# Problem 2. d.

## $Z_{Odd}$

```
f[l_] := (4 l + 3) e^-((4 l^2+6 l+2) θ/T)
a = Assuming[Re[θ/T] > 0, Integrate[f[l], {l, 0, ∞}]];
Expand[FullSimplify[a + f[0]/2 - f'[0]/12 + f'''[0]/720]]
```

$$\frac{7}{6} e^{-\frac{2\theta}{T}} + \frac{e^{-\frac{2\theta}{T}} T}{2\theta} + \frac{41\, e^{-\frac{2\theta}{T}} \theta}{30\, T} + \frac{6\, e^{-\frac{2\theta}{T}} \theta^2}{5\, T^2} - \frac{9\, e^{-\frac{2\theta}{T}} \theta^3}{10\, T^3}$$

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## Now taylor approx exp term and keep up to $O\left(\frac{\theta}{T}\right)$

```
Expand[(1 - 2 θ/T + 2 θ^2/T^2) (T/(2 θ) + 7/6 + 41/30 θ/T)]
```

$$\frac{1}{6} + \frac{T}{2\theta} + \frac{\theta}{30\, T} - \frac{2\theta^2}{5\, T^2} + \frac{41\, \theta^3}{15\, T^3}$$

## $Z_{Even}$

```
feven[l_] := (4 l + 1) e^-2 l (2 l+1) θ/T
b = Assuming[Re[θ/T] > 0, Integrate[feven[l], {l, 0, ∞}]];
Expand[FullSimplify[b + feven[0]/2 - feven'[0]/12 + feven'''[0]/720]]
```

$$\frac{1}{6} + \frac{T}{2\theta} + \frac{\theta}{30\, T} + \frac{2\theta^2}{15\, T^2} - \frac{\theta^3}{90\, T^3}$$