Distinguishing NFW and Isothermal Density Profiles with Weak Gravitational Lensing

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Abstract

We examine the feasibility of distinguishing NFW and cored isothermal density profiles using weak gravitational lensing shear. To do so, we use Python to model lenses in the two different profiles, as well as background galaxies to be lensed. Analyzing the data of these lensed galaxies gives us insight into how we can distinguish the differences between the two profiles.

1 Introduction

Two commonly used density profiles in weak gravitational lensing are the isothermal profile and the Navarro-Frenk-White (NFW) profile. We start with introducing the general characteristics of spherical density profiles, followed by cored isothermal and NFW profiles. We then describe the purpose of our project, to distinguish between NFW and isothermal profiles, in detail.

1.1 General spherical density profile $\rho(r)$

We describe the common characteristics of spherical density profiles. Here we define our conventions for various lensing quantities, which mainly conform to those used by Dodelson [2017]. We use the thin lens approximation and assume spherical lens profiles.

The projected surface density at radius R is defined by

$$\Sigma(R) = \int_{-\infty}^{\infty} dz \ \rho(\sqrt{R^2 + z^2})$$

while the average projected surface density within R is

$$\overline{\Sigma}(R) = \frac{1}{\pi R^2} \int_0^{2\pi} d\phi \int_0^R dR' \ \Sigma(R') R'$$

The critical surface density is

$$\Sigma_{\text{crit}} = \frac{c^2}{4\pi G} \frac{D_S}{D_{SL} D_L}$$

Convergence

$$\kappa(\theta) = \frac{\Sigma(\theta)}{\Sigma_{\text{crit}}}$$

Tangential shear

$$\gamma_t(\theta) = \overline{\kappa}(\theta) - \kappa(\theta)$$

$$\gamma_1 = -\gamma_t \cos 2\phi$$
$$\gamma_2 = -\gamma_t \sin 2\phi$$
$$\gamma = \gamma_t = \sqrt{{\gamma_1}^2 + {\gamma_2}^2} = -\gamma_1 \cos 2\phi - \gamma_2 \sin 2\phi$$

Deflection angle

$$\vec{\alpha}(\vec{\theta}) = \overline{\kappa}(\theta)\vec{\theta}$$
$$\vec{\beta} = \vec{\theta} - \vec{\alpha} = (1 - \overline{\kappa}(\theta))\vec{\theta}$$

Ellipticity

$$\epsilon_i = \frac{2\gamma_i/(1-\kappa)}{1 + \gamma^2/(1-\kappa)^2}$$
$$\epsilon = -\epsilon_1 \cos 2\phi - \epsilon_2 \sin 2\phi$$

In small angle approximation, any length $R = D_L\theta$.

Prove that spherical density profiles only have tangential shear and ellipticity

1.2 Cored Isothermal Sphere Profile

$$\rho_{\text{iso}}(r) = \frac{\sigma^2}{2\pi G(r^2 + {r_c}^2)}$$

$$\Sigma_{\text{iso}}(\theta) = \frac{\sigma^2}{2GD_L\sqrt{\theta^2 + {\theta_c}^2}}$$

$$\overline{\Sigma}_{\text{iso}}(\theta) = \frac{\sigma^2\left(\sqrt{\theta^2 + {\theta_c}^2} - \theta_c\right)}{GD_L\theta^2}$$

$$\gamma_{\text{iso}}(\theta) = \frac{\sigma^2\left(\sqrt{\theta^2 + {\theta_c}^2} - \theta_c\right)}{\Sigma_{\text{crit}}GD_L\theta^2} - \frac{\sigma^2}{2\Sigma_{\text{crit}}GD_L\sqrt{\theta^2 + {\theta_c}^2}}$$

We switch dependence from σ^2 to M_{200} with:

$$\sigma^2 = \frac{M_{200}G}{2\left(r_{200} - r_c \arctan\left(\frac{r_{200}}{r_c}\right)\right)}$$

This is derived from the definition of M_{200}:

$$M_{200} = 200\rho_{\text{crit}}\frac{4}{3}\pi {r_{200}}^3$$
$$M_{200} = M_{\text{enc}}(r_{200}) = \frac{2\sigma^2}{G}\left(r_{200} - r_c \arctan\left(\frac{r_{200}}{r_c}\right)\right)$$
$$r_{200} = \left(\frac{3M_{200}}{800\pi\rho_{\text{crit}}}\right)^{1/3}$$

Ellipticity equations are very ugly but trivial to calculate from the shear.

