

Distinguishing NFW and Isothermal Density Profiles with Gravitational Lensing

April 17, 2018

Abstract

We attempt to distinguish NFW and cored isothermal density profiles using weak gravitational lensing shear.

1 Introduction

1.1 General spherical density profile $\rho(r)$

Using thin lens approximation

Projected surface density at radius R

$$\Sigma(R) = \int_{-\infty}^{\infty} \rho(\sqrt{R^2 + z^2}) dz$$

Average projected surface density within radius R

$$\bar{\Sigma}(R) = \frac{1}{\pi R^2} \int_0^{2\pi} d\phi \int_0^R dR' \Sigma(R') R'$$

Critical surface density

$$\Sigma_{\text{crit}} = \frac{c^2}{4\pi G} \frac{D_S}{D_{SL} D_L}$$

Convergence

$$\kappa(\theta) = \frac{\Sigma(\theta)}{\Sigma_{\text{crit}}}$$

Tangential shear

$$\begin{aligned} \gamma_t(\theta) &= \bar{\kappa}(\theta) - \kappa(\theta) \\ \gamma_1 &= -\gamma_t \cos 2\phi \\ \gamma_2 &= -\gamma_t \sin 2\phi \\ \gamma &= \gamma_t = \sqrt{\gamma_1^2 + \gamma_2^2} = -\gamma_1 \cos 2\phi - \gamma_2 \sin 2\phi \end{aligned}$$

Deflection angle

$$\begin{aligned} \vec{\alpha}(\vec{\theta}) &= \bar{\kappa}(\theta) \vec{\theta} \\ \vec{\beta} &= \vec{\theta} - \vec{\alpha} = (1 - \bar{\kappa}(\theta)) \vec{\theta} \end{aligned}$$

Ellipticity

$$\epsilon_i = \frac{2\gamma_i/(1-\kappa)}{1+\gamma^2/(1-\kappa)^2}$$

In small angle approximation, any length $R = D_L \theta$.

Prove that spherical density profiles only have tangential shear and ellipticity

1.2 Cored Isothermal Sphere Profile

$$\rho_{\text{iso}}(r) = \frac{\sigma^2}{2\pi G(r^2 + r_c^2)}$$

$$\Sigma_{\text{iso}}(R) = \frac{\sigma^2}{2G\sqrt{R^2 + r_c^2}}$$

$$\bar{\Sigma}_{\text{iso}}(R) = \frac{\sigma^2 (\sqrt{R^2 + r_c^2} - r_c)}{GR^2}$$

In terms of angles:

$$\Sigma_{\text{iso}}(\theta) = \frac{\sigma^2}{2GD_L\sqrt{\theta^2 + \theta_c^2}}$$

$$\bar{\Sigma}_{\text{iso}}(\theta) = \frac{\sigma^2 (\sqrt{\theta^2 + \theta_c^2} - \theta_c)}{GD_L\theta^2}$$

$$\gamma_{\text{iso}}(\theta) = \frac{\sigma^2 (\sqrt{\theta^2 + \theta_c^2} - \theta_c)}{\Sigma_{\text{crit}}GD_L\theta^2} - \frac{\sigma^2}{2\Sigma_{\text{crit}}GD_L\sqrt{\theta^2 + \theta_c^2}}$$

We switch dependence from σ^2 to M_{200} with:

$$\sigma^2 = \frac{M_{200}G}{2 \left(r_{200} - r_c \arctan \left(\frac{r_{200}}{r_c} \right) \right)}$$

This is derived from the definition of M_{200} :

$$\begin{aligned} M_{200} &= 200\rho_{\text{crit}} \frac{4}{3}\pi r_{200}^3 \\ M_{200} &= M_{\text{enc}}(r_{200}) = \frac{2\sigma^2}{G} \left(r_{200} - r_c \arctan \left(\frac{r_{200}}{r_c} \right) \right) \\ r_{200} &= \left(\frac{3M_{200}}{800\pi\rho_{\text{crit}}} \right)^{1/3} \end{aligned}$$

Ellipticity equations are very ugly but trivial to calculate from the shear.

1.3 Navarro-Frenk-White (NFW) Profile

$$\rho_{\text{NFW}}(r) = \frac{\rho_{\text{crit}} \delta_C}{(r/r_s)(1 + r/r_s)^2}$$

$$\Sigma_{\text{NFW}}(R) = \frac{2\rho_{\text{crit}}\delta_C r_s}{(R/r_s)^2 - 1} \left(1 - \frac{2}{\sqrt{(R/r_s)^2 - 1}} \right) \arctan \left(\sqrt{\frac{R/r_s - 1}{R/r_s + 1}} \right)$$

[Bartelmann 2001]

$$\bar{\Sigma}_{\text{NFW}}(R) = \frac{4\rho_{\text{crit}}\delta_C r_s}{(R/r_s)^2} \left(\frac{2}{\sqrt{(R/r_s)^2 - 1}} \arctan \left(\sqrt{\frac{R/r_s - 1}{R/r_s + 1}} \right) + \ln \left(\frac{R/r_s}{2} \right) \right)$$

In terms of angles:

$$\Sigma_{\text{NFW}}(\theta) = \frac{2\rho_{\text{crit}}\delta_C D_L \theta_s}{(\theta/\theta_s)^2 - 1} \left(1 - \frac{2}{\sqrt{(\theta/\theta_s)^2 - 1}} \right) \arctan \left(\sqrt{\frac{\theta/\theta_s - 1}{\theta/\theta_s + 1}} \right)$$

$$\bar{\Sigma}_{\text{NFW}}(\theta) = \frac{4\rho_{\text{crit}}\delta_C D_L \theta_s}{(\theta/\theta_s)^2} \left(\frac{2}{\sqrt{(\theta/\theta_s)^2 - 1}} \arctan \left(\sqrt{\frac{\theta/\theta_s - 1}{\theta/\theta_s + 1}} \right) + \ln \left(\frac{\theta/\theta_s}{2} \right) \right)$$

$$\gamma_{\text{NFW}}(\theta) = \frac{\bar{\Sigma}_{\text{NFW}}(\theta) - \Sigma_{\text{NFW}}(\theta)}{\Sigma_{\text{crit}}}$$

Can calculate ellipticities from tangential shear.

We switch the dependence to M_{200} and C with:

$$\delta_C = \frac{200}{3} \frac{C^3}{\ln(1 + C) - C/(1 + C)}$$

$$r_s = \frac{r_{200}}{C}$$

$$r_{200} = \left(\frac{3M_{200}}{800\pi\rho_{\text{crit}}} \right)^{1/3}$$

$$C = \frac{r_{200}}{r_s}$$

2 Methods

2.1 Current plan

- Calculate tangential shear and deflection angle for NFW and SIS
- Consider single foreground lens halo with many background galaxies

- How to arrange and distribute background halos?
 - Initial ellipticity
 - angular density on sky
 - sizes
- Apply shear and deflection angle to background galaxies
- Attempt to fit both profiles, subtracting intrinsic shear, see if the fit is distinguishable
- How many foreground halos to test? - start with 1
- What redshift is ρ_{crit} evaluated at? - at halo redshift

Fake data: N sets of e1, e2, theta1, theta2

Analyzing data: make histogram in annulus mean and standard deviation use log bins for theta

50 gals/square arcminute 5 arcminutes $z_L = 0.3$ $z_S = 1$ $M_{\text{halo}} = 10^{15}$ solar masses intrinsic ellipticity from gaussian with width 0.2