# (Original) Math Using Circular Plate Simplification

Dimensions - (6.22 x 4.74 x .33)in

E (Young's Modulus) = 200 GPa →2e11Pa

V (Poisson Ratio) = .27

h (Thickness) = .33 in = .0084 m

D for this plate = 10655 Pa·$m^4$

Flexural Rigidity $D = \frac{Eh_e^3}{12(1-\nu^2)}$

**I will treat the affected area of plate as a secured circular plate for easier calculations**

a (radius of plate * I will use half of the shorter side) = 2.37in = 0.0602 m

r (radius) = infinitely small

$$w = \frac{F}{16\pi D}\left[\left(a^2 - r^2\right) + 2r^2 \ln\left(r/a\right)\right]$$

$w_y$ → (F = 100) = 6.76e-3mm

(F = 150) = 10.15e-3mm

Change = 3.38 e-3mm

# Boundary Conditions for Math Used in Paper

**For simplicity assume the edges will be bounded and have rotational restraints**

Displacement in x -direction wx = 0 along all edges.

Displacement in y -direction wy = 0 along all edges

**Goal is to find wy at center of plate**

a = width of plate b = height of plate

x= a/2 y= b/2

**Point force P is applied at p**

P = q(x, y)

Q represents shear forces and M represents moments.

$$Q_{x^*} = -D\frac{\partial\left(\nabla^2 w^*\right)}{\partial x^*},$$

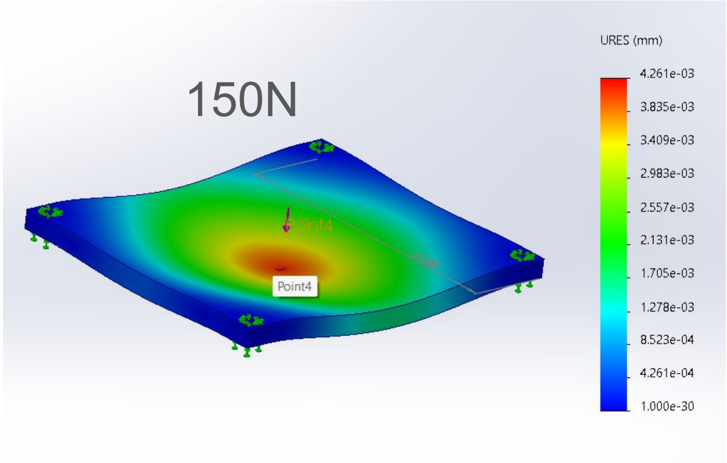
$$Q_{y^*} = -D\frac{\partial\left(\nabla^2 w^*\right)}{\partial y^*},$$

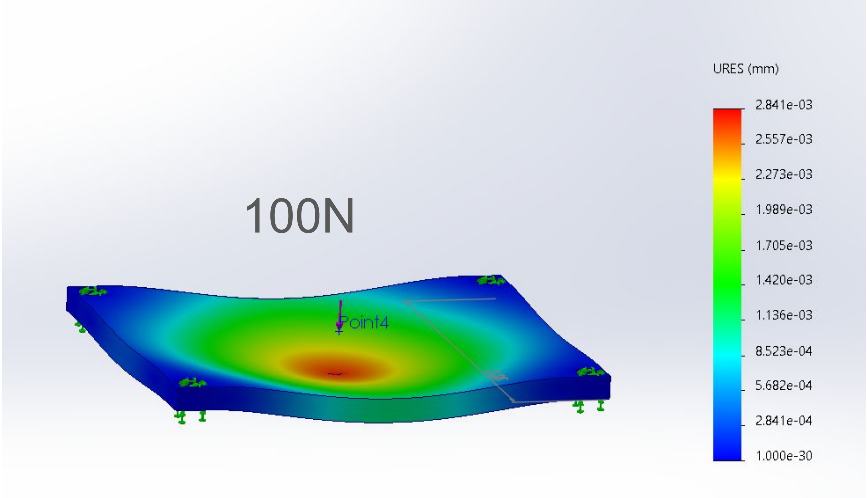
$$M_{x^*} = -D\left(\frac{\partial^2 w^*}{\partial x^{*2}} + \mu\frac{\partial^2 w^*}{\partial y^{*2}}\right)$$

$$M_{y^*} = -D\left(\frac{\partial^2 w^*}{\partial y^{*2}} + \mu\frac{\partial^2 w^*}{\partial x^{*2}}\right)$$

# FEA (Solidworks Simulation)

- Boundary Conditions: fixed position in corner holes to model securing poles
- Alloy steel preset used for material (unsure of actual steel type)
- Point force used in geometric center
- Greatest deformation will oscillate from 4.261e-3 mm to 2.841e-3 mm. With a difference of 1.421e-3 mm.



# Discrepancy from Calculations to Simulation Data

1. Percent Errors:

At 100 N, the percent error is 137.94%.

At 150 N, the percent error is 138.21%.

2. Deformation Percent Error (100 to 150 N):

The percent error in the deformation for a transition from 100 N to 150 N is 138.03%.

