

from Quantum Design

This section is meant to serve both as an introduction and as a reference section on magnetic units. The units of magnetism are complicated by the fact that over the years they have been defined in several different ways. There has been a great deal of reluctance for parts of the magnetics community to convert to SI units. SI stands for *Système Internationale d'Unités*, which are the official units of measure agreed upon by most nations. However, the most common system of units used among physicists, for reporting magnetic measurement results, is the Gaussian cgs system, which stands for "centimeter, gram, second." To add to the complexity, most physical properties can be reported in a variety of ways. For example, even in cgs units, the magnetization might be given in emu/g, emu/cm<sup>3</sup>, emu/mole, emu/atom, or any one of several other possibilities. Here, emu stands for *electromagnetic units*.

We must first choose a definition for the different "fields" involved. We can define three fields:

H, the applied magnetic field,

M, the magnetization, and

B, the flux density.

In cgs Gaussian units, the different fields are related by the equation

$$B = H + 4\pi M. \quad (1)$$

In our chosen definition, H is the field applied by the superconducting magnet residing outside of the sample space. This field is determined by the electric currents running in the magnet and, by our definition, it does not change when we place our sample into the magnet.

One way to look at the definition in Eq. 1 is to think of B as the net local field, H is the field from the magnet, and M is the field which changes the local field from H to B. The MPMS moves the sample through the pickup coils in order to change B within the pickup coil, and thereby changes the current flowing in the pickup coils. The amount of current induced is related to the total *magnetic moment* of the sample.

The units of magnetic moment are emu (in cgs) and A-m<sup>2</sup> (in SI units, where A is amperes and m is meters). The MPMS reports values of magnetic moment in emu. We get the magnetization M by dividing the value of the magnetic moment by volume, mass, or the number of moles in the sample.

The units for B are gauss (denoted G), in cgs units, and tesla (denoted T), in SI units. There is 1 G in 10<sup>-4</sup> T. The units for H are oersted (denoted Oe) in cgs units, and A/m in SI units. An Oersted and a Gauss have the same dimensions, and gauss

is often used for both B and H in conversation. There is 1 Oe in  $1000/4\pi$  A/m. From Eq. 1 we expect that M might also be reported in units of G. It is the volume magnetization, which is the magnetic moment of the sample divided by its volume ( $\text{emu}/\text{cm}^3$ ), that can be reported in units of G. There is 1  $\text{emu}/\text{cm}^3$  in 1000 A/m and there is 1  $\text{emu}/\text{cm}^3$  in  $4\pi$  G. We may also report M in units of  $\text{emu}/\text{g}$ , by dividing the magnetic moment by the mass of the sample in grams, or in  $\text{emu}/\text{mole}$ , by dividing the moment by the number of moles. Sometimes it is the magnetism associated with a certain atom in the compound that is to be measured, in which case we might report, for example,  $\text{emu}/\text{Cu-atom}$  where we divided the moment measured by the number of moles of Cu atoms in the sample (similar units exist in the SI unit system). A very useful table of units and conversions, compiled by R. B. Goldfarb and F. R. Fickett of the National Institute of Standards and Technology in Boulder, Colorado, is included as Appendix A.

Two other quantities frequently used in magnetism are the magnetic susceptibility and the permeability. The susceptibility is given by  $\chi = M/H$  and the permeability by  $\mu = B/H$ . These quantities are often used incorrectly. If an  $M(H)$  curve is not linear (a straight line),  $\chi$  will depend on the value of H. Some do not consider an H-dependent  $\chi$  value to be a legitimate quantity. However, whenever an H-dependent  $\chi$  is to be reported, it is important that the H value associated with the  $\chi$  measurement be included. The proper use of these quantities will be discussed further in later sections.

Every material exhibits some kind of magnetic behavior. However, the term "magnetic" is usually used to refer to something that will attract a piece of iron or a permanent magnet. This is a particular type of magnetism called *ferromagnetism* and is only one of the many types of magnetism. We use a magnetometer to measure the magnetization (amount of magnetism) of a sample. By studying how the magnetization changes with temperature and how it changes with the size of the magnetic field we apply to the sample, we can determine the type of magnetism and important related parameters.

There are two principal magnetic measurements:

$M(H)$  - magnetization as a function of applied magnetic field, and

$M(T)$  - magnetization as a function of temperature.

Here,  $H$  is the *applied magnetic field* which is the magnetic field applied to the sample by the superconducting magnet coil. An  $M(H)$  measurement is made by fixing the temperature  $T$  and measuring  $M$  at a series of  $H$  values. An  $M(T)$  measurement is made by fixing the applied field  $H$  and measuring  $M$  at a series of  $T$  values. There are other less common measurements, like magnetization as a function of time  $M(t)$ , that will also be discussed later. There are many different origins for the magnetic behavior observed in materials, and  $M(H)$  and  $M(T)$  measurements provide valuable information about the different possible types of magnetic behavior.

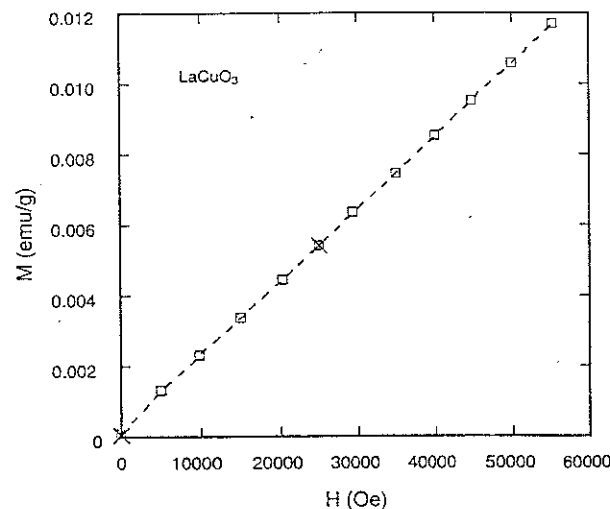
## A. PARAMAGNETISM

### INTRODUCTION

Probably the simplest type of magnetic behavior is known as *paramagnetism*. Shown in Figure 6 is a plot of  $M(H)$  for a typical paramagnet at a fixed temperature. The key features are that 1) the curve is linear, 2) the line intersects zero, and 3) the magnetization is *reversible*. Reversible means that the same curve is followed when going up in field as when going back down in field. When the  $M(H)$  curve is linear, the *magnetic susceptibility*  $\chi$ , which is given by  $\chi = M/H$ , is often an important property.

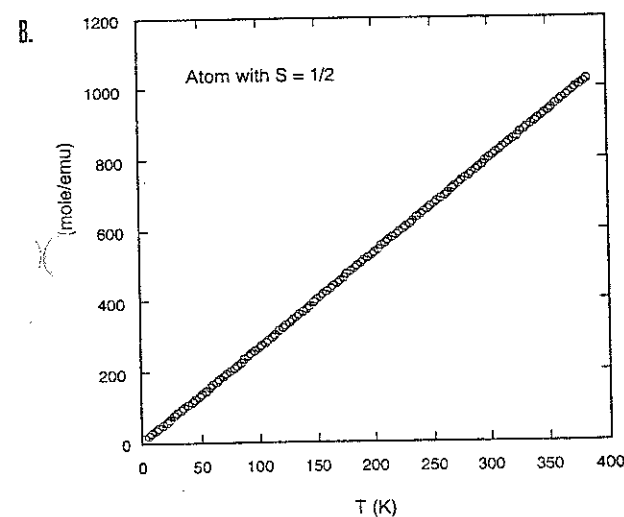
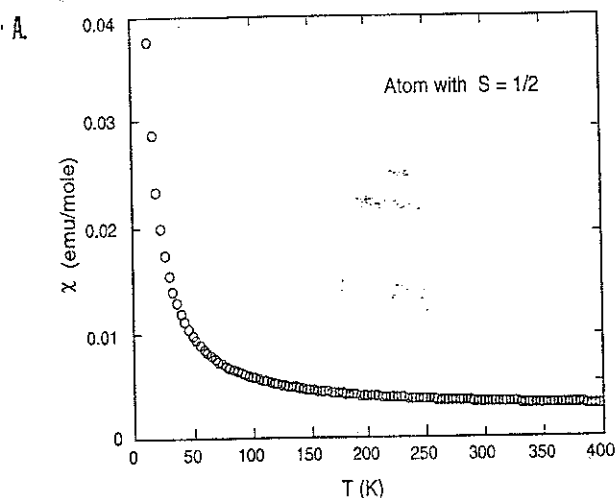
## A. PARAMAGNETISM

**6** Magnetization ( $M$ ) as function of applied magnetic field ( $H$ ) for a paramagnetic compound,  $\text{LaCuO}_3$ , at  $T=100\text{ K}$ . The squares are data collected on increasing field, the x-symbols are data collected on decreasing field. The curve is linear in  $H$  and reversible (i.e., the same curve is followed on increasing and decreasing field.)



## A. PARAMAGNETISM

A) Magnetic susceptibility ( $\chi$ ) as a function of temperature (T) for a hypothetical material with nearly ideal Curie paramagnetic behavior having a mole of  $S=1/2$  atoms. B) The inverse of  $\chi$  (in mole/emu) as a function of T. The slope of the curve is proportional to  $1/C$ .



