

Ranking Matched Pairs: The Wilcoxon Signed-Ranks Test

With matched pairs data, for each pair the sign test merely observes which treatment does better, but not *how much* better. The **Wilcoxon signed-ranks test** is a nonparametric test designed for cases in which the comparisons of the paired observations can themselves be ranked. For each matched pair of responses, it measures the difference between the responses. It tests the hypothesis,

$$H_0: \text{population median of difference scores is 0.}$$

The test uses the quantitative information provided by the n difference scores by ranking their magnitudes, in absolute value. (Like the sign test, it ignores observation pairs for which the difference equals 0.) The test statistic is the sum of the ranks for the differences that are positive.

SUMMARY: Wilcoxon Signed-Ranks Test for Matched Pairs

- 1. Assumptions:** Random sample of matched pairs for which the differences of observations have a symmetric population distribution and can be ranked.
- 2. Hypotheses:**
 - H_0 : Population median of difference scores is 0.
 - H_a : Population median of difference scores is not 0 (one-sided also possible).
- 3. Test statistic:** Rank the absolute values of the difference scores for the matched pairs and then find the sum of ranks of the differences that were positive.
- 4. P-value:** Software can find a P-value based on all the possible samples with the given absolute differences. (For large samples, it uses an approximate normal sampling distribution, as discussed following.)
- 5. Conclusion:** Report the P-value and interpret in context.

Wilcoxon signed-rank test

Example 8

GRE Test Scores

Picture the Scenario

If you want to attend graduate school, taking the Graduate Record Examination (GRE) is usually a requirement. Many graduate schools consider GRE scores for admittance, to qualify for financial aid, to determine fellowships and grants, and for other program research or teaching assignments. The GRE includes three sections designed to test verbal, quantitative, and writing skills.

The verbal and quantitative sections are each scored between 200 and 800. The analytical writing portion of the GRE is given a score between 0 and 6 in half point increments.

In our example, three students volunteered for a study to determine if taking a two-day workshop on GRE preparation improved their GRE analytical writing score from a previous score. Note: The original data was larger ($n = 12$) but a small sample is used in this example to make it easier to explain. The results are shown in the following table:

	Subject		
	1	2	3
Before	2.5	4	1.5
After	3	3.5	3