Multiclass Support Vector Machine exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission. For more details see the assignments page (https://compsci697l.github.io/assignments.html) on the course website.

In this exercise you will:

- implement a fully-vectorized **loss function** for the SVM
- implement the fully-vectorized expression for its **analytic gradient**
- **check your implementation** using numerical gradient
- use a validation set to **tune the learning rate and regularization** strength
- **optimize** the loss function with **SGD**
- **visualize** the final learned weights

In [1]:

```
# Run some setup code for this notebook.

import random
import numpy as np
from asgn1.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

# This is a bit of magic to make matplotlib figures appear inline in the
# notebook rather than in a new window.
%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# Some more magic so that the notebook will reload external python modules;
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2
```

CIFAR-10 Data Loading and Preprocessing

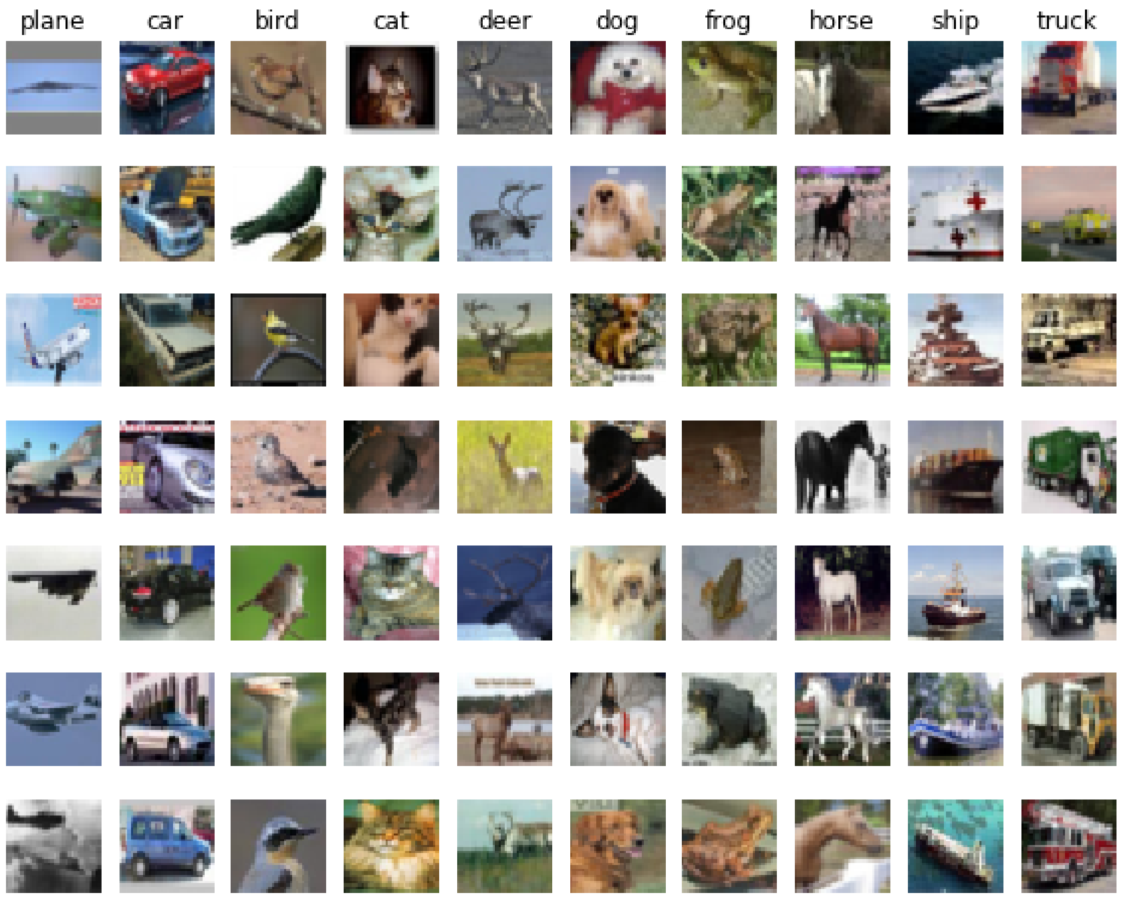
In [2]:

```
# Load the raw CIFAR-10 data.
cifar10_dir = 'datasets/cifar-10-batches-py'
X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

# As a sanity check, we print out the size of the training and test data.
print 'Training data shape: ', X_train.shape
print 'Training labels shape: ', y_train.shape
print 'Test data shape: ', X_test.shape
print 'Test labels shape: ', y_test.shape
```

```
Training data shape:  (50000, 32, 32, 3)
Training labels shape:  (50000,)
Test data shape:  (10000, 32, 32, 3)
Test labels shape:  (10000,)
```

