Logistic regression - introduction

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Introduction

These notes are a basic introduction to binary logistic regression used to analyse data with binary - yes/no - outcomes.

In these notes we'll aim to cover the following learning outcomes:

- **Refresher** on logs, odds, probability and linear regression
- Understand why linear regression not sensible for **binary data**
- Explain how **logit** and binomial model let us **extend linear regression**
- Be able to run a **simple logistic regression in R**
- Be able to explain basic R glm **output**
- Be able to explain **estimates** with categorical and continuous variables
- Explain **significance test results** on variables
- Introduce some basic ideas for **selecting variables and models**
- **Things to watch out for!**
- **Know where to go next!**

Prerequisites

We'll cover a couple of background topics but to follow these notes you'll need to be able to load packages in R, run simple code in R and have a basic understanding of linear regression and statistical hypothesis tests. To run the R code in these notes you'll need to load in the example data-set and have a few packages downloaded and loaded into your R session.

Downloading the packages (you only need to do this if you haven't already downloaded them)

```
install.packages("tidyverse")
install.packages("boot")
install.packages("broom")
install.packages("skimr")
install.packages("sjPlot")
```

Loading the packages...

```
library(tidyverse)
library(boot)
library(broom)
library(skimr)
library(sjPlot)
```

Loading the data (from the csv file)...

```r
dat <- read_csv("logreg_data_01_20190530.csv")
```

note for stella: need to say how to get the data

Revision / background topics

Logarithms ('logs')

Skip this if you are happy with logs (including base 'e')

$$log_{10}(10) = 1$$

$$log_{10}(1000) = 3$$

$$log_{10}(0.01) = -2$$

We can have other bases e.g. e

$$log_e(2.718) \simeq 1$$

And reversing this...

$$10^3 = 1000$$

$$e^2 \simeq 7.389$$

Odds and probability

Probabilities have values from 0 ('never happens') to 1 ('always happens')

'events of interest' ÷ 'all events'

What is the probability that a fair coin lands on heads?

$$1/2 = 0.5$$

What is the probability that a 6 sided die lands on 4?

$$1/6 \simeq 0.166$$

Odds have values from 0 ('never happen') to infinity ('always happens')

'events of interest' ÷ 'other events'

What is the odds that a fair coin lands on heads?

