Allison hypothesized that the tanning studio would give a better tan than the tanning lotion. She wanted to test the null hypothesis,

H_0: The treatments are identical in tanning quality,

against the alternative hypothesis

H_a: Better tanning quality results with the tanning studio.

This alternative hypothesis is one-sided. Another way to state H_a is, "The expected value of the sample mean rank is smaller for the tanning studio than for the tanning lotion." On average, we expect the ranks to be smaller (better) for the tanning studio. Thus, we expect the difference between the sample mean rank for the tanning lotion and the sample mean rank for the tanning studio to be positive.

To find the P-value, we presume that H_0 is true. Then, all samples in Table 15.1 are equally likely, and the sampling distribution is the one shown in Table 15.2. The P-value is the probability of a difference between the sample mean rankings like the observed difference or even more extreme, in terms of giving even more evidence in favor of H_a. The test comparing two groups based on the sampling distribution of the difference between the sample mean ranks is called the **Wilcoxon test**. It is named after the chemist-turned-statistician, Frank Wilcoxon, who devised it in 1945.

SUMMARY: Wilcoxon Nonparametric Test for Comparing Two Groups

1. **Assumptions:** Independent random samples from two groups, either from random sampling or a randomized experiment.
2. **Hypotheses:**
   H_0: Identical population distributions for the two groups (this implies equal expected values for the sample mean ranks)
   H_a: Different expected values for the sample mean ranks (two-sided), or
   H_a: Higher expected value for the sample mean rank for a specified group (one-sided)
3. **Test statistic:** Difference between sample mean ranks for the two groups (equivalently, can use sum of ranks for one sample, as discussed after Example 2).
4. **P-value:** One-tail or two-tail probability, depending on H_a, that the difference between the sample mean ranks is as extreme or more extreme than observed.
5. **Conclusion:** Report the P-value and interpret in context. If a decision is needed, reject H_0 if the P-value $\leq$ significance level, such as 0.05.

P-value for Wilcoxon test

Example 2

Tanning Studio Versus Tanning Lotion

Picture the Scenario

Example 1 describes Allison's experiment to determine whether a tanning lotion or a tanning studio produced a better tan. Table 15.1 showed the possible rankings for five tans. Table 15.2 showed the sampling distribution of the difference between the sample mean ranks, presuming the null hypothesis is true that the tanning treatments have identical effects. For Allison's actual experiment, the ranks were (2, 4, 5) for the three using the tanning lotion and (1, 3) for the two using the tanning studio.

Questions to Explore

a. Find and interpret the P-value for comparing the treatments, using the one-sided alternative hypothesis that the tanning studio gives a better tan than the tanning lotion. That is, H_a states that the expected mean