In [3]:

```
# Visualize some examples from the dataset.
# We show a few examples of training images from each class.
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', '
num_classes = len(classes)
samples_per_class = 7
for y, cls in enumerate(classes):
    idxs = np.flatnonzero(y_train == y)
    idxs = np.random.choice(idxs, samples_per_class, replace=False)
    for i, idx in enumerate(idxs):
        plt_idx = i * num_classes + y + 1
        plt.subplot(samples_per_class, num_classes, plt_idx)
        plt.imshow(X_train[idx].astype('uint8'))
        plt.axis('off')
        if i == 0:
            plt.title(cls)
plt.show()
```



In [4]:

```
# Split the data into train, val, and test sets. In addition we will
# create a small development set as a subset of the training data;
# we can use this for development so our code runs faster.
num_training = 49000
num_validation = 1000
num_test = 1000
num_dev = 500

# Our validation set will be num_validation points from the original
# training set.
mask = range(num_training, num_training + num_validation)
X_val = X_train[mask]
y_val = y_train[mask]

# Our training set will be the first num_train points from the original
# training set.
mask = range(num_training)
X_train = X_train[mask]
y_train = y_train[mask]

# We will also make a development set, which is a small subset of
# the training set.
mask = np.random.choice(num_training, num_dev, replace=False)
X_dev = X_train[mask]
y_dev = y_train[mask]

# We use the first num_test points of the original test set as our
# test set.
mask = range(num_test)
X_test = X_test[mask]
y_test = y_test[mask]

print 'Train data shape: ', X_train.shape
print 'Train labels shape: ', y_train.shape
print 'Validation data shape: ', X_val.shape
print 'Validation labels shape: ', y_val.shape
print 'Test data shape: ', X_test.shape
print 'Test labels shape: ', y_test.shape
```

```
Train data shape:  (49000, 32, 32, 3)
Train labels shape:  (49000,)
Validation data shape:  (1000, 32, 32, 3)
Validation labels shape:  (1000,)
Test data shape:  (1000, 32, 32, 3)
Test labels shape:  (1000,)
```

In [5]:

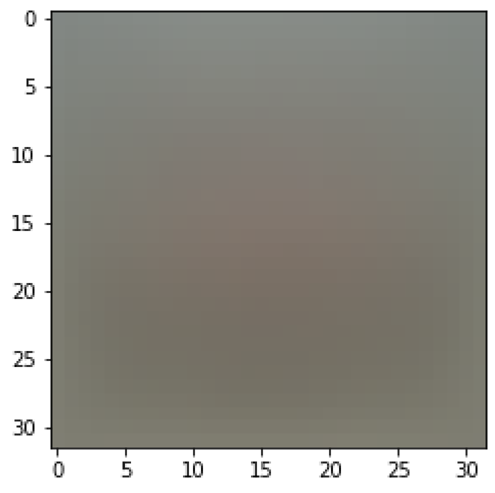
```
# Preprocessing: reshape the image data into rows
X_train = np.reshape(X_train, (X_train.shape[0], -1))
X_val = np.reshape(X_val, (X_val.shape[0], -1))
X_test = np.reshape(X_test, (X_test.shape[0], -1))
X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

# As a sanity check, print out the shapes of the data
print 'Training data shape: ', X_train.shape
print 'Validation data shape: ', X_val.shape
print 'Test data shape: ', X_test.shape
print 'dev data shape: ', X_dev.shape
```

```
Training data shape:  (49000, 3072)
Validation data shape:  (1000, 3072)
Test data shape:  (1000, 3072)
dev data shape:  (500, 3072)
```