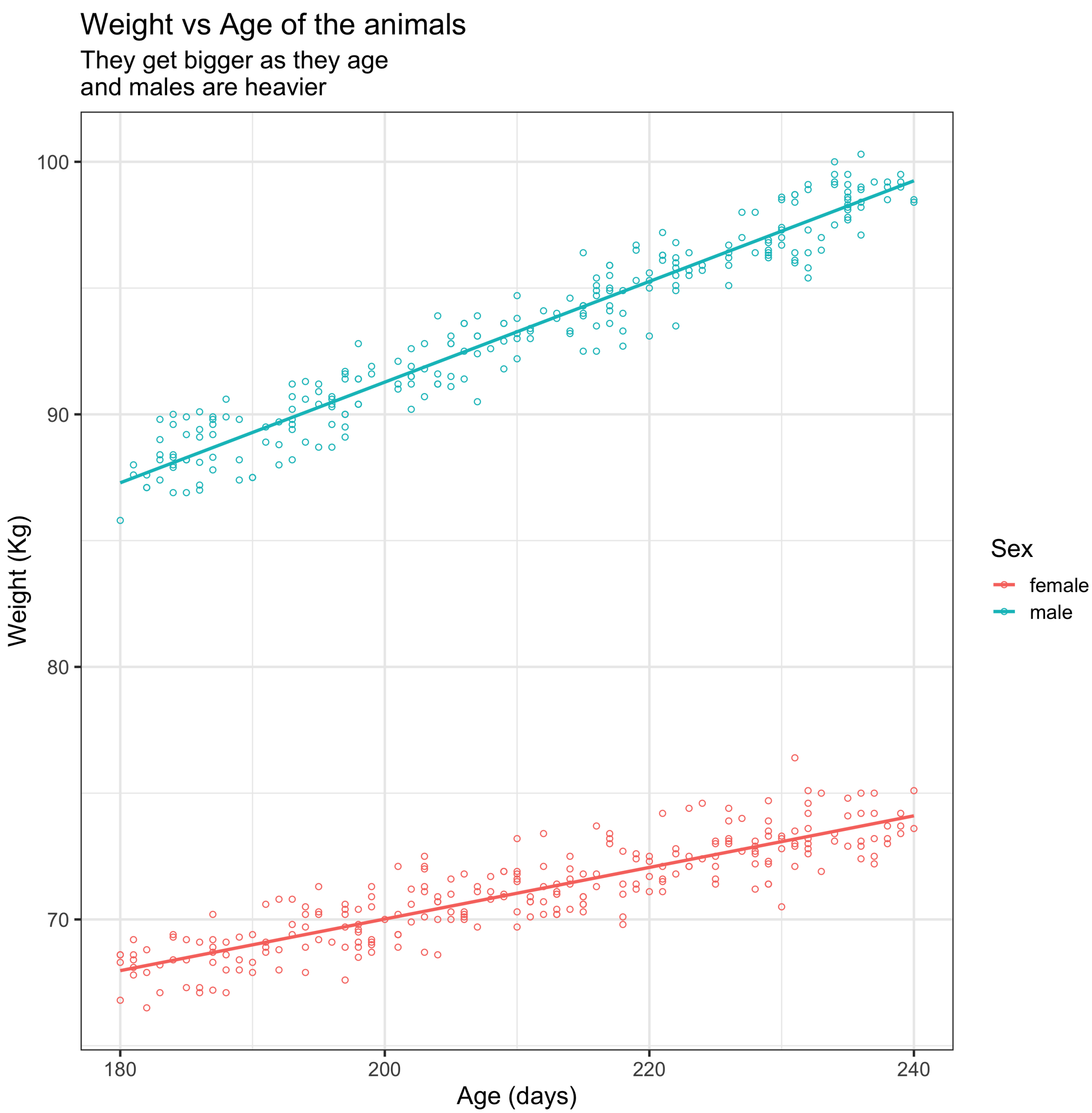
$$1/1 = 1$$

What is the odds that a 6 sided die lands on 4?

