Distinguishing NFW and Isothermal Density Profiles with Weak Gravitational Lensing

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Abstract

We examine the feasibility of distinguishing NFW and cored isothermal density profiles using weak gravitational lensing shear.

1 Introduction

Background/history of this research topic, goals for this section

1.1 General spherical density profile $\rho(r)$

Here we define our conventions for various lensing quantities, which mainly conform to those used by Dodelson [2017]. We use the thin lens approximation and assume spherical lens profiles.

Projected surface density at radius R

$$\Sigma(R) = \int_{-\infty}^{\infty} dz \; \rho(\sqrt{R^2 + z^2})$$

Average projected surface density within radius R

$$\overline{\Sigma}(R) = \frac{1}{\pi R^2} \int_0^{2\pi} d\phi \int_0^R dR' \; \Sigma(R')R'$$

Critical surface density

$$\Sigma_{\text{crit}} = \frac{c^2}{4\pi G} \frac{D_S}{D_{SL} D_L}$$

Convergence

$$\kappa(\theta) = \frac{\Sigma(\theta)}{\Sigma_{\text{crit}}}$$

Tangential shear

$$\gamma_t(\theta) = \overline{\kappa}(\theta) - \kappa(\theta)$$

$$\gamma_1 = -\gamma_t \cos 2\phi$$

$$\gamma_2 = -\gamma_t \sin 2\phi$$

$$\gamma = \gamma_t = \sqrt{{\gamma_1}^2 + {\gamma_2}^2} = -\gamma_1 \cos 2\phi - \gamma_2 \sin 2\phi$$

Deflection angle

$$\vec{\alpha}(\vec{\theta}) = \overline{\kappa}(\theta)\vec{\theta}$$

$$\vec{\beta} = \vec{\theta} - \vec{\alpha} = (1 - \overline{\kappa}(\theta))\vec{\theta}$$

Ellipticity

$$\epsilon_i = \frac{2\gamma_i/(1-\kappa)}{1+\gamma^2/(1-\kappa)^2}$$

$$\epsilon = -\epsilon_1 \cos 2\phi - \epsilon_2 \sin 2\phi$$

In small angle approximation, any length $R = D_L\theta$.

Prove that spherical density profiles only have tangential shear and ellipticity

1.2 Cored Isothermal Sphere Profile

$$\rho_{\text{iso}}(r) = \frac{\sigma^2}{2\pi G(r^2 + {r_c}^2)}$$

$$\Sigma_{\text{iso}}(\theta) = \frac{\sigma^2}{2GD_L\sqrt{\theta^2 + {\theta_c}^2}}$$

$$\overline{\Sigma}_{\text{iso}}(\theta) = \frac{\sigma^2\left(\sqrt{\theta^2 + {\theta_c}^2} - \theta_c\right)}{GD_L\theta^2}$$

$$\gamma_{\text{iso}}(\theta) = \frac{\sigma^2\left(\sqrt{\theta^2 + {\theta_c}^2} - \theta_c\right)}{\Sigma_{\text{crit}}GD_L\theta^2} - \frac{\sigma^2}{2\Sigma_{\text{crit}}GD_L\sqrt{\theta^2 + {\theta_c}^2}}$$

We switch dependence from σ^2 to M_{200} with:

$$\sigma^2 = \frac{M_{200}G}{2\left(r_{200} - r_c \arctan\left(\frac{r_{200}}{r_c}\right)\right)}$$

This is derived from the definition of M_{200}:

$$M_{200} = 200\rho_{\text{crit}}\frac{4}{3}\pi {r_{200}}^3$$

$$M_{200} = M_{\text{enc}}(r_{200}) = \frac{2\sigma^2}{G}\left(r_{200} - r_c \arctan\left(\frac{r_{200}}{r_c}\right)\right)$$

$$r_{200} = \left(\frac{3M_{200}}{800\pi\rho_{\text{crit}}}\right)^{1/3}$$

Ellipticity equations are very ugly but trivial to calculate from the shear.

1.3 Navarro-Frenk-White (NFW) Profile

$$\rho_{\mathrm{NFW}}(r) = \frac{\rho_{\mathrm{crit}}\delta_c}{(r/r_s)\left(1 + r/r_s\right)^2}$$

$$\Sigma_{\mathrm{NFW}}(\theta) = \frac{2\rho_{\mathrm{crit}}\delta_c D_L \theta_s}{(\theta/\theta_s)^2 - 1}\left(1 - \frac{2}{\sqrt{(\theta/\theta_s)^2 - 1}} \arctan\left(\sqrt{\frac{\theta/\theta_s - 1}{\theta/\theta_s + 1}}\right)\right)$$

$$\overline{\Sigma}_{\mathrm{NFW}}(\theta) = \frac{4\rho_{\mathrm{crit}}\delta_c D_L \theta_s}{(\theta/\theta_s)^2}\left(\frac{2}{\sqrt{(\theta/\theta_s)^2 - 1}}\ \arctan\left(\sqrt{\frac{\theta/\theta_s - 1}{\theta/\theta_s + 1}}\right) + \ln\left(\frac{\theta/\theta_s}{2}\right)\right)$$

Similar convention used by Bartelmann et al. [2001]

$$\gamma_{\mathrm{NFW}}(\theta) = \frac{\overline{\Sigma}_{\mathrm{NFW}}(\theta) - \Sigma_{\mathrm{NFW}}(\theta)}{\Sigma_{\mathrm{crit}}}$$

Can calculate ellipticities from tangential shear.

We switch the dependence to M_{200} and c with:

$$\delta_c = \frac{200}{3}\frac{c^3}{\ln(1+c) - c/(1+c)}$$

$$r_s = \frac{r_{200}}{c}$$

$$r_{200} = \left(\frac{3M_{200}}{800\pi\rho_{\mathrm{crit}}}\right)^{1/3}$$

$$c = \frac{r_{200}}{r_s}$$

Estimating ellipticity has well-documented issues [cite] due to noise and PSF

2 Methods

1. Calculate tangential shear and deflection angle for NFW and cored isothermal profiles.
2. Consider single foreground lens halo with many background galaxies.
   - Start with one NFW halo, then maybe consider more tests.
3. Construct background galaxies:
   - Number density on sky: 50 galaxies/square arcminute
   - Intrinsic ellipticity drawn from Gaussian distribution with $\mu = 0, \sigma = 0.2$
   - Assume they are all that the same distance D_S since this can be determined by redshift (neglecting some noise)
4. Recommended values:
   - $z_L = 0.3$
   - $z_S = 1.0$
   - $M_{halo} = 10^{15} M_\odot$
5. Apply shear and deflection angle to background galaxies, get fake data: N sets of ϵ_1, ϵ_2, θ_1, θ_2
6. Bin galaxies in annuli by θ value (use log bins for theta)
7. Calculate mean and standard deviation of ellipticity
8. Attempt to fit both NFW and isothermal profiles, see if the fit is distinguishable

2.1 Questions

- How to properly do sigma contours?
- What theta range should we look at? (5 arcminutes?)
- What redshift is ρ_{crit} evaluated at? - at halo redshift - can we use current time?
- Using $c = 10$?
- Where to go from here?

3 Results

References

M. Bartelmann, L. J. King, and P. Schneider. Weak-lensing halo numbers and dark-matter profiles. *A&A*, 378(2):361–369, nov 2001. ISSN 0004-6361. doi: 10.1051/0004-6361:20011199. URL `http://www.edpsciences.org/10.1051/0004-6361:20011199`.

S Dodelson. *Gravitational Lensing*. Cambridge University Press, jun 2017.