)\right] \approx 27.4^\circ \tag{17}$$

This is a good setup for a well-tuned Bell state.

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Results and Analysis

We chose 4 angles for both α and β. This gives us 16 total measurements for combinations of α and β. Our results can be seen in the following graphs

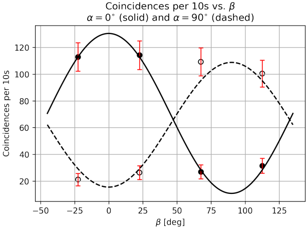

Figure: Coincidence counts for $\alpha = 0^\circ, 90^\circ$. The predicted counts for 90° are lower than the actual counts based on our previous tuning and instrumentation handling competency.

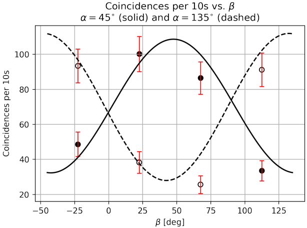

Figure: Coincidence counts for $\alpha = 45^\circ, 135^\circ$. For the 45° counts, it's clear that there is a systematic error in measurement of what 45° was. The anomalous counts for 135° could be due to incompetency.

Results and Analysis cont.

Using the measurements obtained, S was calculated using equations (2) and (3)

$$S = 2.088 \pm 0.104 \tag{18}$$

Bell's inequality was violated, but by more than less a standard deviation.

Counting Errors

Errors:

- Ambient light $\Rightarrow$ artificially increased counts
- BBO imperfections e.g., imperfect phase matching, spacings, etc. $\Rightarrow$ undercounting
- Imperfect alignment (arm angles, beam-splitter placement, etc.) $\Rightarrow$ undercounting
- Detector imperfections $\Rightarrow$ go either way
- The most likely source: user error

Error Propagation

What kinds of errors are expected? With accidental counts we expect Poissonian statistics

- constant background flux $\Rightarrow$ constant rate
- background count events independent of each other

Since Poissonian statistics are expected this means

$$\sigma_N = \sqrt{N}$$

and so

$$\sigma_S = \sqrt{\sum_i \sigma_{N_i} \left(\frac{\partial S}{\partial N_i}\right)^2} = \sqrt{\sum_i N_i \left(\frac{\partial S}{\partial N_i}\right)^2} \tag{19}$$

Interpreting Our Results

Although our results only violate Bell's inequality to within 1 s.d., can something be said of our results?

- We still measured photon polarization correlations that followed a sinusoidal pattern
- Probability has stationary points as predicted by QM unlike HVT
- Cannot confirm nor deny QM or HVT hypotheses due to low confidence, but our data does align more with QM hypothesis

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Concluding Remarks

- The predictions of QM have enjoyed much success since their inception nearly 100 years ago
- Bell had showed that these predictions could not be recreated in a local hidden variable framework
- CHSH showed this to be the case with an explicit experiment
- We carried this experiment out and measured a value of S that violates Bell's inequality
- This method is robust for violating Bell's inequality, but proper care must be taken when tuning the state and taking measurements to ensure accuracy as well as precision