### Curie-Type Paramagnetism

Paramagnetism can have many different origins. Since the  $M(H)$  curves are linear, two features are often used to determine the origin of the paramagnetism: the magnitude of  $\chi$  and the temperature dependence of the susceptibility  $\chi(T)$ . Shown in Figure 7a is a plot of  $\chi(T)$  for a Curie-type paramagnet, which is a type of magnetism resulting from the presence of atoms with unpaired electrons. Curie-type paramagnetism has a particular temperature dependence  $\chi(T) = C/T$ , where  $C$  is a constant. A plot of  $1/\chi$  versus  $T$ , as shown in Figure 7b, is very useful for characterizing Curie paramagnets. The slope of the curve is equal to  $1/C$  and the Curie constant is given as

$$C = b \mu_{\text{eff}}^2 N,$$

where  $\mu_{\text{eff}}$  is known as the *effective magnetic moment*,  $b$  is a universal constant, and  $N$  is the concentration of magnetic atoms with that moment. Thus, the constant  $C$  can be used to determine the *product* of the effective magnetic moment of an atom and the number of magnetic atoms present. If the number of magnetic atoms is known (from a mass measurement, for example), it is then possible to determine  $\mu_{\text{eff}}$  associated with the magnetic atom.

### UNITS

When  $M(H)$  is linear and reversible it is possible to define the magnetic susceptibility as  $\chi = M/H$ . The MPMS reports the magnetic moment  $m$  in units of emu, a cgs unit. If this is divided by the volume (in  $\text{cm}^3$ ) one will have the volume magnetization  $M$  (in  $\text{emu}/\text{cm}^3$ ). Dividing this  $M$  by the applied field  $H$  (in Oe) gives the volume susceptibility, which is dimensionless, but often expressed as  $\text{emu}/\text{cm}^3$  or  $\text{emu}/(\text{cm}^3 \text{ Oe})$ . Similarly dividing  $m$  by the mass in grams (g) gives the mass magnetization  $M_g$  (or  $\sigma$ ). Dividing  $M_g$  by  $H$  in Oe gives the mass susceptibility, which is expressed as  $\text{emu}/\text{g}$ . Dividing  $m$  by the number of moles gives the molar magnetization  $M_m$  in  $\text{emu}/\text{mole}$ . Dividing this by  $H$  in Oe gives the molar susceptibility, which is also expressed as  $\text{emu}/\text{mole}$ . These can be converted to SI units using the table in Appendix A.

### EXAMPLES AND ADVANCED TOPICS

#### Curie Paramagnetism

To determine the  $\mu_{\text{eff}}$  for a Curie-paramagnetic sample will first require calculating the molar susceptibility values (often denoted  $\chi_m$ ). Next  $1/\chi_m$  is plotted as a function of temperature, like the data shown in Figure 7b. If the  $M(H)$  curves are all linear, the  $\chi_m$  value at any magnetic field can be used. For a Curie paramagnet the  $1/\chi_m$  curve will intercept zero and have a slope  $1/C_m$ , where  $C_m$  is the molar Curie

units of emu-K/mole. To get  $p_{\text{eff}}$  from  $C_m$ , we use the complete formula

$$C_m = (N p_{\text{eff}}^2) / 3 k$$

where  $N$  is Avogadro's number ( $6.02 \times 10^{23}$ ) and  $k$  is Boltzmann's constant ( $1.38 \times 10^{-16}$  erg/K). Rearranging this gives

$$p_{\text{eff}} = (3 k C_m / N)^{1/2},$$

with  $p_{\text{eff}}$  in units of erg/Oe. The effective moment is usually reported in units of Bohr magnetons (denoted  $\mu_B$ ). Dividing by  $0.927 \times 10^{-20}$  (erg/Oe)/ $\mu_B$  will give  $p_{\text{eff}}$  in units of  $\mu_B$ . A shortcut to getting  $p_{\text{eff}}$  (in  $\mu_B$ ) from  $C_m$  (in emu K/mole) is to use

$$p_{\text{eff}} = 2.82 C_m^{1/2}. \quad (2)$$

The  $\chi$  data in Figure 7b are in units of emu/mole. The slope of this curve is 2.66 and since the slope equals  $1/C_m$ , this gives  $C_m = 0.376$ . Using Eq. 2 we find that  $p_{\text{eff}} = 1.73 \mu_B$ , which is the effective moment associated with spin  $s = 1/2$  atoms like those of  $\text{Cu}^{2+}$ .

## OTHER TYPES OF PARAMAGNETISM

### *Curie-Weiss Paramagnetism*

For a Curie-type paramagnet, there is a force that tries to align the magnetic moments on atoms with the magnetic field ( $\mathbf{p}_{\text{eff}} \cdot \mathbf{H}$ ). The  $1/T$  (or Curie) temperature dependence is a result of a competition between the force aligning the moments parallel to the field and the tendency for heat to disrupt the alignment. As the temperature increases, the associated increase in heat reduces the relative effect of the field.

What is different about a *Curie-Weiss* paramagnet is that, in addition to the interaction with the applied magnetic field, there is an interaction between the magnetic moments on different atoms. This interaction between moments (exchange interaction) can help align adjacent moments in the same direction or it can help align neighboring moments in opposite directions.

The Curie-Weiss susceptibility is given by

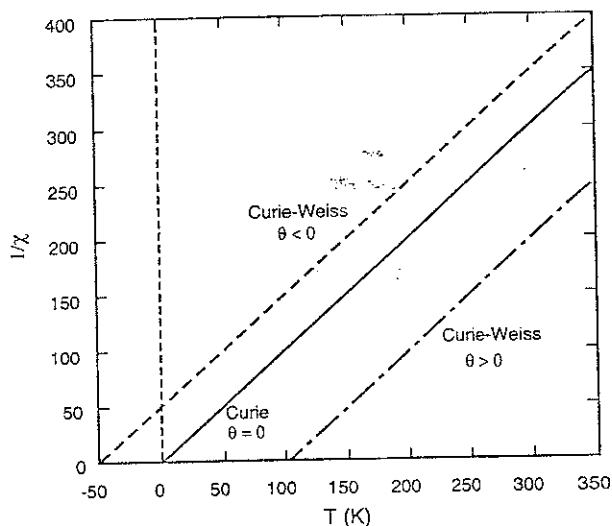
$$\chi_{\text{cw}} = C/(T - \theta), \quad (3)$$

where  $\theta$  is called the *Curie-Weiss temperature*. The Curie-Weiss  $\theta$  is related to the strength of the interaction between moments, and its sign depends on whether the interaction helps align adjacent moments in the same direction or opposite one another. Using the definition in Eq. 3, for  $\theta > 0$  the interaction helps to align adjacent moments in the same direction, and for  $\theta < 0$  the interaction helps to align adjacent moments opposite each other. Other terminology, which will become

## B. DIAMAGNETISM



The inverse of  $\chi$  as a function of  $T$  for systems exhibiting Curie ( $\chi = C/T$ ) and Curie-Weiss behavior ( $\chi = C/(T-\theta)$ ). When  $\theta > 0$  the interaction between moments helps to align neighboring moments in the same and when  $\theta < 0$  moments are aligned in opposite directions. When  $\theta = 0$  the moments act completely independent of one another.



clearer in later sections, is that for  $\theta > 0$  there is a net *ferromagnetic* interaction between moments and for  $\theta < 0$ , there is a net *antiferromagnetic* interaction between moments.

The characteristic plot for Curie behavior was the  $1/\chi$  versus  $T$  plot, and the same is true for Curie-Weiss behavior. But as shown in Figure 8, instead of a straight line through zero, as in Figure 7b, we get an x-axis intercept at a positive or negative  $\theta$ . In the case of the ferromagnetic  $\theta$  ( $\theta > 0$ ), it can be seen that Eq. 3 diverges (goes to infinity) at  $T = \theta$ . This is the approximate location of a ferromagnetic transition, also known as the *Curie temperature* (denoted  $T_C$ ). For an antiferromagnetic  $\theta$  ( $\theta < 0$ ), Eq. 3 will not diverge at  $T = \theta$ . However, the system may now have an antiferromagnetic transition, known as a Néel transition (denoted  $T_N$ ), near  $T = |\theta|$ .

### Pauli Paramagnetism and Van Vleck Paramagnetism

Pauli paramagnetism is observed in metals and is due to the fact that conduction electrons have magnetic moments that can be aligned with an applied field. The key characteristics of Pauli paramagnetism are that the  $\chi$  value is nearly independent of temperature and in most cases it has a very small value.

Van Vleck paramagnetism is another type of paramagnetism that is also nearly independent of temperature and usually has a small value. Van Vleck paramagnetism is associated with thermal excitations to low-lying states.

### Combinations

Often the temperature dependence of the paramagnetism does not follow any particular dependence. In these cases it can be useful to see if the behavior can be fit by an equation that combines a temperature independent part and a Curie, or Curie-Weiss, part. Antiferromagnets also have  $M(H)$  curves like paramagnets, even below  $T_N$ . The  $M(T)$  behavior is distinctive for well-behaved antiferromagnets, and is confusing in other cases. Antiferromagnetism will be discussed in more detail below.