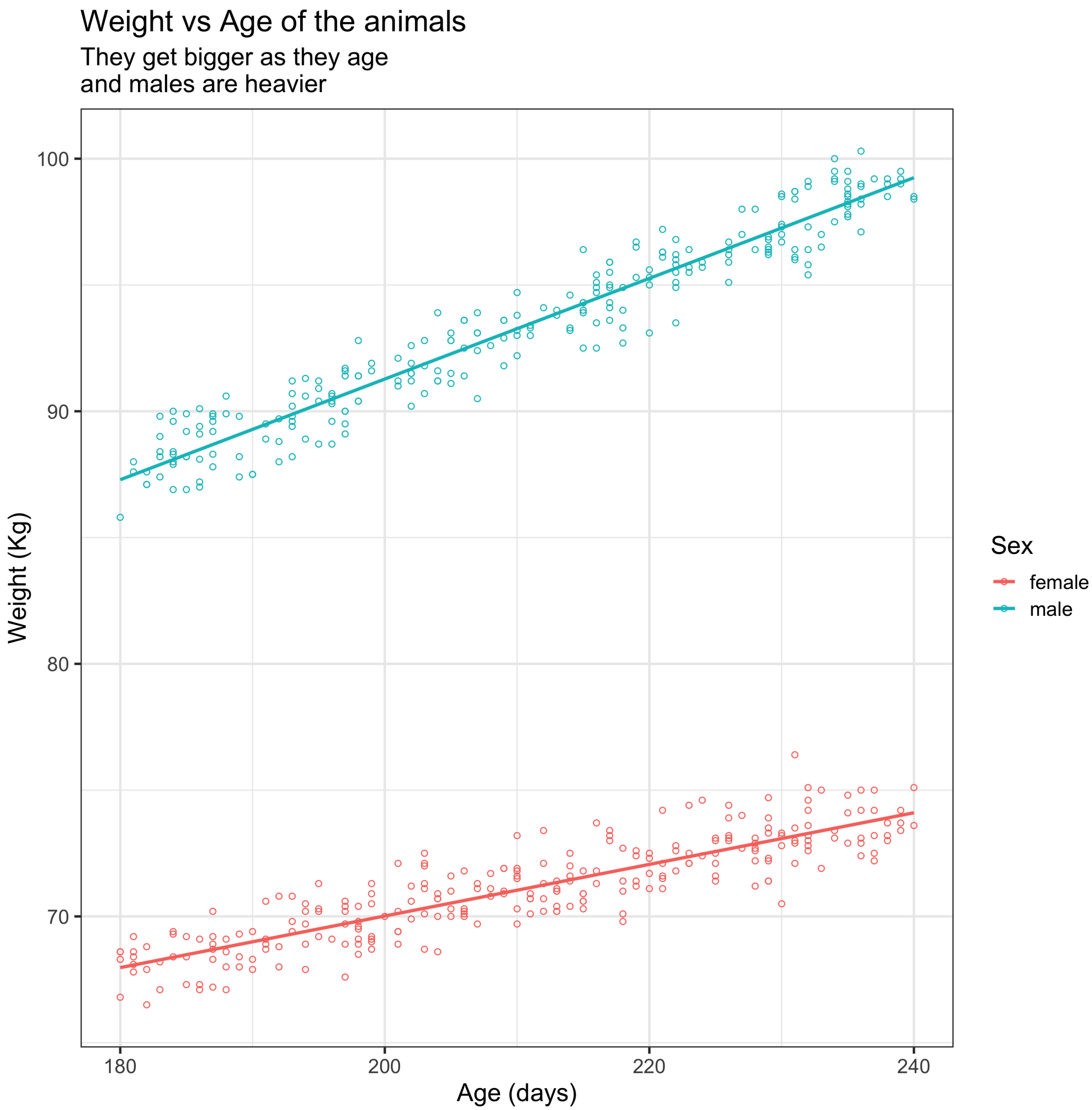
$$1/5 = 0.2$$

Linear regression

- numerical outcome
- numerical / categorical predictors
- linear relationship



Linear regression in R




```
mod <- lm(weight ~ age + sex, data = dat
```

```
Call:
lm(formula = weight ~ age + sex, data = dat)

Residuals:
   Min     1Q Median     3Q    Max
 -3.54  -0.88  -0.02   0.89   3.07

Coefficients:
```

```
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  39.1901     0.7026      56   <2e-16 ***
age           0.1515     0.0033      46   <2e-16 ***
sexmale      22.2796     0.1136     196   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.3 on 497 degrees of freedom
Multiple R-squared:  0.99,  Adjusted R-squared:  0.99
F-statistic: 2e+04 on 2 and 497 DF,  p-value: <2e-16
```

Analysing binary data

Binary data common in epidemiology e.g.

- alive/dead
- healthy/diseased

ID	treatment	age	region	sex	weight	status
A0049	control	221	D	female	71.6	healthy
A0485	treated	220	D	male	93.1	healthy
A0321	treated	238	D	male	99.0	healthy
A0153	control	183	A	female	67.1	diseased
A0074	control	187	B	female	68.3	healthy
A0228	treated	206	A	female	70.3	healthy

Univariable analysis

Status vs treatment

treatment	diseased	healthy
control	43	168
treated	34	255

```r
with(dat,
     {{fisher.test(status, treatment)}})
```

```

    Fisher's Exact Test for Count Data

data:  status and treatment
p-value = 0.01175
alternative hypothesis: true odds ratio is not equal to 1
95 percent confidence interval:
 1.142387 3.238178
sample estimates:
odds ratio
  1.917107
```