1.3 Navarro-Frenk-White (NFW) Profile

$$\rho_{\mathrm{NFW}}(r) = \frac{\rho_{\mathrm{crit}}\delta_c}{(r/r_s)\left(1 + r/r_s\right)^2}$$

$$\Sigma_{\mathrm{NFW}}(\theta) = \frac{2\rho_{\mathrm{crit}}\delta_c D_L \theta_s}{(\theta/\theta_s)^2 - 1}\left(1 - \frac{2}{\sqrt{(\theta/\theta_s)^2 - 1}}\arctan\left(\sqrt{\frac{\theta/\theta_s - 1}{\theta/\theta_s + 1}}\right)\right)$$

$$\overline{\Sigma}_{\mathrm{NFW}}(\theta) = \frac{4\rho_{\mathrm{crit}}\delta_c D_L \theta_s}{(\theta/\theta_s)^2}\left(\frac{2}{\sqrt{(\theta/\theta_s)^2 - 1}}\ \arctan\left(\sqrt{\frac{\theta/\theta_s - 1}{\theta/\theta_s + 1}}\right) + \ln\left(\frac{\theta/\theta_s}{2}\right)\right)$$

Similar convention used by Bartelmann et al. [2001]

$$\gamma_{\mathrm{NFW}}(\theta) = \frac{\overline{\Sigma}_{\mathrm{NFW}}(\theta) - \Sigma_{\mathrm{NFW}}(\theta)}{\Sigma_{\mathrm{crit}}}$$

Can calculate ellipticities from tangential shear.

We switch the dependence to M_{200} and c with:

$$\delta_c = \frac{200}{3}\frac{c^3}{\ln(1+c) - c/(1+c)}$$

$$r_s = \frac{r_{200}}{c}$$

$$r_{200} = \left(\frac{3M_{200}}{800\pi\rho_{\mathrm{crit}}}\right)^{1/3}$$

$$c = \frac{r_{200}}{r_s}$$

Estimating ellipticity has well-documented issues [cite] due to noise and PSF

1.4 Project Purpose

The goal of this project is to devise a method to analyze lensed galaxy cluster data and find which density profile is more probable between the isothermal and NFW profiles.

2 Methods

2.1 Calculation of Shear and Deflection Angles for Each Profile

2.2 Modelling of Foreground Lens and Background Galaxies

1. Calculate tangential shear and deflection angle for NFW and cored isothermal profiles.
2. Consider single foreground lens halo with many background galaxies.
    - Start with one NFW halo, then maybe consider more tests.
3. Construct background galaxies:
    - Number density on sky: 50 galaxies/square arcminute
    - Intrinsic ellipticity drawn from Gaussian distribution with $\mu = 0, \sigma = 0.2$
    - Assume they are all that the same distance D_S since this can be determined by redshift (neglecting some noise)
4. Recommended values:
    - $z_L = 0.3$
    - $z_S = 1.0$
    - $M_{halo} = 10^{15} M_\odot$
5. Apply shear and deflection angle to background galaxies, get fake data: N sets of ϵ_1, ϵ_2, θ_1, θ_2
6. Bin galaxies in annuli by θ value (use log bins for theta)
7. Calculate mean and standard deviation of ellipticity
8. Attempt to fit both NFW and isothermal profiles, see if the fit is distinguishable

2.3 Questions

- How to properly do sigma contours?
- What theta range should we look at? (5 arcminutes?)
- What redshift is $\rho_{\rm crit}$ evaluated at? - at halo redshift - can we use current time?
- Using $c = 10$?
- Where to go from here?

3 Results

References

M. Bartelmann, L. J. King, and P. Schneider. Weak-lensing halo numbers and dark-matter profiles. *A&A*, 378(2):361–369, nov 2001. ISSN 0004-6361. doi: 10.1051/0004-6361:20011199. URL `http://www.edpsciences.org/10.1051/0004-6361:20011199`.

S Dodelson. *Gravitational Lensing.* Cambridge University Press, jun 2017.