For small n, we can conduct the sign test by using the *binomial* distribution. The next example illustrates this case as well as a situation in which we can order responses for each pair but do not have quantitative information about how different the responses are.

Sign test for matched pairs (small sample)

Example 7

Crossover Experiment Comparing Tanning Methods

Picture the Scenario

When Allison told another student in the class (Megan) about her planned experiment to compare tanning methods, Megan decided to do a separate tanning experiment. She used a crossover design for a different sample of five untanned female friends. The results of her experiment were that the tanning studio gave a better tan than the tanning lotion for four of the five participants.

Question to Explore

Find and interpret the P-value for testing that the population proportion p of participants for whom the tanning studio gives a better tan than the tanning lotion equals 0.50. Use the alternative hypothesis that this population proportion is larger than 0.50, because Megan predicted that the tanning studio would give better tans.

Think It Through

The null hypothesis is H_0: $p = 0.50$. For H_a: $p > 0.50$, the P-value is the probability of the observed sample outcome or an even larger one. The sample size ($n = 5$) was small, so we use the binomial distribution rather than its normal approximation to find the P-value.

If $p = 0.50$, from the margin Recall box the binomial probability that $x = 4$ of the $n = 5$ participants would get better tans with the tanning studio is

$$P(4) = \frac{5!}{4!(5-4)!}(0.50)^4(0.50)^1 = 0.156.$$

The more extreme result that all five participants would get better tans with the tanning studio has probability $P(5) = (0.50)^5 = 0.031$. The P-value is the right-tail probability of the observed result and the more extreme one, that is, $0.156 + 0.031 = 0.187$. See the margin figure. In summary, the evidence is not strong that more participants get a better tan from the tanning studio than the tanning lotion.

Insight

Megan would instead use the two-sided alternative, H_a: $p \neq 0.50$, if she did not make a prior prediction about which tanning method would be better. The P-value would then be $2(0.19) = 0.38$. With only $n = 5$ observations, the smallest possible two-sided P-value would be $2(0.031) = 0.06$, which occurs when $x = 0$ or when $x = 5$.

Try Exercise 15.11

Recall

Section 6.3 presented the **binomial** distribution. With probability p of success on a trial, out of n independent trials the probability of x successes is

$$P(x) = \frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}. \blacktriangleleft$$