In [6]:

```
# Preprocessing: subtract the mean image
# first: compute the image mean based on the training data
mean_image = np.mean(X_train, axis=0)
print mean_image.shape
print mean_image[:10] # print a few of the elements
plt.figure(figsize=(4,4))
plt.imshow(mean_image.reshape((32,32,3)).astype('uint8')) # visualize the mean imag
plt.show()
```

```
(3072,)
[ 130.64189796  135.98173469  132.47391837  130.05569388  135.34804082
  131.75402041  130.96055102  136.14328571  132.47636735  131.4846734
7]
```



In [7]:

```
# second: subtract the mean image from train and test data
X_train -= mean_image
X_val -= mean_image
X_test -= mean_image
X_dev -= mean_image
```

```
In [8]:
```

```python
# third: append the bias dimension of ones (i.e. bias trick) so that our SVM
# only has to worry about optimizing a single weight matrix W.
X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

print X_train.shape, X_val.shape, X_test.shape, X_dev.shape
```

```
(49000, 3073) (1000, 3073) (1000, 3073) (500, 3073)
```

SVM Classifier

Your code for this section will all be written inside **asgn1/classifiers/linear_svm.py**.

As you can see, we have prefilled the function `compute_loss_naive` which uses for loops to evaluate the multiclass SVM loss function.

```
In [9]:
```

```python
# Evaluate the naive implementation of the loss we provided for you:
from asgn1.classifiers.linear_svm import svm_loss_naive
import time

# generate a random SVM weight matrix of small numbers
W = np.random.randn(3073, 10) * 0.0001

loss, grad = svm_loss_naive(W, X_dev, y_dev, 0.00001)
print 'loss: %f' % (loss, )
```

```
loss: 8.496232
```

The `grad` returned from the function above is right now all zero. Derive and implement the gradient for the SVM cost function and implement it inline inside the function `svm_loss_naive`. You will find it helpful to interleave your new code inside the existing function.

To check that you have correctly implemented the gradient correctly, you can numerically estimate the gradient of the loss function and compare the numeric estimate to the gradient that you computed. We have provided code that does this for you:

In [10]:

```
# Once you've implemented the gradient, recompute it with the code below
# and gradient check it with the function we provided for you

# Compute the loss and its gradient at W.
loss, grad = svm_loss_naive(W, X_dev, y_dev, 0.0)

# Numerically compute the gradient along several randomly chosen dimensions, and
# compare them with your analytically computed gradient. The numbers should match
# almost exactly along all dimensions.
from asgn1.gradient_check import grad_check_sparse
f = lambda w: svm_loss_naive(w, X_dev, y_dev, 0.0)[0]
grad_numerical = grad_check_sparse(f, W, grad)

# do the gradient check once again with regularization turned on
# you didn't forget the regularization gradient did you?
loss, grad = svm_loss_naive(W, X_dev, y_dev, 1e2)
f = lambda w: svm_loss_naive(w, X_dev, y_dev, 1e2)[0]
grad_numerical = grad_check_sparse(f, W, grad)
```

```
numerical: 3.755606 analytic: 3.755606, relative error: 4.816142e-11
numerical: -1.722810 analytic: -1.722810, relative error: 1.887013e-10
numerical: -52.673726 analytic: -52.673726, relative error: 1.316374e-
12
numerical: 7.232368 analytic: 7.232368, relative error: 3.136122e-11
numerical: 21.754696 analytic: 21.754696, relative error: 2.350017e-11
numerical: 19.399478 analytic: 19.399478, relative error: 1.397737e-11
numerical: -15.439231 analytic: -15.439231, relative error: 7.533037e-
12
numerical: 24.554115 analytic: 24.554115, relative error: 2.416743e-12
numerical: -20.004721 analytic: -20.004721, relative error: 3.526302e-
12
numerical: 23.868994 analytic: 23.868994, relative error: 3.201000e-12
numerical: 15.192910 analytic: 15.192910, relative error: 2.153669e-12
numerical: 16.511404 analytic: 16.511404, relative error: 8.486089e-12
numerical: 21.110470 analytic: 21.110470, relative error: 6.379933e-12
numerical: 17.243599 analytic: 17.243599, relative error: 1.326375e-11
numerical: 23.924730 analytic: 23.924730, relative error: 1.275554e-11
numerical: 9.351478 analytic: 9.351478, relative error: 4.844319e-12
numerical: -22.985801 analytic: -22.985801, relative error: 6.755640e-
12
numerical: -31.719649 analytic: -31.719649, relative error: 8.276839e-
12
numerical: -12.236951 analytic: -12.236951, relative error: 7.473010e-
13
numerical: 6.905856 analytic: 6.905856, relative error: 6.517109e-12
```

