Sampling Distribution of Rank Sum for the Positive Differences

Rank Sum	Probability
0	1/8
1.5	2/8
3	2/8
4.5	2/8
6	1/8

c. The larger the sum of ranks for the positive differences, the greater the evidence that the workshop has a positive effect. So, the P-value is the probability that this sum of ranks is at least as large as observed. Since three of the eight possible samples had a rank sum for the positive differences of at least 4.5 (the observed value), the P-value is $3/8 = 0.375$.

Insight

Suppose we instead used the sign test. We then observe that two of the three differences are positive. For the alternative hypothesis that the workshop has a positive effect, the P-value is the probability that at least two of the three differences are positive, when the chance is 0.50 that any particular difference is positive. Using the binomial distribution, you can find that the P-value is 0.50 (Exercise 15.12).

The sign test ignores the fact that the two positive differences are larger than the negative difference. The Wilcoxon signed-ranks test uses this information. By taking this extra information into account, its P-value of 0.375 is smaller than the P-value of 0.50 from the sign test. However, the P-value is still not small. With only three observations, the one-sided P-value can be no smaller than one-eighth, which is the P-value for the largest possible value (which is 6) for the rank sum of positive differences.

The MINITAB output for the complete data set is shown below. The test of medians between the two groups results in a statistically significant P-value of 0.018.

Wilcoxon Signed Rank CI: Before, After

	N	Estimated Median	Achieved Confidence	Confidence Interval Lower	Confidence Interval Upper
Before	12	3.00	94.5	2.25	3.75
After	12	4.00	94.5	3.00	4.75

Wilcoxon Signed Rank Test: Diff

Test of median $= 0.000000$ versus median not $= 0.000000$

	N	Test	Statistic	P	Median
Diff	12	11	60.0	0.018	1.000

Try Exercise 15.12

Caution

For independent samples, rank all of the data combined together, and then compare the sum of the ranks of the two groups. For dependent samples, find the difference between the two groups and then rank the differences. These methods often are confused—especially if the independent samples have equal sample sizes. ◀

Although the Wilcoxon signed-ranks test has the advantage compared to the sign test that it can take into account the *sizes* of the differences and not merely their *sign*, it also has a disadvantage. For the possible samples (such as the eight samples shown in Table 15.9) to be equally likely, it must make an additional assumption: The population distribution of the difference scores must be *symmetric*.