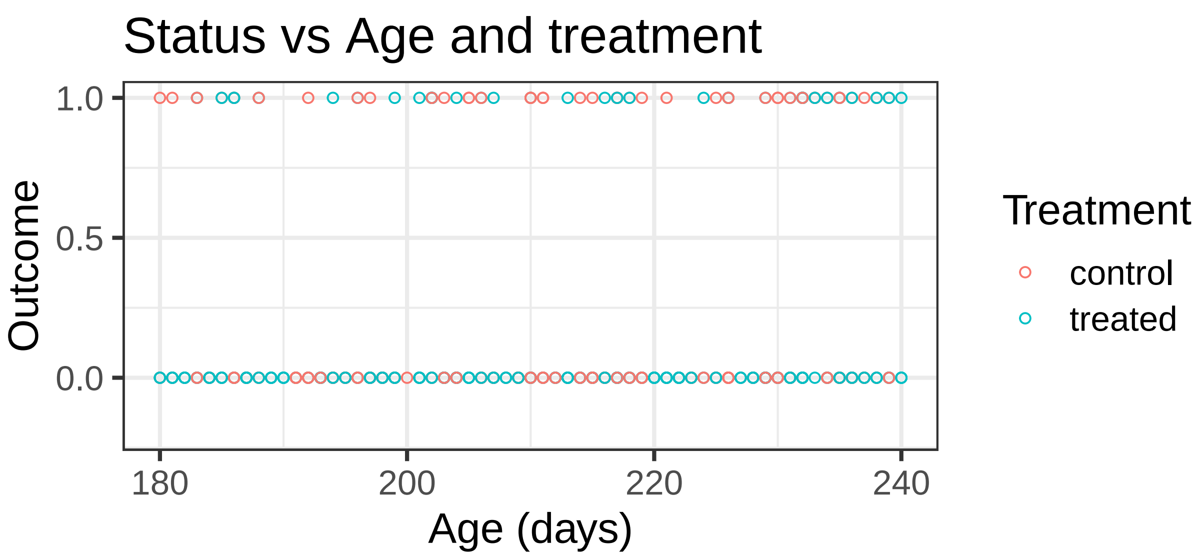
Multivariable analysis

How about recoding the outcome as 0/1?

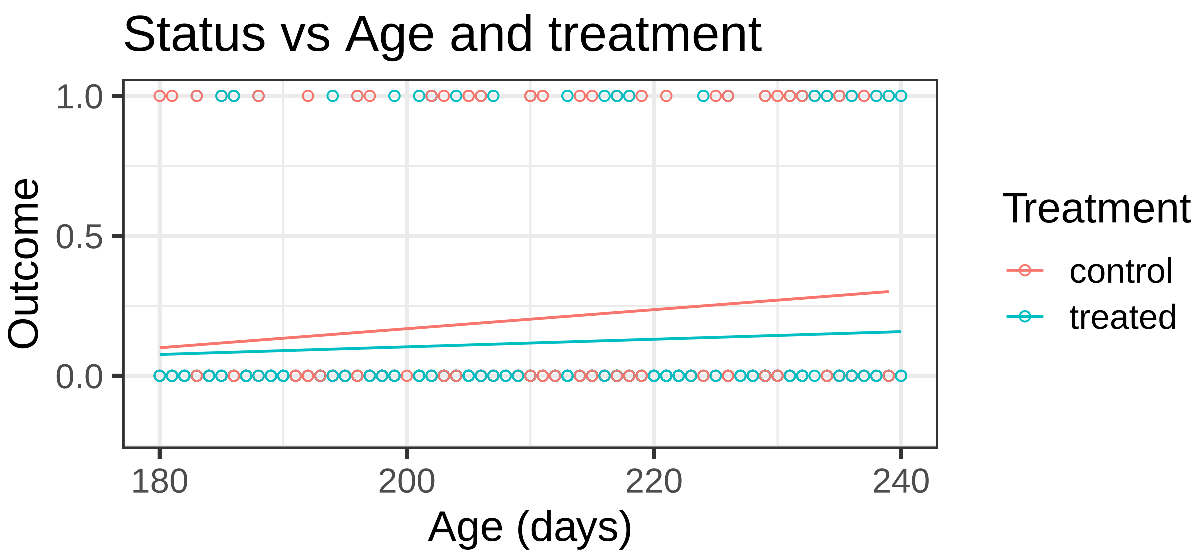
ID	treatment	age	region	sex	weight	status	status01
A0049	control	221	D	female	71.6	healthy	0
A0485	treated	220	D	male	93.1	healthy	0
A0321	treated	238	D	male	99.0	healthy	0
A0153	control	183	A	female	67.1	diseased	1
A0074	control	187	B	female	68.3	healthy	0
A0228	treated	206	A	female	70.3	healthy	0