Inline Question 1:

It is possible that once in a while a dimension in the gradcheck will not match exactly. What could such a discrepancy be caused by? Is it a reason for concern? What is a simple example in one dimension where a gradient check could fail? *Hint: the SVM loss function is not strictly speaking differentiable*

Your Answer: *fill this in.*

In [11]:

```
# Next implement the function svm_loss_vectorized; for now only compute the loss;
# we will implement the gradient in a moment.
tic = time.time()
loss_naive, grad_naive = svm_loss_naive(W, X_dev, y_dev, 0.00001)
toc = time.time()
print 'Naive loss: %e computed in %fs' % (loss_naive, toc - tic)

from asgn1.classifiers.linear_svm import svm_loss_vectorized
tic = time.time()
loss_vectorized, _ = svm_loss_vectorized(W, X_dev, y_dev, 0.00001)
toc = time.time()
print 'Vectorized loss: %e computed in %fs' % (loss_vectorized, toc - tic)

# The losses should match but your vectorized implementation should be much faster.
print 'difference: %f' % (loss_naive - loss_vectorized)
```

```
Naive loss: 8.496232e+00 computed in 0.258584s
Vectorized loss: 8.496232e+00 computed in 0.042408s
difference: 0.000000
```

In [12]:

```
# Complete the implementation of svm_loss_vectorized, and compute the gradient
# of the loss function in a vectorized way.

# The naive implementation and the vectorized implementation should match, but
# the vectorized version should still be much faster.
tic = time.time()
_, grad_naive = svm_loss_naive(W, X_dev, y_dev, 0.00001)
toc = time.time()
print 'Naive loss and gradient: computed in %fs' % (toc - tic)

tic = time.time()
_, grad_vectorized = svm_loss_vectorized(W, X_dev, y_dev, 0.00001)
toc = time.time()
print 'Vectorized loss and gradient: computed in %fs' % (toc - tic)

# The loss is a single number, so it is easy to compare the values computed
# by the two implementations. The gradient on the other hand is a matrix, so
# we use the Frobenius norm to compare them.
difference = np.linalg.norm(grad_naive - grad_vectorized, ord='fro')
print 'difference: %f' % difference
```

```
Naive loss and gradient: computed in 0.249803s
Vectorized loss and gradient: computed in 0.015189s
difference: 0.000000
```

Stochastic Gradient Descent

We now have vectorized and efficient expressions for the loss, the gradient and our gradient matches the numerical gradient. We are therefore ready to do SGD to minimize the loss.

