Distinguishing NFW and Isothermal Density Profiles with Gravitational Lensing

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Abstract

We attempt to distinguish NFW and cored isothermal density profiles using weak gravitational lensing shear.

1 Introduction

1.1 General spherical density profile $\rho(r)$

Using thin lens approximation

Projected surface density at radius R

$$\Sigma(R) = \int_{-\infty}^{\infty} \rho(\sqrt{R^2 + z^2}) dz$$

Average projected surface density within radius R

$$\overline{\Sigma}(R) = \frac{1}{\pi R^2} \int_0^{2\pi} d\phi \int_0^R dR' \; \Sigma(R') R'$$

Critical surface density

$$\Sigma_{\rm crit} = \frac{c^2}{4\pi G} \frac{D_S}{D_{SL} D_L}$$

Convergence

$$\kappa(\theta) = \frac{\Sigma(\theta)}{\Sigma_{\rm crit}}$$

Tangential shear

$$\gamma_t(\theta) = \overline{\kappa}(\theta) - \kappa(\theta)$$

$$\gamma_1 = -\gamma_t \cos 2\phi$$

$$\gamma_2 = -\gamma_t \sin 2\phi$$

$$\gamma = \gamma_t = \sqrt{{\gamma_1}^2 + {\gamma_2}^2} = -\gamma_1 \cos 2\phi - \gamma_2 \sin 2\phi$$

Deflection angle

$$\vec{\alpha}(\vec{\theta}) = \overline{\kappa}(\theta) \vec{\theta}$$

$$\vec{\beta} = \vec{\theta} - \vec{\alpha} = (1 - \overline{\kappa}(\theta)) \vec{\theta}$$

Ellipticity

$$\epsilon_i = \frac{2\gamma_i/(1-\kappa)}{1+\gamma^2/(1-\kappa)^2}$$

In small angle approximation, any length $R = D_L\theta$.

Prove that spherical density profiles only have tangential shear and ellipticity

1.2 Cored Isothermal Sphere Profile

$$\rho_{\text{iso}}(r) = \frac{\sigma^2}{2\pi G(r^2 + {r_c}^2)}$$

$$\Sigma_{\text{iso}}(R) = \frac{\sigma^2}{2G\sqrt{R^2 + {r_c}^2}}$$

$$\overline{\Sigma}_{\text{iso}}(R) = \frac{\sigma^2\left(\sqrt{R^2 + {r_c}^2} - r_c\right)}{GR^2}$$

In terms of angles:

$$\Sigma_{\text{iso}}(\theta) = \frac{\sigma^2}{2GD_L\sqrt{\theta^2 + {\theta_c}^2}}$$

$$\overline{\Sigma}_{\text{iso}}(\theta) = \frac{\sigma^2\left(\sqrt{\theta^2 + {\theta_c}^2} - \theta_c\right)}{GD_L\theta^2}$$

$$\gamma_{\text{iso}}(\theta) = \frac{\sigma^2\left(\sqrt{\theta^2 + {\theta_c}^2} - \theta_c\right)}{\Sigma_{\text{crit}}GD_L\theta^2} - \frac{\sigma^2}{2\Sigma_{\text{crit}}GD_L\sqrt{\theta^2 + {\theta_c}^2}}$$

We switch dependence from σ^2 to M_{200} with:

$$\sigma^2 = \frac{GM_{200}}{2\left(\frac{3M_{200}}{800\pi\rho_{\text{crit}}} - r_c \arctan\left(\frac{3M_{200}}{800\pi\rho_{\text{crit}}{r_c}^3}\right)\right)}$$

$$\sigma^2 = \frac{GM_{200}}{2\left(\frac{3M_{200}}{800\pi\rho_{\text{crit}}} - D_L\theta_c \arctan\left(\frac{3M_{200}}{800\pi\rho_{\text{crit}}{D_L}^3{\theta_c}^3}\right)\right)}$$

This is derived from the definition of M_{200}:

$$M_{200} = M_{enc}(r_{200}) = 200\rho_{\text{crit}}\frac{4}{3}\pi{r_{200}}^3$$

$$r_{200} = \left(\frac{3M_{200}}{800\pi\rho_{\text{crit}}}\right)^{1/3}$$

Ellipticity equations are very ugly but trivial to calculate from the shear.

