La transformada de Laplace de una función $f(t)$ se define como:

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

donde s es una variable compleja, t es la variable de la función $f(t)$, y $e^{-st}f(t)$ debe ser integrable en el intervalo $[0,\infty)$.

$$L\{cos(t)\} = \int_0^\infty e^{-st}\cos(t)\,dt$$

$$\int e^{-st}\cos(t)\,dt$$

$$\int e^{-st}\sin(t)\,dt$$

La transformada de Laplace de $t^n \cdot F(t)$ se expresa como:

$$L\{t^n \cdot F(t)\} = (-1)^n \frac{d^n}{ds^n} L\{F(t)\}$$

Para la integral $\int e^{-st}\cos(t)\,dt$, aplicamos integración por partes:

$$\begin{aligned} u &= e^{-st} & dv &= \cos(t)\,dt \\ du &= -se^{-st}\,dt & v &= \sin(t) \end{aligned}$$

La fórmula de integración por partes es:

$$\int u\,dv = uv - \int v\,du$$

Aplicando esto, obtenemos:

$$\int e^{-st}\cos(t)\,dt = e^{-st}\sin(t) - \int \sin(t)\,(-se^{-st})\,dt$$

$$\int e^{-st}\cos(t)\,dt = e^{-st}\sin(t) + s\int \sin(t)\,e^{-st}\,dt$$

Integraremos por segunda ves:
Para la integral $\int e^{-st}\sin(t)\,dt$, aplicamos integración por partes:

$$\begin{aligned} u &= e^{-st} & dv &= \sin(t)\,dt \\ du &= -se^{-st}\,dt & v &= -\cos(t) \end{aligned}$$

$$\int e^{-st}\sin(t)\,dt = -e^{-st}\cos(t) - \int -\cos(t)\,(-se^{-st})\,dt$$
$$\int e^{-st}\sin(t)\,dt = -e^{-st}\cos(t) - s\int \cos(t)\,e^{-st}\,dt$$

Reemplazamos

$$\int e^{-st}\cos(t)\,dt = e^{-st}\sin(t) + s\int \sin(t)\,e^{-st}\,dt$$
$$\int e^{-st}\cos(t)\,dt = e^{-st}\sin(t) + s(-e^{-st}\cos(t) - s\int \cos(t)\,e^{-st}\,dt)$$
$$\int e^{-st}\cos(t)\,dt = e^{-st}\sin(t) + -se^{-st}\cos(t) - s^2\int \cos(t)\,e^{-st}\,dt)$$
$$(1+s^2)\int e^{-st}\cos(t)\,dt = e^{-st}\sin(t) + -se^{-st}\cos(t)$$
$$\int_0^{\infty} e^{-st}\cos(t)\,dt = \frac{e^{-st}\sin(t) + -se^{-st}\cos(t)}{1+s^2}$$

$$\int_0^{\infty} e^{-st}\cos(t)\,dt = \left.\frac{e^{-st}\sin(t) - se^{-st}\cos(t)}{1+s^2}\right|_0^{\infty}$$
$$\int_0^{\infty} e^{-st}\cos(t)\,dt = \frac{e^{-s(\infty)}\sin(\infty) + -se^{-s(\infty)}\cos(\infty)}{1+s^2} - \frac{e^{-s(0)}\sin(0) + -se^{-s(0)}\cos(0)}{1+s^2}$$

exponencial elevado al -infinito es 0

$$\int_0^{\infty} e^{-st}\cos(t)\,dt = -\frac{-s}{1+s^2}$$
$$\int_0^{\infty} e^{-st}\cos(t)\,dt = \frac{s}{1+s^2}$$

$$L\{cos(t)\} = \frac{s}{1+s^2}$$

$$L\{t^2cos(t)\} = \frac{s}{1+s^2}$$

$$L\{t^n \cdot F(t)\} = (-1)^n \frac{d^n}{ds^n} L\{F(t)\}$$

$$L\{t^2 cos(t)\} = (-1)^2 \frac{d^2}{ds^2}(\frac{s}{s^2+1})$$

$$L\{t^2 cos(t)\} = \frac{d^2}{ds^2}(\frac{s}{s^2+1})$$

$$L\{t^2 cos(t)\} = \frac{2s(s^2-3)}{(s^2+1)^3}$$

$$\frac{d^2}{ds^2}(\frac{s}{s^2+1}) = \frac{d}{ds}(\frac{s^2+1-s(2s)}{(s^2+1)^2})$$

$$\frac{d^2}{ds^2}(\frac{s}{s^2+1}) = \frac{d}{ds}(\frac{1-s^2}{(s^2+1)^2})$$

$$\frac{d}{ds}(\frac{1-s^2}{(s^2+1)^2}) = \frac{(-2s)(s^2+1)^2-(1-s^2)2(2s)(s^2+1)}{(s^2+1)^4}$$

$$\frac{d}{ds}(\frac{1-s^2}{(s^2+1)^2}) = \frac{2s^5-4s^3-6s}{(s^2+1)^4}$$

$$\frac{d}{ds}(\frac{1-s^2}{(s^2+1)^2}) = \frac{2s(s^2-3)(s^2+1)}{(s^2+1)^4}$$

$$\frac{d}{ds}(\frac{1-s^2}{(s^2+1)^2}) = \frac{2s(s^2-3)}{(s^2+1)^3}$$