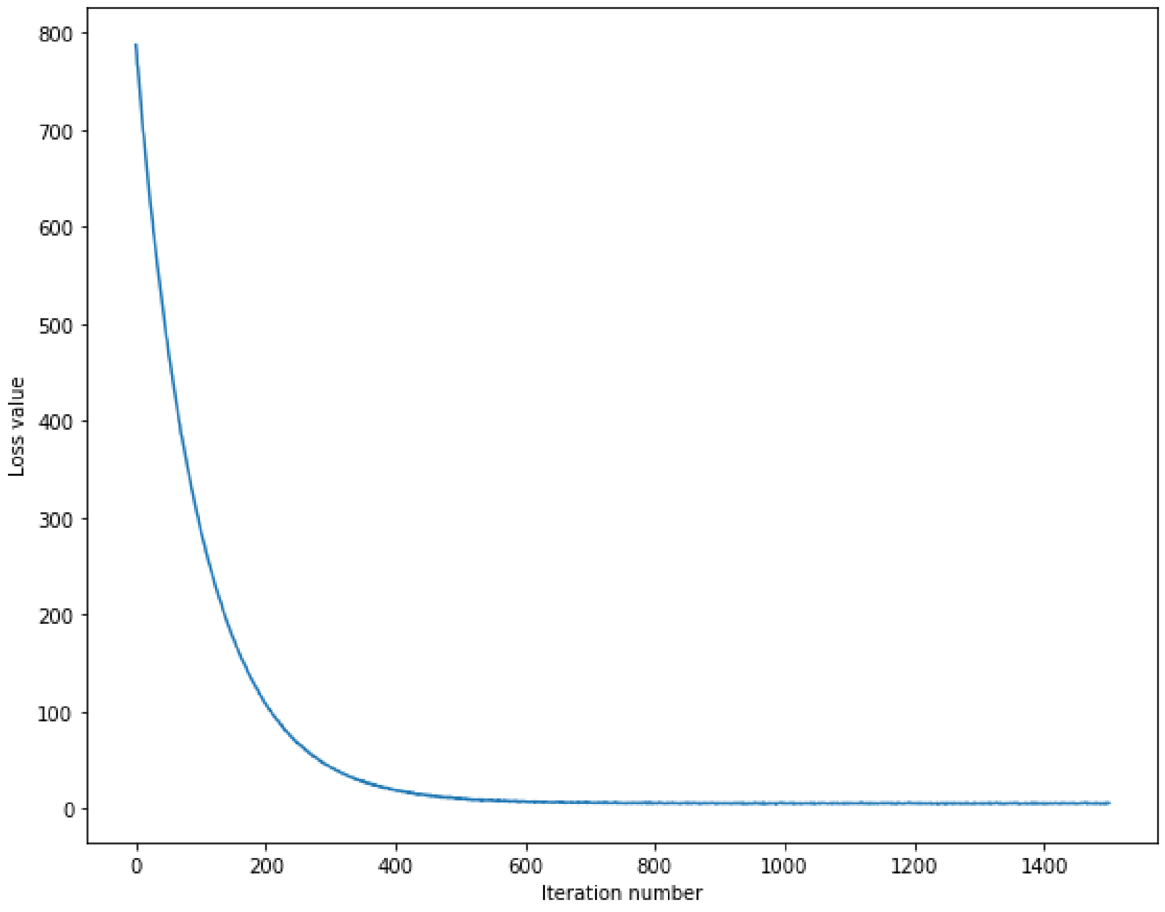
In [13]:

```
# In the file linear_classifier.py, implement SGD in the function
# LinearClassifier.train() and then run it with the code below.
from asgn1.classifiers import LinearSVM
svm = LinearSVM()
tic = time.time()
loss_hist = svm.train(X_train, y_train, learning_rate=1e-7, reg=5e4,
                      num_iters=1500, verbose=True)
toc = time.time()
print 'That took %fs' % (toc - tic)
```

```
iteration 0 / 1500: loss 787.428510
iteration 100 / 1500: loss 286.737068
iteration 200 / 1500: loss 108.357066
iteration 300 / 1500: loss 42.467751
iteration 400 / 1500: loss 19.026980
iteration 500 / 1500: loss 10.833890
iteration 600 / 1500: loss 6.940751
iteration 700 / 1500: loss 5.857895
iteration 800 / 1500: loss 5.851322
iteration 900 / 1500: loss 5.592318
iteration 1000 / 1500: loss 5.274975
iteration 1100 / 1500: loss 5.599379
iteration 1200 / 1500: loss 5.249657
iteration 1300 / 1500: loss 5.494946
iteration 1400 / 1500: loss 5.227184
That took 7.994031s
```