There are also a variety of much more complicated magnetic behaviors that are active areas of research and are not easily identified by an  $M(T)$  curve. These include heavy-fermion systems, mixed-valence systems, spin-density-wave systems, spin glass systems, as well as others.

## B. DIAMAGNETISM

When an  $M(H)$  plot is linear and reversible but has a negative slope, we describe the magnetic behavior as diamagnetism. This means that  $\chi$  is negative. In most cases a diamagnetic contribution to the magnetism arises from paired electrons and the value of this contribution is usually extremely small. Tables of the contributions

due to certain atoms and atom combinations, usually called *Pascal's Constant tables*, used for approximating these contributions. The other important type of diamagnetism is superconductivity, which can have the largest possible value of diamagnetism. This will be discussed at length below.

There is an interesting observation one can make about diamagnets: they are repelled by a magnetic field. On the other hand, a paramagnet is attracted into a magnetic field. A simple picture for this is to think of the magnetic field to be composed of lines of magnetic field. The density of lines at any point is proportional to the field  $B$ . A diamagnet will push field lines out while a paramagnet will pull them in. Thus, for a diamagnet  $B < H$  inside the sample and for a paramagnet  $B > H$  inside the sample.

## C. FERROMAGNETISM

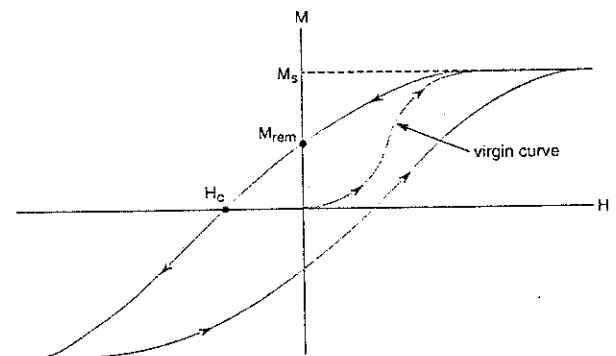
### INTRODUCTION

Ferromagnetism is a very important type of magnetism. Ferromagnets, and the related ferrimagnets, are the materials we usually call magnets - they are attracted to a piece of iron or a permanent magnet. The strongest type of magnetism found in materials is ferromagnetism. Ferromagnets have very distinctive  $M(H)$  and  $M(T)$  curves. Figure 9 shows a typical  $M(H)$  curve for a ferromagnet at a fixed temperature. The key features are that the curve is *not* linear and the behavior is *not* reversible. The lack of reversibility is often called *magnetic hysteresis*. Figure 9 shows that as  $H$  is increased the magnetization gradually reaches a maximum value, known as the *saturation magnetization*, which is usually denoted  $M_s$ . The saturation magnetization is an important property of a ferromagnet that is the same for any piece of a particular compound (it is an *intrinsic property*) no matter what the material's processing history has been.

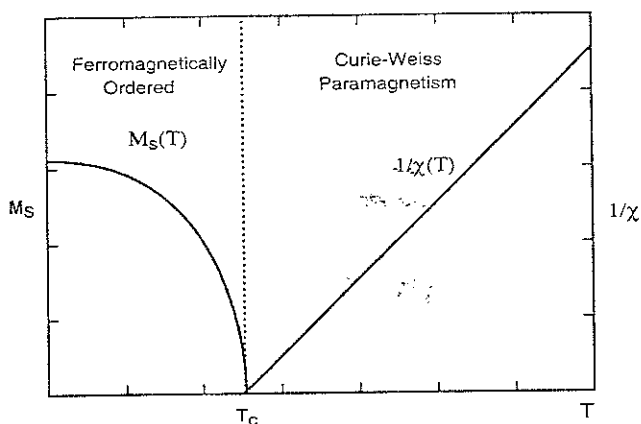
As  $H$  is reduced back to zero from saturation, we can see in Figure 9, that a different curve is followed on the way down and that  $M$  does not go to zero when  $H$  returns to zero. The value of the *magnetization* when  $H$  is returned to zero is called the *remanent magnetization*, denoted  $M_{rem}$ . The remanent magnetization is often confused with the saturation magnetization, but they are very different properties. Whereas the saturation magnetization is an intrinsic property of a compound, the remanent magnetization depends on the way a sample of the material has been prepared and treated (its history). A large remanent magnetization is desirable for applications like magnetic recording, whereas a small remanent magnetization is desirable for applications like magnetic transformer cores. Materials with large remanent magnetizations are called *hard ferromagnets*, and those with small remanent magnetizations are called *soft ferromagnets*.



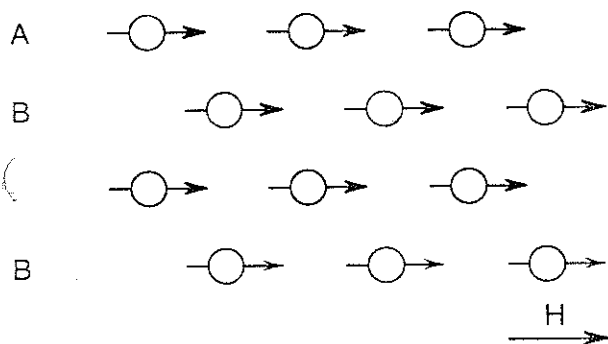
Magnetization as a function of field at  $T < T_c$ , after cooling the sample from above  $T_c$ . Identified are the initial magnetization curve (or virgin curve), the saturation magnetization ( $M_s$ ), the remanent magnetization ( $M_{rem}$ ), and the coercive field ( $H_c$ ).



**10** Above the Curie temperature ( $T_C$ ) a ferromagnet is paramagnetic exhibiting Curie-Weiss behavior ( $\chi = C/(T-\theta)$ ) with  $\theta > 0$ . Below  $T_C$ ,  $\chi$  is no longer a useful parameter since  $\chi$  is both field and history dependent. Instead, the saturation magnetization  $M_s$  is an important intrinsic property.



**11** Schematic diagram showing how the atomic moments in a ferromagnet are locked together and aligned in the same direction below  $T_C$ .



The magnetization of a ferromagnet does not return to zero by reducing  $H$  to zero. However, by applying a sufficiently large magnetic field in the opposite direction, the magnetization can be returned to zero. The magnetic field required to return the magnetization to zero is called the *coercive field*, usually denoted  $H_c$ . The coercive field is not an intrinsic property and the value of  $H_c$  has an additional dependence on the rate of change of the magnetic field  $dH/dt$ . The size of the coercive field determines how useful the material is for various applications. Although  $H_c$  is related to the remanent magnetization (i.e., if  $M_{rem}$  were zero,  $H_c$  would also be zero), it is a different property.

So far the description of a ferromagnet has been an operational one. A major distinction between ferromagnets and paramagnets is that the ferromagnetic state is a state of *long range order*. This long range order sets in at a phase transition which occurs at the Curie temperature. The Curie temperature is close to the same point where a  $1/\chi$  versus  $T$  plot will extrapolate to zero for a Curie-Weiss paramagnet that becomes ferromagnetic. An example is shown in Figure 10 where Curie-Weiss paramagnetic behavior ( $\chi = C/(T-\theta)$ ) with  $\theta > 0$  is observed above the transition, while below  $T_C$  the system is ferromagnetically ordered. Below  $T_C$ ,  $\chi$  is no longer a useful parameter, since  $\chi$  is both field and history dependent, and instead, it is the saturation magnetization  $M_s$  that is an important intrinsic property.

In a ferromagnet below  $T_C$ , the individual magnetic moments of the atoms are all lined up in the same direction and essentially locked together as shown schematically in Figure 11. Instead of the magnetic moments acting individually, they act together like one very large magnetic moment. The term "long range order" means that if we know the orientation of one moment at a particular position, we can determine the orientation of any other moment a long distance away. For a ferromagnet, all of the moments within a domain are aligned in the same direction. On the other hand, for a paramagnet the orientation of any moment is random and even though there is a higher probability for a moment to align with the magnetic field, each moment acts nearly independently of the others.

## UNITS

The units for magnetization and applied magnetic field are the same for ferromagnets as they are for all types of magnetic systems. It is important to remember that  $M_{rem}$  and  $M_s$  have values of magnetization, while  $H_c$  is a value of the applied magnetic field.



## EXAMPLES AND ADVANCED TOPICS

### History Dependence

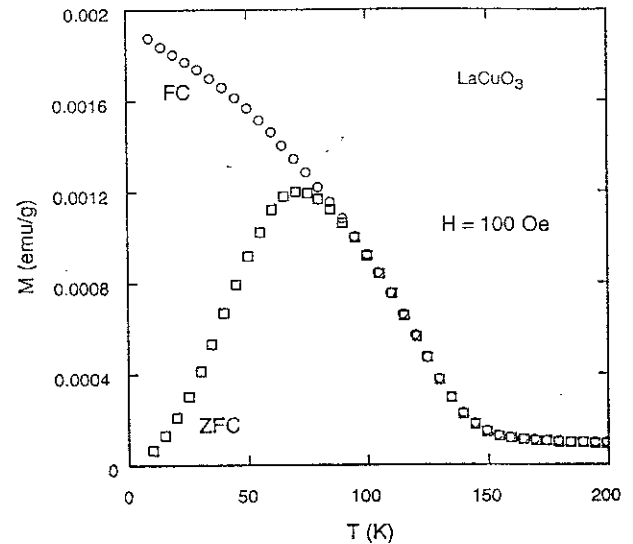
Ferromagnets usually exhibit some amount of irreversibility (or hysteresis) in their  $M(H)$  behavior. This irreversibility in the  $M(H)$  curves is a part of a more general type of behavior often referred to as *history dependent* behavior. Here, history dependent means that the value of the magnetic moment observed is dependent on the sequence of magnetic field changes and temperature changes that were involved in getting the sample to the condition in which it was measured. In addition to the hysteresis in  $M(H)$  curves at fixed temperature,  $M(T)$  behavior can also show a dependence on the magnetic field history if  $M$  is not saturated on every half-cycle of a  $M(H)$  loop.

