the ranks for the entire sample and then finds the sample mean rank for each group. The test statistic is based on the between-groups variability in the sample mean ranks. To calculate this test statistic, denote the sample mean rank by $\bar{R}_i$ for group i and by $\bar{R}$ for the combined sample of g groups. The Kruskal-Wallis test statistic is

$$\left(\frac{12}{n(n+1)}\right)\Sigma n_i(\bar{R}_i - \bar{R})^2.$$

The constant $12/(n(n+1))$ is there so that the test statistic values have approximately a chi-squared sampling distribution. Software easily calculates it for us.

The sampling distribution of the test statistic indicates whether the variability among the sample mean ranks is large compared to what's expected under the null hypothesis that the groups have identical population distributions. With g groups, the test statistic has an approximate chi-squared distribution with $g - 1$ degrees of freedom. (Recall that for the chi-squared distribution, the df is the mean.) The approximation improves as the sample sizes increase. The larger the differences among the sample mean ranks, the larger the test statistic and the stronger the evidence against H_0. The P-value is the right-tail probability above the observed test statistic value.

When would we use this test? The ANOVA F test assumes normal population distributions. The Kruskal-Wallis test does not have this assumption. It's a safer method to use with small samples when not much information is available about the shape of the distributions. It's also useful when the data are merely ranks and we don't have a quantitative measurement of the response variable. Here's a basic summary of the test:

SUMMARY: Kruskal-Wallis Nonparametric Test for Comparing Several Groups

1. **Assumptions:** Independent random samples from several (g) groups, either from random sampling or a randomized experiment

2. **Hypotheses:**
   H_0: Identical population distributions for the g groups
   H_a: Population distributions not all identical

3. **Test statistic:** Uses between-groups variability of sample mean ranks

4. **P-value:** Right-tail probability above observed test statistic value from chi-squared distribution with $df = g - 1$

5. **Conclusion:** Report the P-value and interpret in context.

Example 5

Kruskal-Wallis test

Frequent Dating and College GPA

Picture the Scenario

Tim decided to study whether dating was associated with college GPA. He wondered whether students who date a lot tend to have poorer GPAs. He asked the 17 students in the class to anonymously fill out a short questionnaire in which they were asked to give their college GPA (0 to 4 scale) and to indicate whether during their college careers they had dated regularly, occasionally, or rarely.

Figure 15.4 shows the dot plots that Tim constructed of the GPA data for the three dating groups. Since the dot plots showed evidence of severe