In [14]:

```
# A useful debugging strategy is to plot the loss as a function of
# iteration number:
plt.plot(loss_hist)
plt.xlabel('Iteration number')
plt.ylabel('Loss value')
plt.show()
```



In [15]:

```
# Write the LinearSVM.predict function and evaluate the performance on both the
# training and validation set
y_train_pred = svm.predict(X_train)
print 'training accuracy: %f' % (np.mean(y_train == y_train_pred), )
y_val_pred = svm.predict(X_val)
print 'validation accuracy: %f' % (np.mean(y_val == y_val_pred), )
```

```
training accuracy: 0.365020
validation accuracy: 0.371000
```

In [16]:

```
# Use the validation set to tune hyperparameters (regularization strength and
# learning rate). You should experiment with different ranges for the learning
# rates and regularization strengths; if you are careful you should be able to
# get a classification accuracy of about 0.4 on the validation set.
learning_rates = [1e-7,3e-7, 5e-7, 1e-8 ]
regularization_strengths = [1e4, 3e4, 5e4, 1e5]
svm=LinearSVM()
# results is dictionary mapping tuples of the form
# (learning_rate, regularization_strength) to tuples of the form
# (training_accuracy, validation_accuracy). The accuracy is simply the fraction
# of data points that are correctly classified.
results = {}
best_val = -1   # The highest validation accuracy that we have seen so far.
best_svm = None # The LinearSVM object that achieved the highest validation rate.
for i in learning_rates:
    for j in regularization_strengths:
        loss_hist = svm.train(X_train, y_train, learning_rate=i, reg=j,
                      num_iters=2000, verbose=True)
        y_train_pred = svm.predict(X_train)
        y_val_pred = svm.predict(X_val)
        results[i,j]=[np.mean(y_train == y_train_pred),np.mean(y_val == y_val_pred)
        print np.mean(y_val == y_val_pred)
        if (np.mean(y_val == y_val_pred))>best_val:
            best_val=np.mean(y_val == y_val_pred)
            best_svm=svm



# Print out results.
for lr, reg in sorted(results):
    train_accuracy, val_accuracy = results[(lr, reg)]
    print 'lr %e reg %e train accuracy: %f val accuracy: %f' % (
                lr, reg, train_accuracy, val_accuracy)

print 'best validation accuracy achieved during cross-validation: %f' % best_val
```

```
iteration 0 / 2000: loss 177.054429
iteration 100 / 2000: loss 133.638724
iteration 200 / 2000: loss 108.991917
iteration 300 / 2000: loss 89.600929
iteration 400 / 2000: loss 73.790317
iteration 500 / 2000: loss 61.442929
iteration 600 / 2000: loss 50.885164
iteration 700 / 2000: loss 42.030139
iteration 800 / 2000: loss 34.827579
iteration 900 / 2000: loss 28.541460
iteration 1000 / 2000: loss 25.532737
iteration 1100 / 2000: loss 21.084897
iteration 1200 / 2000: loss 18.058355
iteration 1300 / 2000: loss 15.615121
iteration 1400 / 2000: loss 13.115609
iteration 1500 / 2000: loss 11.904148
iteration 1600 / 2000: loss 10.754154
iteration 1700 / 2000: loss 9.065807
iteration 1800 / 2000: loss 9.132477
```