One way of determining if irreversibility exists is to do a *zero-field-cooled/field-cooled* (ZFC/FC) set of measurements. This is done by cooling the sample to the lowest measurement temperature in  $H = 0$ . Once stabilized, a magnetic field is applied and the moment is measured as a function of temperature up to the highest desired temperature. This is the ZFC part. Next the sample is cooled in this same field to the lowest temperature and again measured as a function of temperature. This FC part can also be done by collecting data as the sample is cooled, however, most cryogenic systems are more time efficient when collecting on warming. At least in the case of superconductors, the FC data should be collected on cooling, not on warming. This ZFC/FC method is very useful for determining the temperature range over which systems are irreversible. Results of a ZFC/FC measurement are shown in Figure 12.

Another important result of irreversibility in ferromagnets is that the shape of an  $M(H)$  curve will change unless saturation is attained on every half cycle of a hysteresis loop. If a magnetic field below saturation is used, a different  $M(H)$  curve will be traced. These other curves are usually called *minor loops*. Running a magnetic material through a series of minor loops of decreasing size (smaller and smaller peak values of  $H$ ) is a method for "demagnetizing" a ferromagnet. Demagnetization can be a misleading term because the sample is still ferromagnetic. When demagnetized, a ferromagnet consists of many small domains within which magnetic moments point in the same direction. Adjacent domains however, have their magnetization pointing in different directions. In this way, the magnetic field lines can close inside the sample and the remanent magnetization is zero. Domains are discussed in the next section.

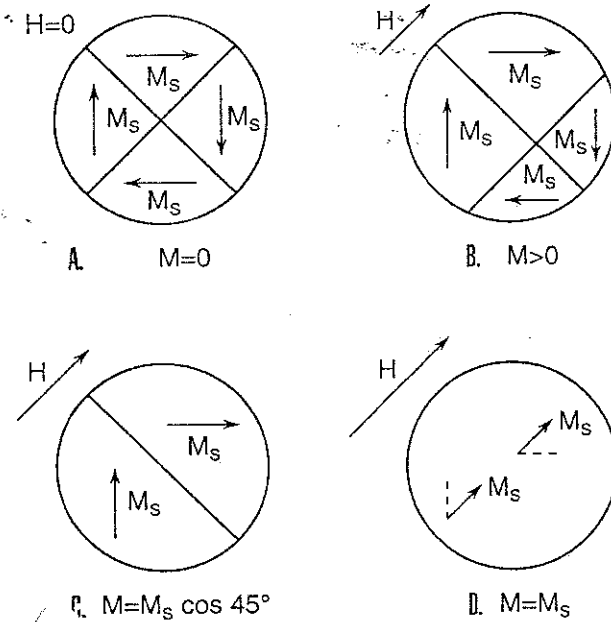
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Magnetization as a function of temperature for the cases where a) the sample is first cooled in  $H = 0$ , a field turned on, and then data collected on warming (ZFC) and, b) when data are collected as the sample is cooled in the same field (FC).





The magnetization process for a ferromagnetic sample that was cooled from above its  $T_C$ . A. In  $H = 0$  the magnetic domains form a closed loop with the moments aligned along preferred crystallographic directions. Within each domain the magnetization is saturated. B. As  $H$  is applied, the domain walls move to allow the domains aligned with the field to grow. C. At a sufficiently high field only a small number of domains exist. D. The last stage of the magnetization process is the rotation of the moments away from the preferred crystallographic direction and parallel with the applied field to form one saturated domain.



## Magnetic Domains

When a ferromagnet is cooled from a temperature above its Curie temperature in  $H = 0$ , it will usually show very little evidence of having a large magnetization value. This is due to domain formation. Instead of all the magnetic moments in the sample lining up in the same direction in one single domain, the lower energy configuration is for the sample to be divided up into several or many magnetic domains. Within a domain, all of the moments are aligned in the same direction (the magnetization is saturated within a domain). It is important to note that the magnetic domains are not the same as crystallographic domains—the magnetic domains cannot be seen without magnetic imaging techniques. In the border region between different magnetic domains, the direction of the magnetization changes. This border region is called the *domain wall*. It is the way that domain walls move that causes much of the irreversibility in ferromagnets.

When a magnetic field is applied to a sample that has been cooled in  $H = 0$  from above  $T_C$ , the sample will initially seem to be nonmagnetic. As a magnetic field is initially applied, the  $M(H)$  behavior looks like the initial curve identified in Figure 9. This is called the “virgin curve” and the behavior associated with this curve is shown in Figure 13. Initially, the domains are directed so that the spontaneous magnetic fields form a closed magnetic loop. When a magnetic field is applied, more of the moments start to align with the field. This does not happen randomly within all domains, but rather the domains with a component directed along the field direction preferentially grow in size. In Figure 13 this is pictured as the domain wall moving. This selective domain growth continues until the whole sample is one domain. Even though the moments in a domain are parallel to each other, they still may not be aligned with the magnetic field but along some preferred crystallographic direction. This is due to *magnetic anisotropy*, which is a topic for a separate discussion. To reach full saturation requires that the moments turn from their preferred crystallographic direction to be parallel to the magnetic field.

As the field is reduced from saturation, the moments will first return to the preferred crystallographic direction. As the field is decreased further, the domain walls will reform (or nucleate) and try to move. The domain wall motion can, however, be strongly impeded as in the case of hard ferromagnets. This impedance to domain wall motion is caused by domain wall *pinning*. Domain walls can get stuck at various defects, like grain boundaries and inclusions, producing the characteristics of remanence and coercivity.

## Magnetic Recording and Transformer Coils

area within an  $M(H)$  loop is the energy dissipated on cycling that sample through the loop. The magnetic cores of ac powerline transformers, which are cycled through an  $M(H)$  loop at 50 to 60 times per second, is an example in which a soft ferromagnet is very useful. A large mutual inductance is possible with a minimum of *hysteresis* loss.

An important application of hard ferromagnetic materials is their use in magnetic recording media. Two states are required to store binary data. The recording head is a small magnetic pickup coil that can discern regions having either of the two different directions of magnetization. The head is also used to write these regions. To write, the coil applies a magnetic field, larger than  $H_c$ , to the recording media in one or the other direction for a short time. A patch of magnetic material is thereby "switched" into a particular direction. The coercive field ( $H_c$ ) determines the size of the field necessary to switch the field in that region. On the other hand it is the remanent magnetization ( $M_{rem}$ ) of the magnetic media that determines how large a magnetization is available for the head to read.

## D. ANTIFERROMAGNETISM AND FERRIMAGNETISM

### ANTIFERROMAGNETISM

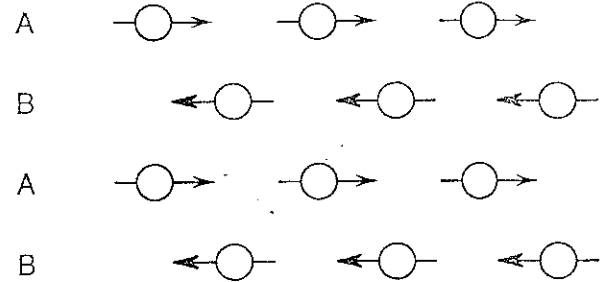
The term *antiferromagnet* often produces considerable confusion. Some people may think that an antiferromagnet is a ferromagnet but with the magnetic moments all lined up opposed to the applied field direction. However, this kind of behavior has never been observed. In an antiferromagnet the magnetic moments line up so that adjacent moments are aligned in opposite directions to each other (see Figure 14). Instead of the enormous combined moments associated with ferromagnets, the moments on neighboring atoms cancel each other, resulting in relatively small values of  $M$ . The  $M(H)$  behavior is more characteristic of a paramagnet; however, the origin of the  $M(H)$  behavior in antiferromagnets is quite different from that of Curie paramagnets, since the antiferromagnetic state is a long range ordered state. The moments are locked together but in the alternating configuration shown in Figure 14.

The temperature dependence of an antiferromagnet is shown in Figure 15. The phase transition to the antiferromagnetic state is known as a Néel transition and occurs at a temperature usually denoted  $T_N$ . Above  $T_N$  an antiferromagnet is often paramagnetic exhibiting Curie-Weiss behavior ( $\chi = C/(T - \theta)$ ) with  $\theta < 0$ . Since  $M(H)$  as are linear below  $T_N$ ,  $\chi$  remains a useful property.