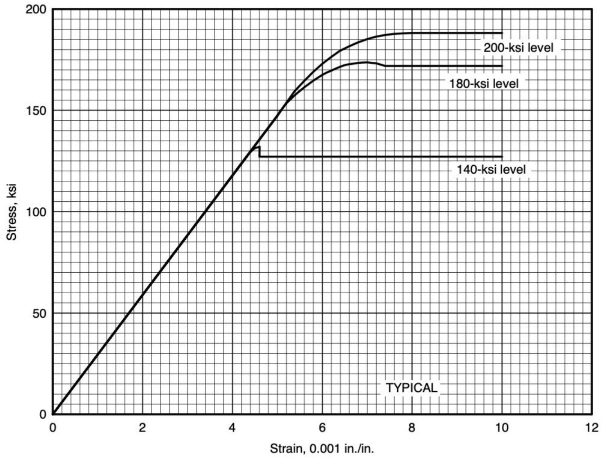
3. Percent Change in Deformation (100 to 150 N):

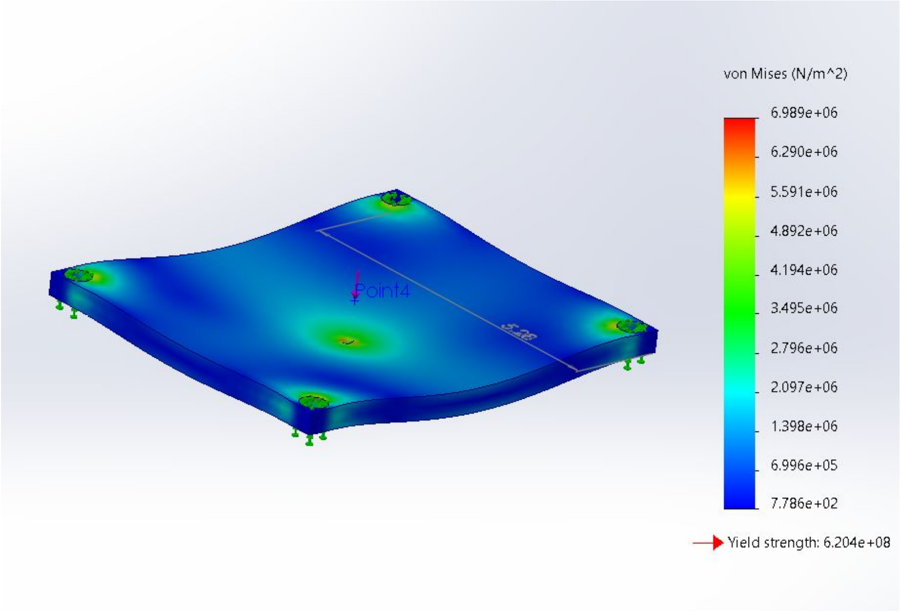
The percent change in deformation from 100 N to 150 N is -0.34% (The percent deformation solidworks simulation and math is similar probably due to linear use of force in calculation and simulation software)

# Stress

- Maximum stress even at a point source is well below the yield strength (provided by solidworks database and chart below)
- Deformation will remain elastic

1 ksi = 68947. $N/m^2$



# Sources of Error in Calculations/Simulations vs. Reality

**1. Calculation Assumptions**

- Edge vs. Corner Constraints: Calculations assume full edge constraints, but in reality, constraints are at four corner holes, creating localized stress differences.
- Point Load Approximation: Calculations use a concentrated point load, while the actual force is distributed over a small area, affecting the deflection profile.
- Linear Material Assumption: Models assume linear elasticity; however, high stress near constraints in reality can introduce non-linear effects, especially under repeated loads.

**2. Simulation Discrepancies in SolidWorks**

- Washer vs. Hole Constraints: Washers in the simulation distribute force differently than internal hole constraints, adding slight flexibility at the constraint points.
- Mesh Resolution and Element Type: Finite element mesh may miss small-scale stress concentrations; mesh resolution and element choice affect the accuracy near constraints.
- Simplified Boundary Conditions: SolidWorks approximates boundary conditions, potentially allowing minor rotational freedom that isn't in the physical setup.

**3. Additional Real-World Factors**

- Material Inhomogeneity: Variations in material properties can affect uniform stiffness and deformation patterns. (The actual kind of steel is unknown different compositions can have very drastic differences)
- Thermal/Environmental Effects: Temperature and humidity can alter dimensions and stresses, unaccounted for in static models.
- Friction at Constraints: Contact friction from washers or bolts can add resistance, impacting deformation differently from idealized models.

# More Info

- While the mathematical approach to solving the plate deformation problem provides a close approximation, SolidWorks Simulation enables finer control over specific parameters and shapes, making it a more accurate method for calculating deformation in the actual process.

# Source

## Equations are from:

https://pkel015.connect.amazon.auckland.ac.nz/SolidMechanicsBooks/Part_II/index.html

## Paper Used

Shi, W., Li, X. F., & Wang, C. (2016). Bending of a rectangular plate with rotationally restrained edges under a concentrated force. *Applied mathematics and computation*, *286*, 265-278.

## Stress Strain Graph

https://ntrl.ntis.gov/NTRL/dashboard/searchResults/titleDetail/PB2003106632.xhtml