rank is larger for the tanning lotion than for the tanning studio, so the difference between the two is positive.

b. What's the smallest possible P-value you could get for this experiment?

Think It Through

a. For the observed sample, the mean ranks are $(2 + 4 + 5)/3 = 3.7$ for the tanning lotion and $(1 + 3)/2 = 2$ for the tanning studio. The test statistic is the difference between the sample mean ranks, $3.7 - 2 = 1.7$. The right tail of the sampling distribution in Figure 15.1 has the large positive differences, for which the ranks tended to be better (lower) with the tanning studio. For the one-sided H_a, the P-value is the probability,

$$\text{P-value} = P(\text{difference between sample mean ranks at least as large as observed}).$$

That is, under the presumption that H_0 is true,

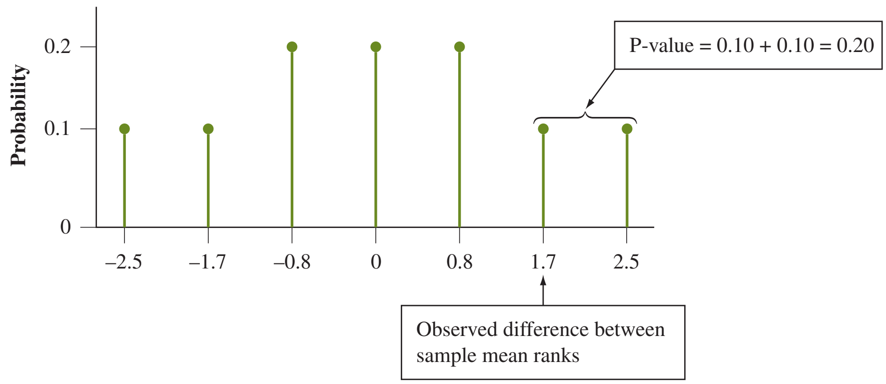
$$\text{P-value} = P(\text{difference between sample mean ranks} \geq 1.7).$$

From Table 15.2 (shown again in the margin) or Figure 15.1 (reproduced below), the probability of a sample mean difference of 1.7 or even larger is $1/10 + 1/10 = 2/10 = 0.20$. This is the P-value. It is not very close to 0. Although there is some evidence that the tanning studio gives a better tan (it *did* have a lower sample mean rank), the evidence is not strong. If the treatments had identical effects, the probability would be 0.20 of getting a sample like we observed or even more extreme.

Recall

Sampling distribution of the difference between mean ranks:

Difference	Probability
−2.5	1/10
−1.7	1/10
−0.8	2/10
0	2/10
0.8	2/10
1.7	1/10
2.5	1/10



b. In this experiment, suppose the tanning studio gave the two most natural-looking tans. The ranks would then be (1, 2) for the tanning studio and (3, 4, 5) for the tanning lotion. The difference of sample means then equals $4 - 1.5 = 2.5$. It is the most extreme possible sample, and (from Table 15.2 or Figure 15.1) its tail probability is 0.10. This is the smallest possible one-sided P-value.

Insight

With sample sizes of only 2 for one treatment and 3 for the other treatment, it's not possible to get a very small P-value. If Allison wanted to make a decision using a 0.05 significance level, she would never be able to get strong enough evidence to reject the null hypothesis. To get informative results, she'd need to conduct an experiment with larger sample sizes.

Try Exercises 15.1 and 15.2