## D. ANTIFERROMAGNETISM AND FERRIMAGNETISM

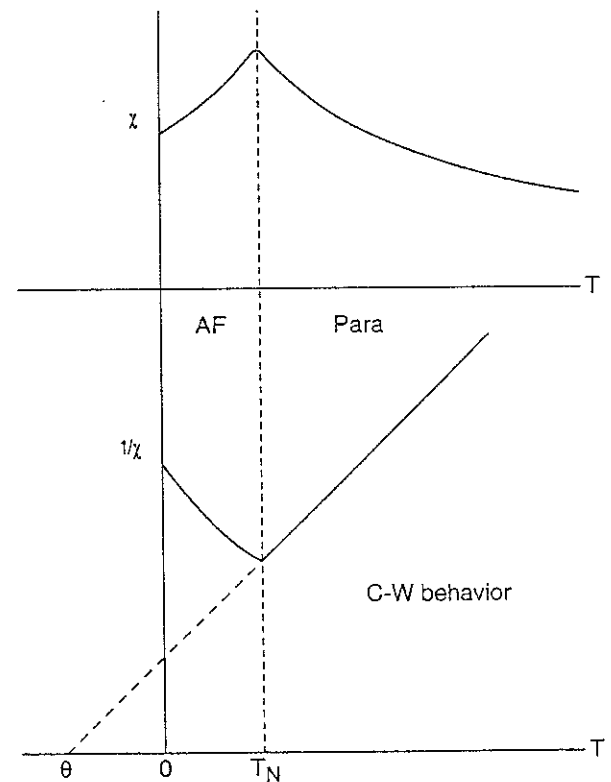


Schematic diagram showing how the atomic moments in an antiferromagnet are locked together and aligned so that adjacent moments have spins in opposite directions. Above  $T_N$ , in the paramagnetic state, the moments behave nearly independently of one another.



Magnetic susceptibility and inverse susceptibility as a function of temperature for an antiferromagnet. Above the Néel temperature ( $T_N$ ) a antiferromagnet often resembles a paramagnet exhibiting Curie-Weiss behavior ( $\chi = C/(T - \theta)$ ) with  $\theta < 0$ .

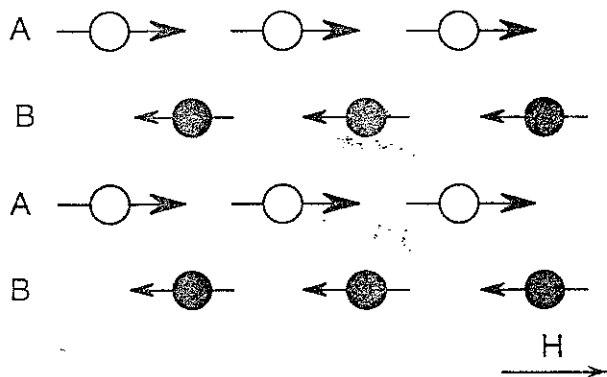
### Antiferromagnetism



## E. SUPERCONDUCTIVITY



Schematic diagram showing how the atomic moments in a ferrimagnet are locked together and aligned so that adjacent moments have spins in opposite directions. The adjacent moments are of different size. The larger moments usually line up with the applied field and the smaller ones oppose the field. Above  $T_c$ , in the paramagnetic state, the moments behave nearly independently of each other.



## FERRIMAGNETISM

Ferrimagnets are often associated with ferromagnets because their  $M(H)$  and  $M(T)$  behavior is nearly identical to that of ferromagnets. However, at the atomic level ferrimagnets are more similar to antiferromagnets because the magnetic moments of the atoms in ferrimagnets are antiferromagnetically coupled; i.e., adjacent magnetic moments are locked in opposite directions. What makes ferrimagnets different from antiferromagnets is that the adjacent moments have different magnitudes. The larger of the two moments tends to align with the applied magnetic field while the smaller moment aligns opposite to the field direction (see Figure 16). The result is that the different moments add up to produce a large net moment aligned with the magnetic field. Some of the most useful materials for making permanent magnets are ferrimagnets. Many of these materials are non-electrically conducting ceramics.

## UNITS

Antiferromagnets behave essentially like paramagnets. On the other hand, ferrimagnets behave essentially like ferromagnets, having the same types of irreversibility possible and the same parameters ( $H_c$ ,  $M_{rem}$ , and  $M_s$ ) used to describe their behavior.

## E. SUPERCONDUCTIVITY

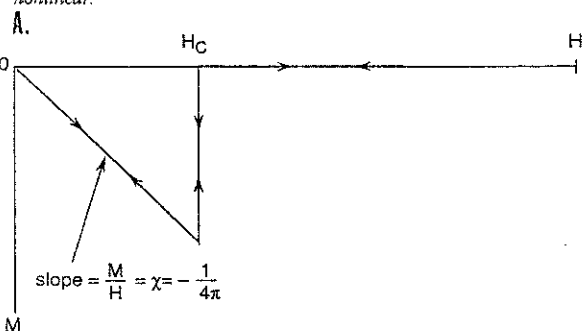
### INTRODUCTION

Another very important type of magnetism is associated with superconductivity. The magnetic properties of superconductors are unique and very complex. Figure 17a is a plot of  $M(H)$  at a fixed temperature for a *type-I* superconductor in the superconducting state. Currents near the surface of a *type-I* superconductor completely screen the inside of the sample, from the applied magnetic field, up to a field called the *critical field* (denoted  $H_c$ ). Screening of the field means that none of the applied magnetic field gets into the sample, and the superconductor acts like a magnetic mirror (with  $B = 0$  inside the superconductor). Up to  $H_c$ , the  $M(H)$  curve is linear having the largest negative slope possible. This volume magnetic susceptibility is that of a perfect diamagnet ( $\chi = -1/4\pi$  in cgs units), which is an enormous value compared to most other diamagnets.

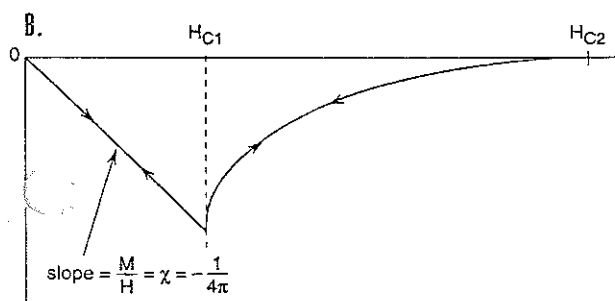
An  $M(H)$  curve for an ideal *type-II* superconductor is shown in Figure 17b. Up to an applied field known as the lower critical field  $H_{c1}$ , a *type-II* superconductor behaves like a *type-I* superconductor. However, when the applied magnetic field  $H$  exceeds  $H_{c1}$  the magnetization begins to decrease in magnitude due to the penetration of the magnetic field into the material in the form of *flux vortices*.



A. Magnetization as a function of field for a *type-I* superconductor. B. Magnetization as a function of field for an ideal *type-II* superconductor. The magnetization is reversible even though the curve is nonlinear.



B.



The magnetization will continue to decrease up to a magnetic field value called the *critical field* (denoted  $H_{c2}$ ). At  $H_{c2}$  and higher fields the superconductivity is suppressed and the system becomes normal (non-superconducting). The curve shown in Figure 17b is the ideal type-II superconductor  $M(H)$  curve and this curve is reversible even though it is non-linear above  $H_{c1}$ . It is interesting to note that this ideal superconductor would be useless for most applications, and, in fact, no supercurrent could flow above  $H_{c1}$  in such a superconductor. For it to be useful in practical applications we must modify the superconductor in ways such that the  $M(H)$  curve of the superconductor becomes irreversible (hysteretic). This can be done by introducing defects into the material, usually through an appropriate choice of preparation and processing conditions. These defects serve to pin the magnetic field lines, thereby restricting their motion. It is the motion of the field lines that usually limits the current density above  $H_{c1}$ .

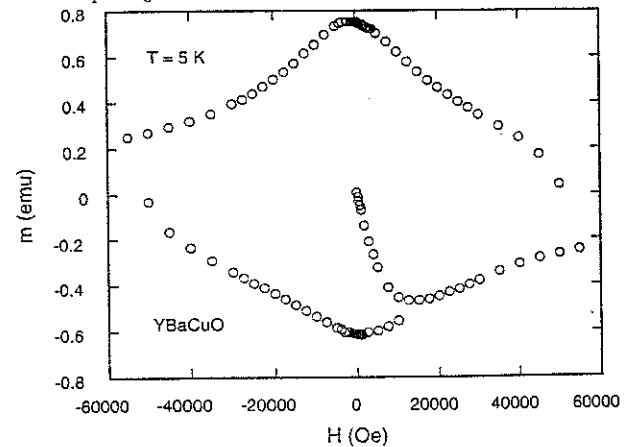
Introducing defects into an ideal superconductor changes the  $M(H)$  curve from that of Figure 17b to one like that of Figure 18 which shows little evidence of  $H_{c1}$ ; furthermore, the  $M(H)$  curve is definitely neither linear nor reversible. Now it is possible for a supercurrent to flow above  $H_{c1}$ , and the amount of supercurrent that can flow can be determined from the value of the magnetization. An important quantity in such superconductors is the critical current density (denoted  $J_c$ ), and it is a remarkable result that a magnetic measurement can be used to determine how much electrical supercurrent (a transport property) can be carried by the superconductor. The relation between the magnetization  $M$  and  $J_c$  is called the *Bean Critical State Model* and is given by

$$J_c = s M / d,$$

where  $d$  is the sample width or diameter and the constant  $s$  is a shape dependent constant having a value  $s = 10/\pi$  for an infinite rectangular slab sample and  $s = 15/\pi$  for a cylindrical sample. (This equation is for units of  $J_c$  in  $A/cm^2$ ,  $M$  in G, and  $d$  in cm.)

