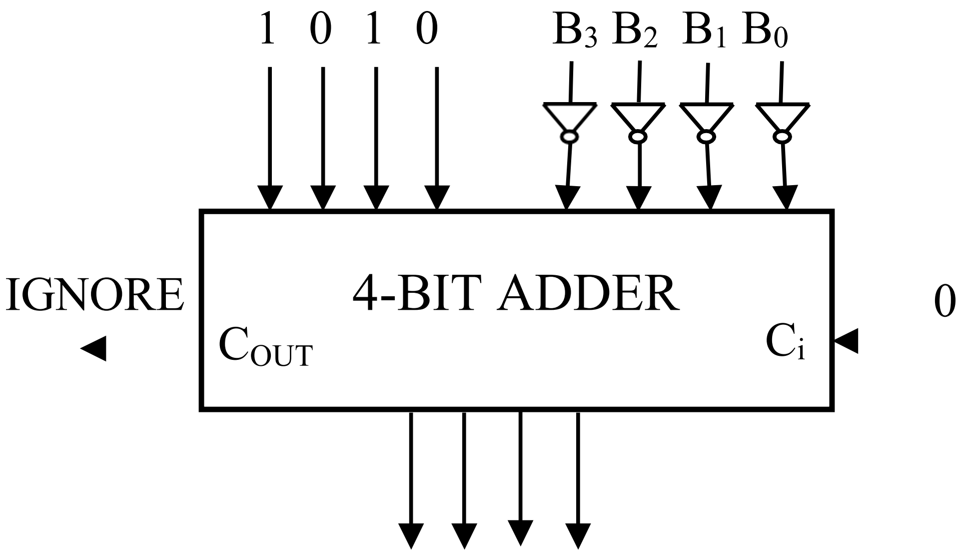
EXERCISE

DESIGN A **9's COMPLEMENT GENERATOR** FOR A BCD DIGIT:



<u>C_3 C_2C_1C</u>

<u>Read the following notes before starting the exercise!!!</u>

<u>BINARY CODED DECIMAL: B.C.D.</u>

- ANOTHER METHOD TO REPRESENT DECIMAL NUMBERS
- USEFUL BECAUSE MANY DIGITAL DEVICES PROCESS + DISPLAY NUMBERS IN TENS

IN **BCD EACH** NUMBER IS DEFINED BY A BINARY CODE OF **<u>4 BITS</u>**.

*** **8 – 4 – 2 – 1** MOST COMMON CODE

8 – 4 – 2 – 1 CODE INDICATES THE **WEIGHT** OF EACH BIT $\mathbf{2^3 - 2^2 - 2^1 - 2^0}$

E.G. 934 = 1001 0011 0100
9 **3** **4**

FOR **EACH** DIGIT A **BINARY** [NORMAL] **CODE** IS ALLOCATED.

OHER REPRESENTATION FORMS ARE **2-4-2-1** AND **EXCESS-3**

BINARY	8-4-2-1	2-4-2-1	EXCESS-3
0000	**0**	**0**	**NOT USED**
0001	**1**	**1**	**NOT USED**
0010	**2**	**2**	**NOT USED**
0011	**3**	**3**	**0**
0100	**4**	**4**	**1**
0101	**5**	**NOT USED**	**2**
0110	**6**	**NOT USED**	**3**
0111	**7**	**NOT USED**	**4**
1000	**8**	**NOT USED**	**5**
1001	**9**	**NOT USED**	**6**
1010	**NOT USED**	**NOT USED**	**7**
1011	**NOT USED**	**5**	**8**
1100	**NOT USED**	**6**	**9**
1101	**NOT USED**	**7**	**NOT USED**
1110	**NOT USED**	**8**	**NOT USED**
1111	**NOT USED**	**9**	**NOT USED**

- WE WILL USE 8-4-2-1 BCD
- DECIMAL NUMBERS > **9** MAY BE OBTAINED WHEN ADDING **TWO** DECIMAL DIGITS (**RANGE**: 0-18) **I.E.** 0 + 0 ÷ 9 + 9. **ONLY 0→9** HAVE THE CORRECT BCD CODE.
- WE NEED TO **CORRECT** THE OTHERS

