Recall

How to handle ties is discussed on page 727. ◀

A negative difference (after − before) represents a lower score. A positive difference represents an improved score. Let's test the null hypothesis that the two-day GRE workshop has no effect, in the sense that the population median gained score is 0, against the alternative hypothesis that the population median gained score is positive.

Observations that have a tie score use a decimal point ranking and are ranked the same. The next ranking includes all of the previous rankings.

Questions to Explore

a. For ranks applied to the absolute values of the differences, find the rank sum for the differences that were positive.

b. Consider each of the possible ways that positive and negative signs could be assigned to these three differences. For each case, find the rank sum for the positive differences. Create the sampling distribution of this rank sum that applies if the workshop truly has no effect.

c. Find the P-value for the Wilcoxon signed-ranks test, using the sampling distribution of the rank sum created in part b.

Think It Through

a. The Wilcoxon test begins by calculating the difference and then the absolute value, of each instance. In most applications of the Wilcoxon procedure, the cases in which there is zero difference are eliminated from consideration because they provide no useful information; the remaining absolute differences are then ranked from lowest to highest, with tied ranks included where appropriate.

After sorting the GRE data, the differences have absolute values and ranks as follows:

Subject	Before	After	Difference	Absolute Value	Rank of Absolute Value	Signed Rank
1	2.5	3	0.5	0.5	1.5	1.5
2	4	3.5	−0.5	0.5	1.5	−1.5
3	1.5	3	1.5	1.5	3	3

b. For the difference values of the three subjects, 0.5, − 0.5, and 1.5, Table 15.9 shows all the possible ways the differences could have been positive or negative. For each sample, this table also shows the sum of ranks for the positive differences. The observed data are Sample 1, which had a rank sum of 4.5.

Table 15.9 Possible Samples with Absolute Difference Values of Sample

Subject	1	2	3	4	5	6	7	8	Rank of Absolute Value
1	0.5	0.5	−0.5	0.5	−0.5	0.5	−0.5	−0.5	1.5
2	−0.5	−0.5	−0.5	0.5	−0.5	0.5	0.5	0.5	1.5
3	1.5	−1.5	1.5	1.5	−1.5	−1.5	1.5	−1.5	3
Sum of Ranks for Positive Differences									
	4.5	1.5	3	6	0	3	4.5	1.5	

If the workshop has no effect, then the eight possible samples in Table 15.9 are equally likely. The table that follows summarizes the sampling distribution of the rank sum for the positive differences, presuming no effect of the workshop. For example, the rank sum was 4.5 for two of the eight samples, so its probability is 2/8.