Another important property of a superconductor is its superconducting transition temperature (usually denoted  $T_c$  for critical temperature). This is the temperature at which the sample goes from the superconducting state to normal state upon warming.  $T_c$  usually decreases as  $H$  is increased. It is common to measure the *Meissner effect* in order to determine  $T_c$ . When an *ideal* superconductor is cooled through its superconducting transition with a very small field applied to the sample (field-cooled (FC) measurement), the magnetic field will be completely expelled from the inside of the superconductor at  $T_c$ . The expulsion of the field at  $T_c$  is the Meissner effect and it is possible to determine  $T_c$  by measuring  $M(T)$  as the point at

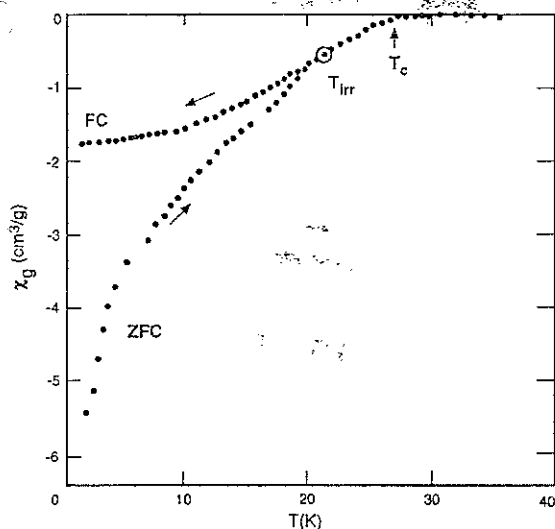
18 Magnetization as a function of field for a superconductor exhibiting irreversible magnetic behavior associated with flux pinning.



## E. SUPERCONDUCTIVITY

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Magnetization as a function of temperature for the high temperature superconductor  $\text{LaSrCuO}$ . Both zero-field-cooled and field-cooled measurements are presented.



which there is a drop in  $M$  (remembering that  $M$  is negative). A plot of  $M(T)$  for a high temperature superconductor is shown in Figure 19.

It is not uncommon to see a different measurement, often called *magnetic shielding* or *screening*, mistakenly called the Meissner effect. Magnetic shielding is measured by first cooling the sample to a low temperature below  $T_c$ , then turning on a magnetic field (which is also called a zero-field-cooled (ZFC) measurement). The changing magnetic field induces currents that flow as long as the sample is superconducting. The Meissner measurement and the shielding measurement both are often mistakenly used as a measure of the amount (quantity) of superconducting material in a sample. A *100% Meissner fraction*, which corresponds to a  $\chi = -1/4\pi$  susceptibility value, can be used as a legitimate measure of a completely superconducting sample. However, any value of the Meissner fraction less than 100% could simply mean that there is flux pinning in the sample. The shielding measurement has other potential problems. For example, if a sample has only a thin superconducting skin and a non-superconducting center, it could conceivably produce a  $\chi = -1/4\pi$  susceptibility value, since the skin could screen the whole interior of the sample to compensate the applied field  $H$  ( $B = 0$  inside the sample).

### UNITS

It is important to realize that the superconducting shielding results from the screening of a **volume**. The magnetization value will depend on the size of the applied field; therefore, it is the volume susceptibility  $\chi$  which is important. In cgs units, perfect diamagnetism, or complete screening, has a value of  $\chi = (-1/4\pi)$  emu/cm<sup>3</sup>. Another way of viewing this is to realize that  $B = 0$  in Eq. (1), requires that  $-4\pi M = H$ , and therefore  $M/H = -1/4\pi$ . In SI units,  $\chi = -1$ .

The critical current density ( $J_c$ ) is a current per unit cross sectional area. The cgs units are A/cm<sup>2</sup> and the SI units are A/m<sup>2</sup>.

### EXAMPLES AND ADVANCED TOPICS

#### Irreversibility Line

The region in the phase diagram between  $H_{c1}$  and  $H_{c2}$  in a type II superconductor is called either the *Abrikosov* or the *flux-vortex phase*. As discussed above, at a fixed temperature and for  $H > H_{c1}$ , the magnitude of  $M$  decreases as  $H$  increases. The decrease in  $M$  is due to the fact that for  $H > H_{c1}$ ,  $B \neq 0$  inside the sample, and some of the magnetic field actually penetrates into the superconductor. However, the magnetic field inside the superconductor is not distributed uniformly as it would be in most other types of material. The field that penetrates into a superconductor is *quantized* into single *quanta* of magnetic flux. A single quantum of magnetic flux

(denoted  $\phi_0$ ) has a value  $\phi_0 = 2.07 \times 10^{-7} \text{ G-cm}^2$  in cgs units. There is a supercurrent associated with each magnetic flux quantum and together this comprises an entity called a *flux vortex*.

Flux vortices can be thought of as discrete lines of  $B$ . When an electrical transport current is applied to a superconductor containing flux vortices, the vortices experience a force, called the Lorentz-force, which acts perpendicular to the current. If the vortices are moved by the Lorentz force, however, they generate a voltage parallel to the current that in turn produces resistive losses. To keep a supercurrent flowing requires zero resistance, therefore the flux vortices must be kept from moving. Defects in the sample can pin the vortices and thereby allow large supercurrents (large  $J_c$  values) to flow.

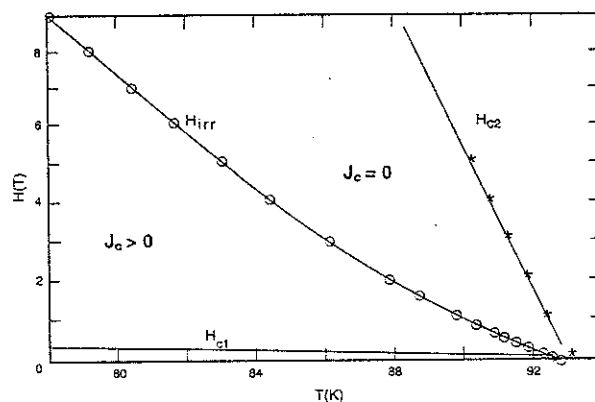
It was a surprise to many when it was observed that for high temperature superconductors the magnetic phase diagram between  $H_{c1}$  and  $H_{c2}$  was split into two regions and separated by a new phase boundary that is commonly referred to as the *irreversibility line* (IRL). On the upper side of the IRL below  $H_{c2}$  there is no pinning, the magnetic behavior is reversible, and no bulk supercurrents can flow. For temperatures and fields below the IRL, the magnetic behavior is irreversible and supercurrents can flow. One way of measuring the IRL is to use the zero-field-cooled/field-cooled (ZFC/FC) method described in the section on ferromagnets. Below the IRL the irreversibility results in two different curves for ZFC and FC, while above the IRL there is only one value of  $M$  at a given  $H$  and  $T$ , and thus the ZFC and FC curves are superimposed as shown in Figure 19. The IRL is an important property that sets the limits on the range where a superconductor can be used in most applications. A plot showing the relationship between the IRL and the critical fields is presented in Figure 20.

### Minor Loops

When trying to measure irreversible magnetic properties in a sample, it is important to be able to control the sample's field history. Determining  $J_c$  using a magnetic measurement requires that the critical state be established in the sample. To establish the critical state the field should be changed monotonically and then should not change at all during the measurement scan.

If the length of an MPMS measuring scan is too long, the sample will experience a significant change in the value of the magnetic field during the scan. This occurs because the field in a superconducting solenoid changes with position. Over a long scan length, the sample experiences non-monotonic field variations, and, in fact, the field at the sample oscillates as the sample moves up and down. The result is that the critical state in the sample is reduced or destroyed (the sample is effectively

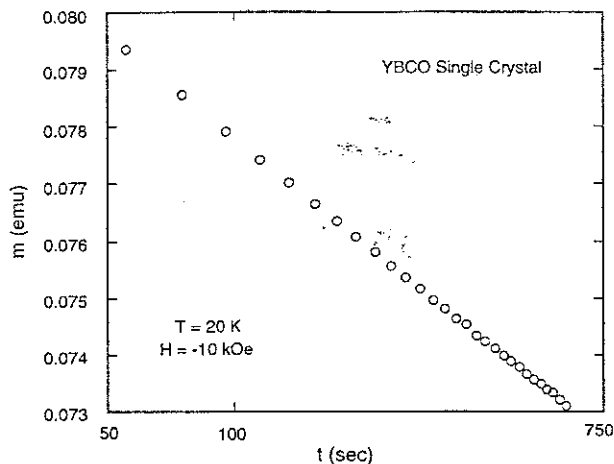
A magnetic phase diagram for the high temperature superconductor  $\text{YBa}_2\text{Cu}_3\text{O}_7$ . The irreversibility line separates a region (lower  $H$  and  $T$ ) of finite critical current density  $J_c$  from one with  $J_c = 0$ .