Then use linear regression...

Linear regression 1



Linear regression 2



Problems

-predicts (impossible) intermediate values

-can predict <0 and >1

So how do we fix this?

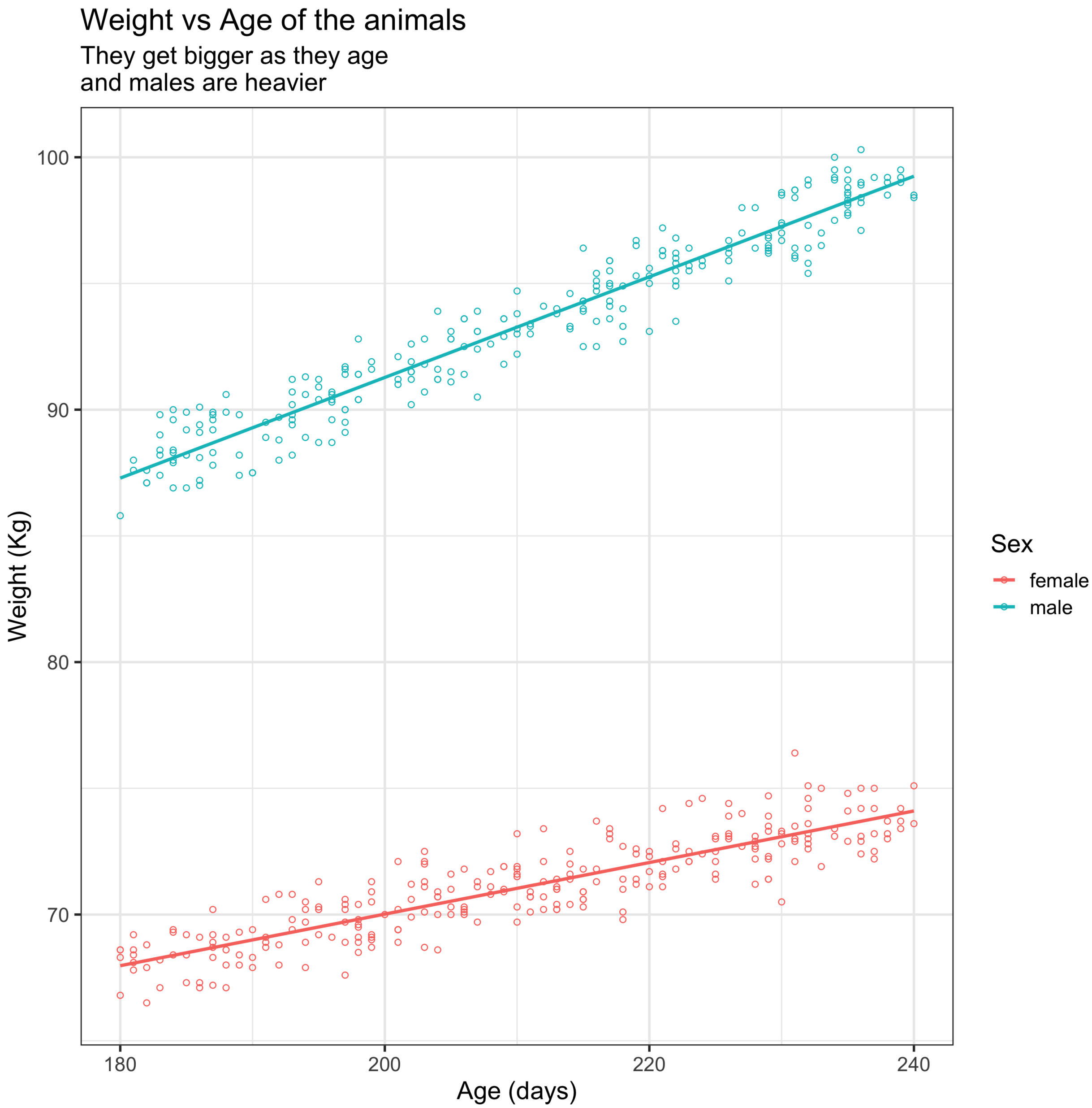
Linear regression does this...