DECIMAL	**UNCORECTED** BCD SUM $C'\ S'_3\ S'_2\ S'_1\ S'_0$	**CORRECTED** BCD SUM $C_N\ S_3\ S_2\ S_1\ S_0$
0	0 0 0 0	0 0 0 0
:	:	⋮
9	1 0 0 1	1 0 0 1
10	1 0 1 0	**1 0 0 0 0**
11	1 0 1 1	**1 0 0 0 1**
12	1 1 0 0	**1 0 0 1 0**
13	1 1 0 1	**1 0 0 1 1**
14	1 1 1 0	**1 0 1 0 0**
15	1 1 1 1	**1 0 1 0 1**
16	1 0 0 0 0	**1 0 1 1 0**
17	1 0 0 0 1	**1 0 1 1 1**
18	1 0 0 1 0	**1 1 0 0 0**
19	1 0 0 1 1	**1 1 0 0 1**

- **0→9** ONLY LEGAL CODES

  E.G. 19 = **1 9** = **0001 1001** = **11001**

THUS, FOR SUMS BETWEEN **10 → 18** MUST **SUBTRACT 10** AND **PRODUCE A CARRY**

SUBTRACT 10 = 1010_2 >> **ADD 2's COMPLEMENT = 0110**

4-BIT BCD ADDER

TO ADD TWO DIGITS

FOR SUMS **>9** WE NEED TO ADD 2's COMPLEMENT of 10_{10} TO THE UNCORRECTED RESULT (**S'_3 S'_2 S'_1 S'_0**)

CORRECTION IS **ALSO** NEEDED WHEN A **CARRY OUT (C'_3)** IS GENERATED [NUMBERS **16 → 18**]