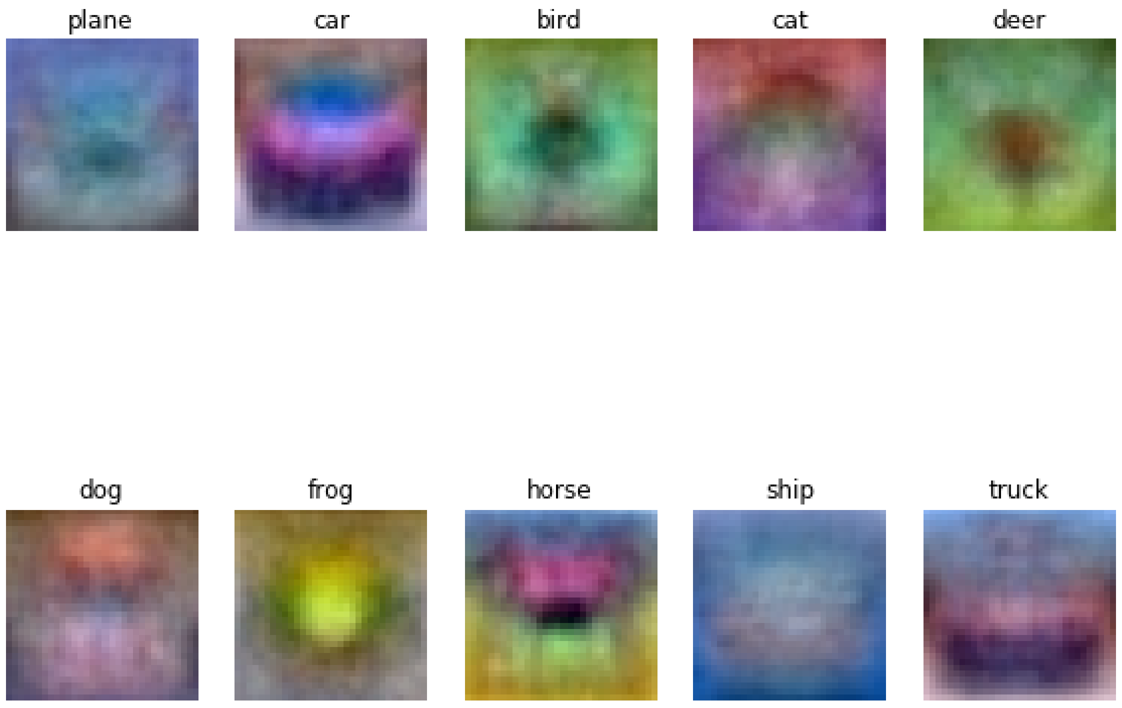
In [23]:

```
# Evaluate the best svm on test set
y_test_pred = best_svm.predict(X_test)
#making a confusion matrix to analize performance
confmatrix = np.zeros((num_classes,num_classes))
for i in range(0,y_test_pred.shape[0]):
    confmatrix[y_test[i],y_test_pred[i]]+=1
test_accuracy = np.mean(y_test == y_test_pred)
print 'linear SVM on raw pixels final test set accuracy: %f' % test_accuracy
```

```
10
(10, 10)
linear SVM on raw pixels final test set accuracy: 0.359000
```

In [24]:

```
# Visualize the learned weights for each class.
# Depending on your choice of learning rate and regularization strength, these may
# or may not be nice to look at.
w = best_svm.W[:-1,:] # strip out the bias
w = w.reshape(32, 32, 3, 10)
w_min, w_max = np.min(w), np.max(w)
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', '
for i in xrange(10):
  plt.subplot(2, 5, i + 1)

  # Rescale the weights to be between 0 and 255
  wimg = 255.0 * (w[:, :, :, i].squeeze() - w_min) / (w_max - w_min)
  plt.imshow(wimg.astype('uint8'))
  plt.axis('off')
  plt.title(classes[i])
```



In [25]:

```
# saving the best svm model to pickel file
import pickle
pickle.dump({"best_svm":best_svm,"cmatrix":confmatrix}, open( "best_svm.p", "wb" )
```

Inline question 2:

Describe what your visualized SVM weights look like, and offer a brief explanation for why they look they way that they do.

Your answer: The visualized SVM weights look like a mean of all images of the class they are trained for. So the weights attain values in such a way that its dot product with the class represented shall result in maximum score.In essence they represent how different they are from the other classes.