

Distinguishing NFW and Isothermal Density Profiles with Gravitational Lensing

April 3, 2018

Abstract

1 Introduction

1.1 General spherical density profile $\rho(r)$

Using thin lens approximation

Projected surface density at radius R

$$\Sigma(R) = \int_{-\infty}^{\infty} \rho(\sqrt{R^2 + z^2}) dz \quad (1)$$

Average projected surface density within radius R

$$\bar{\Sigma}(R) = \frac{1}{\pi R^2} \int_0^{2\pi} d\phi \int_0^R dR' \Sigma(R') R' \quad (2)$$

Critical surface density

$$\Sigma_{\text{crit}} = \frac{c^2}{4\pi G} \frac{D_S}{D_{SL} D_L} \quad (3)$$

Convergence at radius R

$$\kappa(R) = \frac{\Sigma(R)}{\Sigma_{\text{crit}}} \quad (4)$$

Tangential shear at radius R

$$\gamma(R) = \bar{\kappa}(R) - \kappa(R) \quad (5)$$

Deflection angle

$$\vec{\alpha}(\vec{\theta}) = \bar{\kappa}(\theta) \vec{\theta} \quad (6)$$

Ellipticity

$$\epsilon(R) = \frac{2\gamma/(1 - \kappa)}{1 + (\gamma/(1 - \kappa))^2} \quad (7)$$

Prove that spherical density profiles only have tangential shear and ellipticity

1.2 Navarro-Frenk-White (NFW) Profile

$$\rho_{\text{NFW}}(r) = \frac{\rho_{\text{crit}} \delta_c}{(r/r_s)(1+r/r_s)^2} \quad (8)$$

$$\Sigma_{\text{NFW}}(R) = \frac{2\rho_{\text{crit}} \delta_c r_s}{(R/r_s)^2 - 1} \left(1 - \frac{2}{\sqrt{(R/r_s)^2 - 1}} \right) \arctan \left(\sqrt{\frac{R/r_s - 1}{R/r_s + 1}} \right) \quad (9)$$

[Bartelmann 2001]

$$\bar{\Sigma}_{\text{NFW}}(R) = \frac{4\rho_{\text{crit}} \delta_c r_s}{(R/r_s)^2} \left(\frac{2}{\sqrt{(R/r_s)^2 - 1}} \arctan \left(\sqrt{\frac{R/r_s - 1}{R/r_s + 1}} \right) + \ln \left(\frac{R/r_s}{2} \right) \right) \quad (10)$$

$$\gamma(R) = \quad (11)$$

We switch the dependence to M_{200} and c with:

$$\delta_c = \frac{200}{3} \frac{c^3}{\ln(1+c) - c/(1+c)} \quad (12)$$

$$r_s = \frac{1}{c} \left(\frac{3M_{200}}{800\pi\rho_{\text{crit}}} \right)^{\frac{1}{3}} \quad (13)$$

1.3 Cored Isothermal Sphere Profile

$$\rho_{\text{iso}}(r) = \frac{\sigma^2}{2\pi G(r^2 + r_c^2)} \quad (14)$$

$$\Sigma_{\text{iso}}(R) = \frac{\sigma^2}{2G\sqrt{R^2 + r_c^2}} \quad (15)$$

$$\bar{\Sigma}_{\text{iso}}(R) = \frac{\sigma^2 (\sqrt{R^2 + r_c^2} - r_c)}{GR^2} \quad (16)$$

$$\gamma(R) = \frac{\sigma^2 (\sqrt{R^2 + r_c^2} - r_c)}{\Sigma_{\text{crit}} GR^2} - \frac{\sigma^2}{2\Sigma_{\text{crit}} G\sqrt{R^2 + r_c^2}} \quad (17)$$

1.4 Current plan

- Calculate tangential shear and deflection angle for NFW and SIS
- Consider single foreground lens halo with many background galaxies
- How to arrange and distribute background halos?
 - Initial ellipticity
 - angular density on sky
 - sizes
- Apply shear and deflection angle to background galaxies

- Attempt to fit both profiles, subtracting intrinsic shear, see if the fit is distinguishable
- How many foreground halos to test?
- What redshift is ρ_{crit} evaluated at?

2 Methods