>>>> **A DECODER IS REQUIRED TO DETECT WHEN CARRY OUT (C_N) TO THE NEXT STAGE IS NEEDED**

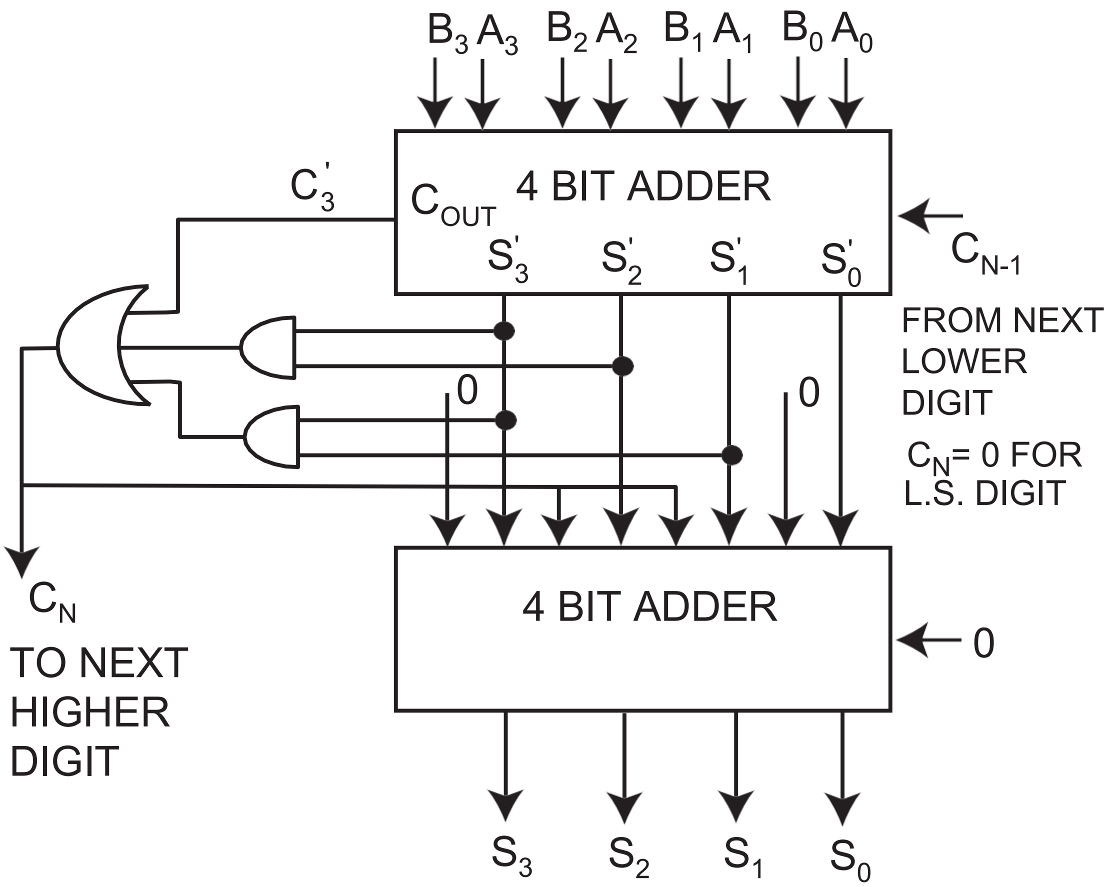
K-MAP FOR **C_N**

$S'_3S'_2$ \ $S'_1S'_0$	0 0	0 1	1 1	1 0
0 0	0	1	3	2
0 1	4	5	7	6
1 1	**12**	**13**	**15**	**14**
1 0	8	9	**11**	**10**

>>> **C_N** $= C'_3 + S'_3S'_2 + S'_3S'_1$

TO IMPLEMENT A 4_BIT BCD ADDER WE NEDD **TWO 4-BIT FULL ADDERS**, **ONE TO ADD TWO 4-BIT BCD NUMBERS** AND THE **OTHER FULL ADDER TO ADD 2's COMPLEMENT OF** $\mathbf{10_{10}}$ TO THE RESULT **IF** $\mathbf{C_N = 1}$

ALSO WE NEED **2 AND GATES** AND **ONE OR** GATE TO GENERATE $\mathbf{C_N}$



- ADD **0110** WHEN $\mathbf{C_N=1}$
- ADD **0000** WHEN $\mathbf{C_N=0}$

BCD SUBTRACTION

9’s COMPLEMENT

THE **9’s COMPLEMENT** OF A **DECIMAL NUMBER** IS FOUND BY **SUBTRACTING EACH DIGIT** IN THE NUMBER FROM **9**

DECIMAL DIGIT	9’s COMPLEMENT
0	9
1	8
2	7
⋮	⋮
9	0

E.G. 9’s COMPLEMENT of 28 = **99** –28
= 71

9’s COMPLEMENT of 562 = **999** –562
= 437

SUBTRACTION OF A **SMALLER DECIMAL** NUMBER FROM A **LARGER** ONE CAN BE DONE BY **ADDING** THE **9’s COMPLEMENT** OF THE **SMALLER** NUMBER TO THE **LARGER** NUMBER AND THEN **ADDING** THE **CARRY** TO THE RESULT (**END AROUND CARRY**).

WHEN **SUBTRACTING** A **LARGER** NUMBER FROM A **SMALLER** ONE THERE IS **NO CARRY** AND THE **RESULT** IS IN **9's COMPLEMENT FORM** AND **NEGTIVE**.

<u>EXAMPLES:</u>

(a)

$$\begin{array}{r} +8 \\ \underline{-3} \\ 5 \end{array} \qquad\qquad \begin{array}{rl} +8 & \\ \underline{+6} & \blacktriangleleft \quad \textbf{9's COMP}\text{. OF 3} \\ (1)\ \ 4 & \\ +\mathbf{1} & \quad \text{END AROUND CARRY} \\ \hline 5 & \end{array}$$

(b)

$$\begin{array}{r} 54 \\ -\underline{21} \\ 33 \end{array} \qquad\qquad \begin{array}{rl} 54 & \\ \underline{78} & \blacktriangleleft \quad \textbf{9's COMP}\text{. OF 3} \\ (1)\ \ 32 & \\ \underline{+\mathbf{1}} & \quad \text{END AROUND CARRY} \\ 33 & \end{array}$$

