

Q1

$$\begin{aligned}1. \quad m(a+bX) &= \frac{1}{N} \sum_{i=1}^N (a + bx_i) \\&= \frac{1}{N} \sum_{i=1}^N a + \frac{1}{N} \sum_{i=1}^N bx_i \\&= \frac{Na}{N} + b \cdot \underbrace{\frac{1}{N} \sum_{i=1}^N x_i}_{=m(x)} \\&= a + b \cdot m(x)\end{aligned}$$

$$\begin{aligned} \text{2. } \text{cov}(X, a+bY) &= \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(y_i - m(a+bY)) \\ &\quad \underbrace{(a+bY_i) - m(a+bY)}_{\text{ }} = a + bm(Y) \\ \text{cov}(X, a+bY) &= \frac{1}{N} \sum_{i=1}^N (x_i - m(X))(a+by_i - a - bm(Y)) \\ &= b \cdot \frac{1}{N} \sum_{i=1}^N (x_i - m(X))(y_i - m(Y)) \\ &\quad \underbrace{(x_i - m(X))(y_i - m(Y))}_{\text{ }} = \text{cov}(X, Y) \\ &= b \cdot \text{cov}(X, Y) \end{aligned}$$

$$\begin{aligned} \text{cov}(X, X) &= s^2 = \frac{1}{N} \sum_{i=1}^N (x_i - m(x))^2 \\ \text{cov}(a+bX, a+bX) &= b \cdot \text{cov}(X, a+bX) \\ &= b \cdot b \cdot \text{cov}(X, X) \\ &= b^2 \text{cov}(X, X) \\ &= b^2 s^2 \end{aligned}$$

4. If the g function is non-decreasing, every value would maintain its relative position to one another, even if it is transformed meaning the median will remain the same. This would also apply to the quantiles as well. The IQR & Range would be affected in that the function could compress or widen the set of values.

5. No, the mean is susceptible to the order in which the operations take place. For example, the moment when a value is squared can change the outcome of the mean. For linear functions it is true, but for non-linear functions, the equality fails.