$weight \sim \beta_0 + \beta_1 age + \beta_2 sex + \epsilon$

or in english...

The outcome, *weight*, is related to the predictors

by one or more straight lines.



For binary data we want

Our outcome to be 0 or 1

So rather than modelling the outcome.

We model the **probability** of something e.g. being diseased…

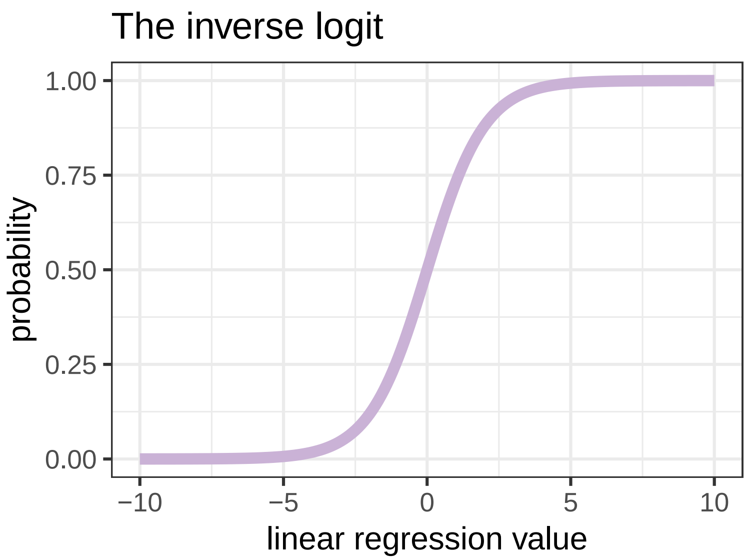
The logistic bit…

Linear regression models model numbers, any numbers!

Probabilities go from…

0 to 1

So we need to turn any number into 0 - 1



In fact the regression value is the log of the odds of the outcome.

#The logistic bit 2

So we have an outcome, e.g. being diseased vs healthy, that is coded 0 or 1

And our model is

$$log_e(\frac{prob}{1-prob}) \sim \beta_0 + \beta_1 age + \beta_2 treatment)$$

or in english

The log of the odds of an animal being diseased are modelled by a linear combination of the predictor variables

class: inverse, middle, center

Worked example in R

R code for logistic regression

```r
head(dat)
```

ID	treatment	age	region	sex	weight	status	status01
A0001	control	219	A	female	71.4	diseased	1
A0002	control	218	A	female	70.1	healthy	0
A0003	treated	214	D	female	71.4	healthy	0
A0004	treated	194	D	female	68.9	healthy	0
A0005	control	185	D	female	67.3	healthy	0
A0006	treated	235	D	male	98.6	healthy	0

A linear model of weight

```
mod_weight <- lm(weight ~ age + sex, data = dat)
```

A logistic regression model of disease status

```
mod_disease <- glm(status01 ~ treatment + age, family = binomial, data = dat)
```