(c)

$$\begin{array}{r} 15 \\ -\underline{28} \\ \hline -13 \end{array} \qquad\qquad \begin{array}{rl} 15 & \\ +\underline{71} & \blacktriangleleft \quad \textbf{9's COMP}\text{. OF 3} \\ 86 & \rightarrow \mathbf{-13} \end{array}$$

NO CARRY >>> NEGATIVE RESULT

86 – **99** = – **13**

BCD SUBTRACTION

RECALL FOR DECIMAL SUBTRACTION:

A – B = **A** + [**9's COMPLEMENT** OF **B**]

- **SIMILARLY FOR BCD**

<u>RULES:</u>

(a) **ADD 9's COMP**. OF **B** TO **A**

(b) **IF RESULT > 9**, **CORRECT** BY ADDING **0110**

(c) **IF MOST SIGNIFICANT** CARRY **IS PRODUCED** [i.e. =1] THEN THE **RESULT IS POSITIVE** AND THE **END ARROUND CARRY** MUST BE **ADDED**.

(d) **IF MOST SIGNIFICANT** CARRY IS **0** [i.e. NO CARRY] THEN THE RESULT IS **NEGATIVE** AND WE GET THE **9's COMP**. OF THE **RESULT**

E.G. 8 – 3 = 8 + [9's COMP. OF 3]

= 8 + 6

1000

0110

1110 → **INVALID** (>9)

0110 → **CORRECTION**

(**1**) 0100

1 → **END AROUND CARRY**

0101 = 5

(b) 3 – 8 = **-**5

0011

0001

0100

NO CARRY >>> **NEGATIVE**

9's COMP. OF 0100 = **0101** = **-5**

(c) 87 –39 >>> 87 + [9's COMP OF 39]

8 7	1000	0111
6 0	0110	0000
	1110	0111
INVALID	0110	
	(**1**) 0100	0111
		1
	0100	1000
=	**4**	**8**

(d) 18 –72 >>> 18 + [27]