## F. DEMAGNETIZATION CORRECTIONS



Magnetization as a function of the logarithm of time for a thin film sample of the high temperature superconductor  $\text{YBa}_2\text{Cu}_3\text{O}_7$ . The slope of the curve gives a flux creep relaxation rate. The sample was cooled in applied field after which a field  $H = 10 \text{ K Oe}$  was applied and the magnetic moment measured.

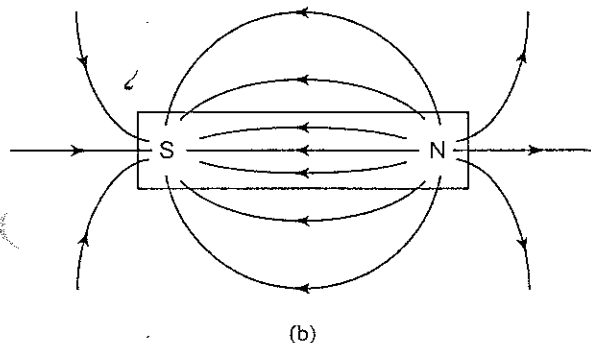
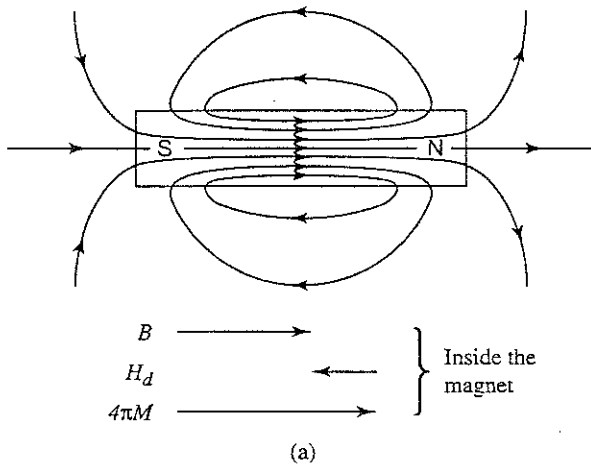


Shown is a hard ferromagnetic bar in zero applied field ( $H_0 = 0$ ).

A. Schematic diagram of the total flux density  $B = H_d + 4\pi M$ .

The vector contributions of  $M$  and  $H_d$  are illustrated below the bar magnet.

B. The demagnetization field  $H_d$  alone for the same ferromagnetic rod. Note that it is directed opposite the magnetization  $M$ . [After B. D. Cullity, "Introduction to Magnetic Materials"]



demagnetized) and the magnetization value is lower than should be observed.

Thus, the  $J_c$  value is underestimated. Hence, the choice of scan length is critical when determining the magnitude of irreversible behavior. In general, the shorter the scan length the better. But from experience a scan length on the order of 2.5 cm on the MPMS seems to be a good compromise. A Technical Advisory on this subject is available.

### Time Dependence

Certain types of magnetic irreversibilities have a measurable time dependence. Two of the more notable examples are high temperature superconductors and spin glasses. In superconductors the decrease in magnetization in time is attributed to flux creep. Flux creep results from the thermal activation of flux vortices out of their pinning sites. By measuring  $M$  as a function of time a flux creep rate can be determined. From a plot of  $M$  versus the logarithm of time, as shown in Figure 21, a slope related to the flux creep rate can be extracted.

## F. DEMAGNETIZATION CORRECTIONS

When measuring a sample with a large magnetization value, it is often important to make *demagnetization corrections*. In this case "demagnetization" does not refer to the process of reducing the remanent magnetization, but rather is associated with the fact that the field  $H$  inside the sample depends on the shape of the sample. The origin of the demagnetization effect is usually described in terms of another magnetic field, the demagnetization field  $H_d$ , that results from the separation of hypothetical magnetic charge associated with the sample's magnetization  $M$  (see recommended reading to find a more complete discussion). The total  $H$  field inside the sample is given by  $H = H_0 + H_d$ , where  $H_0$  is now the applied field produced by the current in a magnet coil and  $H_d$  is the demagnetization field. The demagnetization field is given by  $H_d = -NM$  where  $N$  is the shape-dependent demagnetization factor and  $M$  is the magnetization of the material. This correction can be particularly important when measuring a strongly magnetic material. For a long, thin sample in a field parallel to its long axis,  $N = 0$ . For a short, flat sample in a perpendicular field, on the other hand, the demagnetization correction ( $NM$ ) can be enormous. The value of  $N$  has a range  $0 \leq N \leq 1$  in SI units and  $0 \leq N \leq 4\pi$  in cgs units.

Shown in Figure 22 is a schematic diagram of a magnetized piece of ferromagnetic material in zero applied field ( $H_0 = 0$ ). Figure 22a shows the  $B$  field (sum of  $H$  and  $M$ ) for this ferromagnet while Figure 22b shows the field  $H$  alone. Since  $H_0 = 0$ , the field  $H$  inside the sample is just the demagnetization field  $H_d$ . The vectors  $M$ ,  $B$ , and  $H_d$  inside the sample are also shown in Figure 22. It can be seen that for a ferromagnet  $H_d$  is negative, thereby reducing  $H$  inside the sample. In the case of a



superconductor,  $M$  is negative so  $H_d$  will be positive, leading to an increase of  $H$  in a superconducting sample. When an external field ( $H_0$ ) is applied,  $H_d$  will change with the magnetization. However, it is the total  $H$  field inside the sample ( $H_0 + H_d$ ) that will determine  $M$ . Therefore an  $M(H)$  curve should be plotted as a function of the total internal field  $H = H_0 + H_d$ , not the applied field  $H_0$ .

When measuring an isotropic ferromagnetic film, for example, the  $M(H_0)$  curve observed for  $H_0$  parallel to the plane of the film will be very different than that for  $H_0$  perpendicular to the film plane. When  $H_0$  is perpendicular there is a large demagnetization field in the film that reduces the total  $H$ , requiring a larger applied field  $H_0$  to saturate the film than would be required when  $H_0$  is parallel to the film plane. However, at saturation the same value of magnetization,  $M_s$ , will be observed in both cases.

To be able to accurately apply this simple demagnetization correction to  $H_0$  requires that the demagnetization field be uniform throughout the sample. The only sample shape for which a uniform field can exist throughout the sample is an ellipsoid. Therefore it is common practice to approximate the demagnetizing factor  $N$  for a sample by considering the "maximum-enclosed-ellipsoid," which is the ellipsoid with shape and dimensions that allow it to be enclosed in the sample and fill the maximum volume. Tables and plots of calculations of  $N$  values for ellipsoids are available (see recommended reading). Since most samples are not ellipsoids, this correction method should be considered only approximate. Sometimes materials are shaped into spheres so that this correction method can be used more accurately.

## **G. MAGNETIC CHARACTERIZATION OF A NEW MATERIAL**

When presented with a new or unknown material, one often wants to quickly determine the general magnetic properties of the sample. The following procedure comes with no guarantees, but rather is intended as a systematic plan to help identify the magnetic behavior present in the sample. It is also only a starting point, because the more complete characterization of a sample typically requires detailed measurements once the regions of particular interest have been identified.

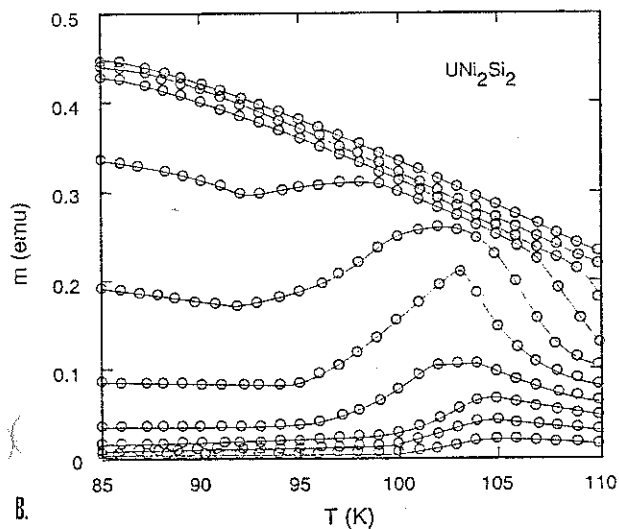
After determining the safe range of temperatures for the sample, an  $M(T)$  sweep with a very low magnetic field ( $H \sim 50$  to  $100$  Oe) should be run over the full temperature range. A temperature resolution of  $5$  K at low temperature and  $10$  K at higher temperatures is often suitable. Both superconductivity and ferromagnetism can stand out in such a scan. If one of these properties is identified, detailed measurements of that property can proceed.

