Distinguishing NFW and Isothermal Density Profiles with Gravitational Lensing

April 17, 2018

Abstract

1 Introduction

1.1 General spherical density profile $\rho(r)$

Using thin lens approximation

Projected surface density at radius R

$$\Sigma(R) = \int_{-\infty}^{\infty} \rho(\sqrt{R^2 + z^2}) dz \tag{1}$$

Average projected surface density within radius R

$$\overline{\Sigma}(R) = \frac{1}{\pi R^2} \int_0^{2\pi} d\phi \int_0^R dR' \; \Sigma(R') R' \tag{2}$$

Critical surface density

$$\Sigma_{\rm crit} = \frac{c^2}{4\pi G} \frac{D_S}{D_{SL} D_L} \tag{3}$$

Convergence at radius R

$$\kappa(R) = \frac{\Sigma(R)}{\Sigma_{\rm crit}} \tag{4}$$

Tangential shear at radius R

$$\gamma_t(R) = \overline{\kappa}(R) - \kappa(R) \tag{5}$$

$$\gamma_1 = -\gamma_t \cos 2\phi$$

$$\gamma_2 = -\gamma_t \sin 2\phi$$

Deflection angle

$$\vec{\alpha}(\vec{\theta}) = \overline{\kappa}(\theta) \vec{\theta} \tag{6}$$

Ellipticity

$$\epsilon(R) = \frac{2\gamma/(1-\kappa)}{1 + (\gamma/(1-\kappa))^2} \tag{7}$$

Prove that spherical density profiles only have tangential shear and ellipticity

1.2 Navarro-Frenk-White (NFW) Profile

$$\rho_{\mathrm{NFW}}(r) = \frac{\rho_{\mathrm{crit}}\delta_c}{(r/r_s)\left(1 + r/r_s\right)^2} \tag{8}$$

$$\Sigma_{\mathrm{NFW}}(R) = \frac{2\rho_{\mathrm{crit}}\delta_c r_s}{(R/r_s)^2 - 1}\left(1 - \frac{2}{\sqrt{(R/r_s)^2 - 1}}\right)\arctan\left(\sqrt{\frac{R/r_s - 1}{R/r_s + 1}}\right) \tag{9}$$

[Bartelmann 2001]

$$\overline{\Sigma}_{\mathrm{NFW}}(R) = \frac{4\rho_{\mathrm{crit}}\delta_c r_s}{(R/r_s)^2}\left(\frac{2}{\sqrt{(R/r_s)^2 - 1}}\ \arctan\left(\sqrt{\frac{R/r_s - 1}{R/r_s + 1}}\right) + \ln\left(\frac{R/r_s}{2}\right)\right) \tag{10}$$

$$\gamma(R) = \tag{11}$$

We switch the dependence to M_{200} and c with:

$$\delta_c = \frac{200}{3}\frac{c^3}{\ln(1+c) - c/(1+c)} \tag{12}$$

$$r_s = \frac{1}{c}\left(\frac{3M_{200}}{800\pi\rho_{\mathrm{crit}}}\right)^{\frac{1}{3}} \tag{13}$$

1.3 Cored Isothermal Sphere Profile

$$\rho_{\mathrm{iso}}(r) = \frac{\sigma^2}{2\pi G(r^2 + {r_c}^2)} \tag{14}$$

$$\Sigma_{\mathrm{iso}}(R) = \frac{\sigma^2}{2G\sqrt{R^2 + {r_c}^2}} \tag{15}$$

$$\overline{\Sigma}_{\mathrm{iso}}(R) = \frac{\sigma^2\left(\sqrt{R^2 + {r_c}^2} - r_c\right)}{GR^2} \tag{16}$$

$$\gamma(R) = \frac{\sigma^2\left(\sqrt{R^2 + {r_c}^2} - r_c\right)}{\Sigma_{\mathrm{crit}}GR^2} - \frac{\sigma^2}{2\Sigma_{\mathrm{crit}}G\sqrt{R^2 + {r_c}^2}} \tag{17}$$

We switch dependence from σ^2 to M_{200} with:

$$r_{200} = \left(\frac{3M_{200}}{800\pi\rho_{\mathrm{crit}}}\right)^{1/3} \tag{18}$$

$$\sigma^2 = \frac{GM_{200}}{2\left(\frac{3M_{200}}{800\pi\rho_{\mathrm{crit}}} - r_c\arctan\left(\frac{3M_{200}}{800\pi\rho_{\mathrm{crit}}{r_c}^3}\right)\right)} \tag{19}$$

M_{200} is defined as:

$$M_{200} = M_{enc}(r_{200}) = 200\rho_{\mathrm{crit}}\frac{4}{3}\pi r_{200}^3 \tag{20}$$

1.4 Current plan

- Calculate tangential shear and deflection angle for NFW and SIS
- Consider single foreground lens halo with many background galaxies
- How to arrange and distribute background halos?
  - Initial ellipticity
  - angular density on sky
  - sizes
- Apply shear and deflection angle to background galaxies
- Attempt to fit both profiles, subtracting intrinsic shear, see if the fit is distinguishable
- How many foreground halos to test? - start with 1
- What redshift is ρ_{crit} evaluated at? - at halo redshift

N sets of e1, e2, theta1, theta2

Analyzing data: make histogram in annulus mean and standard deviation use log bins for theta

50 gals/square arcminute 5 arcminutes $z_L = 0.3$ $z_S = 1$ $M_halo = 10^15$ solar masses instrinsic ellipticity from gaussian with width 0.2

2 Methods