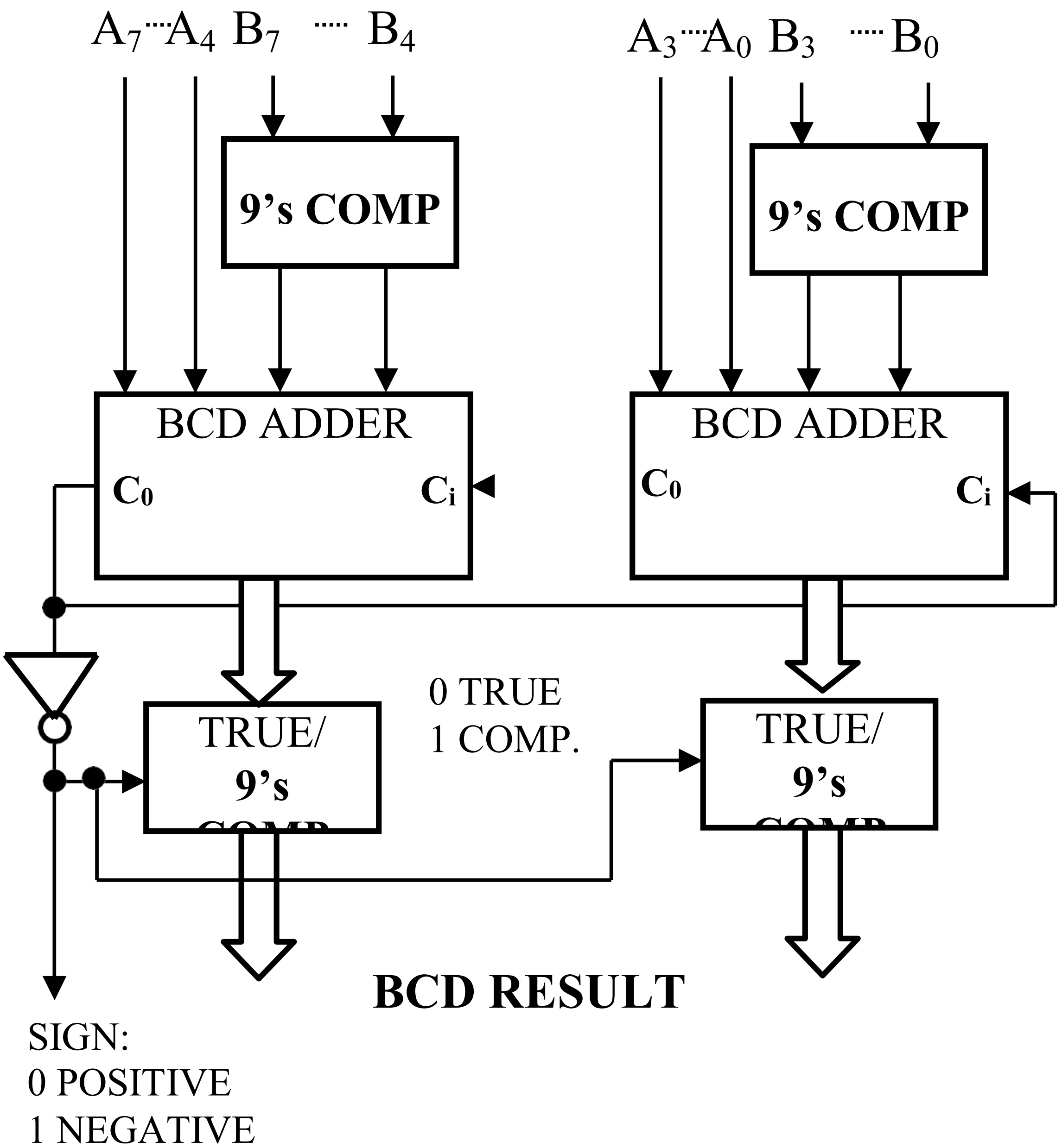
```
0001        1000
0010        0111
----------------
0011        1111  → NO CARRY NEGATIVE
0001        0110
----        ----
0100    (1) 0101      CORRECTION
 4           5   =  -54
```

OUTPUT IS A **NEGATIVE** NUMBER >>
THE RESULT IS IN **9's COMP.** FORM

(e) 65 – 12 >>> 65 + [87]

```
    0110        0101
    1000        0111
    ----------------
    1110        1100      INVALID
    0110        0110      CORRECTION
    ----        ----
(1) 0100    (1) 0010
       1           1
    ----        ----      END AROUND
    0101        0011      CARRY

     5           3
```



A7…A4 B7 … B4
A3…A0 B3 … B0
9's COMP
9's COMP
BCD ADDER
C0
Ci
BCD ADDER
C0
Ci
0 TRUE
1 COMP.
TRUE/
9's
TRUE/
9's
BCD RESULT
SIGN:
0 POSITIVE
1 NEGATIVE

<u>9’s COMPLEMENT</u>

9’s COMPLEMENT OF A NUMBER
= 9 – NUMBER

BUT SUBTRACTORS ARE NOT WIDELY AVAILABLE >>> **WE GENERATE** THE **9’s COMPLEMENT** BY **ADDING 1010** TO THE **<u>INVERTED</u>** **NUMBER**

BCD DIGIT	$\overline{\textbf{DIGIT}}$	$\overline{\textbf{DIGIT}}$ **+ 1010 = 9’s COMP** $C_3 C_2 C_1 C_0$
0 0 0 0	1 1 1 1	1 0 0 1
0 0 0 1	1 1 1 0	1 0 0 0
0 0 1 0	1 1 0 1	0 1 1 1
0 0 1 1	1 1 0 0	0 1 1 0
0 1 0 0	1 0 1 1	0 1 0 1
0 1 0 1	1 0 1 0	0 1 0 0
0 1 1 0	1 0 0 1	0 0 1 1
0 1 1 1	1 0 0 0	0 0 1 0
1 0 0 0	0 1 1 1	0 0 0 1
1 0 0 1	0 1 1 0	0 0 0 0

WE <u>IGNORE</u> THE CARRY OUT