1.3 Navarro-Frenk-White (NFW) Profile

$$\rho_{\mathrm{NFW}}(r) = \frac{\rho_{\mathrm{crit}}\delta_C}{(r/r_s)\,(1+r/r_s)^2}$$

$$\Sigma_{\mathrm{NFW}}(R) = \frac{2\rho_{\mathrm{crit}}\delta_C r_s}{(R/r_s)^2-1}\left(1-\frac{2}{\sqrt{(R/r_s)^2-1}}\right)\arctan\left(\sqrt{\frac{R/r_s-1}{R/r_s+1}}\right)$$

[Bartelmann 2001]

$$\overline{\Sigma}_{\mathrm{NFW}}(R) = \frac{4\rho_{\mathrm{crit}}\delta_C r_s}{(R/r_s)^2}\left(\frac{2}{\sqrt{(R/r_s)^2-1}}\ \arctan\left(\sqrt{\frac{R/r_s-1}{R/r_s+1}}\right)+\ln\left(\frac{R/r_s}{2}\right)\right)$$

In terms of angles:

$$\Sigma_{\mathrm{NFW}}(\theta) = \frac{2\rho_{\mathrm{crit}}\delta_C D_L\theta_s}{(\theta/\theta_s)^2-1}\left(1-\frac{2}{\sqrt{(\theta/\theta_s)^2-1}}\right)\arctan\left(\sqrt{\frac{\theta/\theta_s-1}{\theta/\theta_s+1}}\right)$$

$$\overline{\Sigma}_{\mathrm{NFW}}(\theta) = \frac{4\rho_{\mathrm{crit}}\delta_C D_L\theta_s}{(\theta/\theta_s)^2}\left(\frac{2}{\sqrt{(\theta/\theta_s)^2-1}}\ \arctan\left(\sqrt{\frac{\theta/\theta_s-1}{\theta/\theta_s+1}}\right)+\ln\left(\frac{\theta/\theta_s}{2}\right)\right)$$

$$\gamma_{\mathrm{NFW}}(\theta) = \frac{\overline{\Sigma}_{\mathrm{NFW}}(\theta)-\Sigma_{\mathrm{NFW}}(\theta)}{\Sigma_{\mathrm{crit}}}$$

Can calculate ellipticities from tangential shear.

We switch the dependence to M_{200} and C with:

$$\delta_C = \frac{200}{3}\frac{C^3}{\ln(1+C)-C/(1+C)}$$

$$r_s = \frac{1}{C}\left(\frac{3M_{200}}{800\pi\rho_{\mathrm{crit}}}\right)^{\frac{1}{3}}$$

$$C = \frac{r_{200}}{r_s}$$

2 Methods

2.1 Current plan

- Calculate tangential shear and deflection angle for NFW and SIS
- Consider single foreground lens halo with many background galaxies

- How to arrange and distribute background halos?
  - Initial ellipticity
  - angular density on sky
  - sizes
- Apply shear and deflection angle to background galaxies
- Attempt to fit both profiles, subtracting intrinsic shear, see if the fit is distinguishable
- How many foreground halos to test? - start with 1
- What redshift is $\rho_{\rm crit}$ evaluated at? - at halo redshift

Fake data: N sets of e1, e2, theta1, theta2

Analyzing data: make histogram in annulus mean and standard deviation use log bins for theta

50 gals/square arcminute 5 arcminutes $z_L = 0.3$ $z_S = 1$ $M_{halo} = 10^15$ solar masses instrinsic ellipticity from gaussian with width 0.2