## G. MAGNETIC CHARACTERIZATION OF A NEW MATERIAL

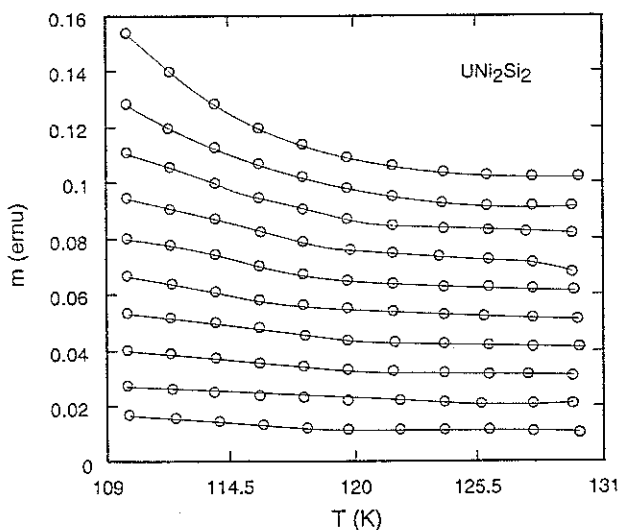
23

Magnetization as a function of temperature for  $\text{UNi}_2\text{Si}_2$  at a series of evenly spaced applied magnetic fields. The fields range from  $H = 5000$  to  $H = 50000$  Oe in steps of 5000 Oe. A: The temperature range from 10 K where a metamagnetic transition region exists (from 85 to about 103 K) and a regular ferromagnetic region exists (from about 103 to about 110 K). In a metamagnetic transition the system goes from antiferromagnetic to ferromagnetic. At the lower temperatures there is a cluster of points at the lower field. The lower field cluster has an  $M(H)$  curve characteristic of an antiferromagnet. Then there is a sharp increase in magnetization followed by another cluster of points at the highest fields. The cluster at the high fields is the magnetic saturation associated with a ferromagnet while in between is the transition from one magnetic behavior to the other. B: In the region near and above the ferromagnetic transition the spacing of the points at a given temperature suggest paramagnetic behavior.

A.



B.



Depending on the available time, the next procedure in a general search would be to measure  $M(T)$  again over the full temperature range but now at a series of equally spaced values of  $H$  (eg.  $\Delta H \sim 5$  kOe). The advantage of this kind of plot can be seen in Figure 23, for a sample of  $\text{UNi}_2\text{Si}_2$ . If the  $M(H)$  curve at a given temperature is linear, then the  $M(T)$  curves taken with even field increments will be evenly spaced. If the  $M(H)$  curve saturates at a certain temperature, then the points will get closer together as  $M$  saturates. Other interesting information, like a *spin flop* or *metamagnetic transitions* can often be identified from sets of  $M(T)$  plots. Detailed measurements including history dependence can then proceed more efficiently with this information at hand.

## A. TABLE OF CONVERSIONS

## UNITS FOR MAGNETIC PROPERTIES

Quantity	Symbol	Gaussian & cgs emu <sup>a</sup>	Conversion factor, C <sup>b</sup>	SI & rationalized mks <sup>c</sup>
Magnetic flux density, magnetic induction	$B$	gauss (G) <sup>d</sup>	$10^{-4}$	tesla (T), Wb/m <sup>2</sup>
Magnetic flux	$\Phi$	maxwell (Mx), G·cm <sup>2</sup>	$10^{-8}$	weber (Wb), volt second (V·s)
Magnetic potential difference, magnetomotive force	$U, F$	gilbert (Gb)	$10/4\pi$	ampere (A)
Magnetic field strength, magnetizing force	$H$	oersted (Oe), <sup>e</sup> Gb/cm	$10^3/4\pi$	A/m <sup>f</sup>
(Volume) magnetization <sup>g</sup>	$M$	emu/cm <sup>3</sup> <sup>h</sup>	$10^3$	A/m
(Volume) magnetization	$4\pi M$	G	$10^3/4\pi$	A/m
Magnetic polarization, intensity of magnetization	$J, I$	emu/cm <sup>3</sup>	$4\pi \times 10^{-4}$	T, Wb/m <sup>2</sup> <sup>i</sup>
(Mass) magnetization	$\sigma, M$	emu/g	$\frac{1}{4\pi \times 10^{-7}}$	A·m <sup>2</sup> /kg Wb·m/kg
Magnetic moment	$m$	emu, erg/G	$10^{-3}$	A·m <sup>2</sup> , joule per tesla (J/T)
Magnetic dipole moment	$j$	emu, erg/G	$4\pi \times 10^{-10}$	Wb·m <sup>i</sup>
(Volume) susceptibility	$\chi, \kappa$	dimensionless, emu/cm <sup>3</sup>	$\frac{4\pi}{(4\pi)^2 \times 10^{-7}}$	dimensionless henry per meter (H/m), Wb/(A·m)
(Mass) susceptibility	$\chi_p, \kappa_p$	cm <sup>3</sup> /g, emu/g	$\frac{4\pi \times 10^{-3}}{(4\pi)^2 \times 10^{-10}}$	m <sup>3</sup> /kg H·m <sup>2</sup> /kg
(Molar) susceptibility	$\chi_{\text{mol}}, \kappa_{\text{mol}}$	cm <sup>3</sup> /mol, emu/mol	$\frac{4\pi \times 10^{-6}}{(4\pi)^2 \times 10^{-13}}$	m <sup>3</sup> /mol H·m <sup>2</sup> /mol
Permeability	$\mu$	dimensionless	$4\pi \times 10^{-7}$	H/m, Wb/(A·m)
Relative permeability <sup>j</sup>	$\mu_r$	not defined		dimensionless
(Volume) energy density, energy product <sup>k</sup>	$W$	erg/cm <sup>3</sup>	$10^{-1}$	J/m <sup>3</sup>
Demagnetization factor	$D, N$	dimensionless	$1/4\pi$	dimensionless

a. Gaussian units and cgs emu are the same for magnetic properties. The defining relation is  $B = H + 4\pi M$ .

b. Multiply a number in Gaussian units by C to convert it to SI (e.g.,  $1 \text{ G} \times 10^{-4} \text{ T/G} = 10^{-4} \text{ T}$ ).

c. SI (*Système International d'Unités*) has been adopted by the National Bureau of Standards. Where two conversion factors are given, the upper one is recognized under, or consistent with, SI and is based on the definition  $B = \mu_0(H + M)$ , where  $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$ . The lower one is not recognized under SI and is based on the definition  $B = \mu_0 H + J$ , where the symbol  $I$  is often used in place of  $J$ .

d.  $1 \text{ gauss} = 10^5 \text{ gamma } (\gamma)$ .

e. Both oersted and gauss are expressed as  $\text{cm}^{-1/2} \cdot \text{g}^{1/2} \cdot \text{s}^{-1}$  in terms of base units.

f. A/m was often expressed as "ampere-turn per meter" when used for magnetic field strength.

g. Magnetic moment per unit volume.

h. The designation "emu" is not a unit.

i. Recognized under SI, even though based on the definition  $B = \mu_0 H + J$ . See footnote c.

j.  $\mu_r = \mu/\mu_0 = 1 + \chi$ , all in SI.  $\mu_r$  is equal to Gaussian  $\mu$ .

k.  $B \cdot H$  and  $\mu_0 M \cdot H$  have SI units J/m<sup>3</sup>;  $M \cdot H$  and  $B \cdot H/4\pi$  have Gaussian units erg/cm<sup>3</sup>.

## B. RECOMMENDED READING

### *Introduction to Magnetic Materials,*

B. D. Cullity, Addison-Wesley, 1972, ISBN# 0-201-01218-9.

This is the simplest introduction to magnetism in materials that is available. It is an engineering textbook that discusses many practical matters; an essential reference for anyone working on the magnetism of materials. Plots and tables of demagnetization coefficients are included.

### *The Physical Principles of Magnetism,*

A. H. Morrish, R. E. Krieger, Wiley (no longer in print), 1983, ISBN# 0-88275-670-2.

This is a very complete physics textbook quantitatively describing the physical properties of magnetic materials.

### *Magneto-Chemistry,*

R. L. Carlin, Springer-Verlag, 1986, ISBN# 0-387-15816-2.

This is an introduction to magnetism primarily in molecular systems.

### *Long Range Order in Solids,*

R. M. White and T. H. Geballe, Academic Press, 1979, ISBN# 0-12-607777-0.

This is an advanced solid state physics textbook that discusses long range order in general and in various specific cases (i.e., magnetic and superconducting transitions). It is an excellent source of references to the literature on magnetism.

### *Introduction to Superconductivity,*

A. C. Rose-Innes and E. H. Rhoderick, Pergamon, 1978, ISBN# 0-08-021651-8.

This is a fairly accessible development of superconductivity requiring a modest knowledge of solid state physics.

### *Superconductivity of Metals and Alloys,*

P. G. DeGennes, Addison-Wesley, 1966, ISBN# 0-201-51007-3.

This is a graduate-level physics textbook on superconductivity and is regarded as one of the classic texts on superconductivity.

### *Introduction to Superconductivity,*

M. Tinkham, R. E. Krieger, Pubnet, 1980, ISBN# 0-89874-049-5.

This is a graduate-level physics textbook on superconductivity and is also regarded as one of the classic texts on superconductivity.