The output

```
print(summary(mod_disease), digits = 3)
```

```
Call:
glm(formula = status01 ~ treatment + age, family = binomial,
    data = dat)

Deviance Residuals:
   Min      1Q  Median      3Q     Max
-0.830  -0.605  -0.532  -0.421   2.284

Coefficients:
                 Estimate Std. Error z value Pr(>|z|)
(Intercept)      -5.07438    1.61382   -3.14   0.0017 **
treatmenttreated -0.66259    0.25164   -2.63   0.0085 **
age               0.01751    0.00752    2.33   0.0199 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 429.59  on 499  degrees of freedom
Residual deviance: 417.16  on 497  degrees of freedom
AIC: 423.2

Number of Fisher Scoring iterations: 4
```

The output

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print(summary(mod_disease), digits = 3)
```

```
Call:
glm(formula = status01 ~ treatment + age, family = binomial,
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Deviance Residuals:
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```
   Min       1Q   Median       3Q      Max  
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AIC: 423.2

Number of Fisher Scoring iterations: 4
```

The output

Lets get 'tidy output…

```
tidy(mod_disease) #tidy from the broom package
```

```
# A tibble: 3 x 5
  term             estimate std.error statistic p.value
  <chr>               <dbl>     <dbl>     <dbl>   <dbl>
1 (Intercept)       -5.07     1.61        -3.14 0.00166
2 treatmenttreated  -0.663    0.252       -2.63 0.00846
3 age                0.0175   0.00752      2.33 0.0199 
```

odds ratios

The estimates = log(odds ratios)

i.e.

$$\frac{odds\ of\ outcome\ if\ have\ factor}{odds\ of\ outcome\ if\ dont\ have\ factor}$$

So we get odds ratios by 'inverse logging them'.

We can remove the intercept.

```
tidy(mod_disease) %>%
  filter(term != "(Intercept)") %>%
  mutate(OR = exp(estimate))
```

```
# A tibble: 2 x 6
  term             estimate std.error statistic p.value    OR
```

```
  <chr>                <dbl>     <dbl>     <dbl>   <dbl> <dbl>
1 treatmenttreated  -0.663    0.252        -2.63 0.00846 0.516
2 age                0.0175   0.00752       2.33 0.0199  1.02
```

A results table

```r
tidy(mod_disease) %>%
  mutate(OR = exp(estimate)) %>%
    bind_cols(exp(confint_tidy(mod_disease)) %>%
    as_tibble()
  ) %>%
  filter(term != "(Intercept)") %>%
  select(term, OR, `conf.low`, `conf.high`, p.value)
```

term	OR	conf.low	conf.high	p.value
treatmenttreated	0.5155152	0.3131803	0.8423683	0.0084623
age	1.0176680	1.0029464	1.0330367	0.0198711

But what does it mean?

Interpreting the odds ratios

term	OR	2.5 %	97.5 %	p.value
treatmenttreated	0.5155152	0.3131803	0.8423683	0.0084623
age	1.0176680	1.0029464	1.0330367	0.0198711

Odds ratios multiply

Categorical predictors

How many times greater the odds of outcome are **if** the risk factor (etc) is present.

So for the treatment variable (which can be control or treatment) the odds of disease if treated are 0.516 **times greater** than if untreated (control).

Interpreting the odds ratios

term	OR	2.5 %	97.5 %	p.value
treatmenttreated	0.5155152	0.3131803	0.8423683	0.0084623
age	1.0176680	1.0029464	1.0330367	0.0198711

Odds ratios multiply

Numerical predictors

How many times greater the odds of outcome are for **each unit change** in the variable

So for the age variable the odds of disease are 1.018 **times greater** for each day older.

So for 3 days it's 1.018 x 1.018 x 1.018 $\simeq$ 1.055.

Things to watch out for

Factor levels

How does R know if you are predicting 'healthy' or 'diseased'?

Perfect predictors

E.g. all the males are diseased and all the females are healthy

Linear on logit

Disease risk might go up and then down

Model selection - a blank page

More help

Veterinary Epi Reasearch - Ian Dahoo