Week 1 Homework (due 27 MAR)

***** All problems from Chapter 7**

5. Reconsider the house fire problem of Example 7.4. In this exercise we will investigate the problem of estimating the rate λ at which emergency calls occur.

(a) Suppose that 2,050 emergency calls are received in a one–year period. Estimate the rate λ of house fires per month.

2050 / 12 = 170.8333 = 170.8333 = λ

(b) Assuming that the true value of λ is 171 calls per month, calculate the range of normal variation for the number of emergency calls received in one year.

$$\frac{n}{\lambda} \pm \frac{2\sqrt{n}}{\lambda} \rightarrow \frac{n}{171} \pm \frac{2\sqrt{n}}{171}$$

Excel table → n(147, 199) w/ 95% confidence

$$\underline{\underline{12 \cdot (147, 199) = (1764, 2388)}}$$

ANS

(c) Calculate the range of λ for which 2,050 calls in one year is within the range of normal variation. How accurate is our estimate of the true rate λ at which house fires occur?

c) $$\frac{2050}{\lambda} \pm \frac{2\sqrt{2050}}{\lambda} \rightarrow \left.\begin{array}{l}\underline{\lambda_L = 1960} \\ \underline{\underline{\lambda_h = 2140}}\end{array}\right\}$$ used excel table

The difference between these numbers and the numbers from part B shows that our assumed rate of 171 calls per month is not great, especially cause the range in 'b' is outside of the range in 'c'. If it was the other way around, lambda would be appropriate.

(d) How many years of data would be required to obtain an estimate of λ accurate to the nearest integer (an error of ±0.5)?

Smallest viable sample size is generally considered to be 30(?)

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6. Reconsider the house fire problem of Example 7.4. The underlying random process is called a Poisson process because it can be shown that the number of arrivals (calls) Nt during a time interval of length t has a Poisson distribution. Specifically,
for all n = 0, 1, 2, . . .

Pr{Nt = n} = e−λt(λt)n n!

(a) Show that ENt = λt and VNt =λt.

$$E(N_T) = \mu_{NT} = \underline{\underline{1}} \quad ; \quad E = \frac{\lambda t}{N_T} = \frac{171(1)}{171} = \underline{\underline{1}}$$

$$V(N_T) = \underline{\underline{1}} = \sigma_N^2 \quad ; \quad V = \frac{\lambda t}{N_t} = \frac{171(1)}{171} = \underline{\underline{1}}$$

(b) Use the Poisson distribution to calculate the probability that the number of calls received in a given month deviates from the mean of 171 by as much as 18 calls.

```
> x = seq(153,189)
> sum(dpois(x,171))
[1] 0.8430889
>
```

(c) Generalize the calculation of part (b) to determine the exact range of normal variation (at the 95% level) for the number of calls in a one–month period.

```
> std = sqrt(((171-18)^2)/(171))
> print(std)
[1] 11.7002
>
> me = qnorm(0.975)*std
> range = c(171-me,171+me)
> print(range)
[1] 148.068 193.932
```

[I used this PDF in helping me to decide on what function to use for "me" (margin of error): stat.ucla.edu/~rgould/110as02/bsci]

(d) Compare the exact method used in part (c) with the approximate calculation of the range of normal variation that is included in the discussion of sensitivity analysis for Example 7.4 in the text. Which method would be more appropriate for determining the range of normal variation in the number of calls received in a single day? In a year?

The text states that normal variation is between 147 and 198, which differs slightly from my range of 148 and 194 that I found in part b. Because the texts method created a larger range, I think their method would be more appropriate for determining the range of calls received in a day, as there is more room for variation in smaller sample sizes, whereas my more precise approximation would be more appropriate for a year, as there is less variation in larger sample sizes.

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11. A squadron of 16 bombers needs to penetrate air defenses to reach its target. They can either fly low and expose themselves to the air defense guns, or fly high and expose themselves to surface–to–air missiles. In either case, the air defense firing sequence proceeds in three stages. First, they must detect the target, then they must acquire the target (lock on target), and finally they must hit the target. Each of these stages may or may not succeed. The probabilities are as follows:

AD Type	P_{detect}	P_{acquire}	P_{hit}
Low	0.90	0.80	0.05
High	0.75	0.95	0.70

The guns can fire 20 shells per minute, and the missile installation can fire three per minute. The proposed flight path will expose the planes for one minute if they fly low, and five minutes if they fly high.

(a) Determine the optimal flight path (low or high). The objective is to maximize the number of bombers that survive to strike the target.

```
> shots_low = 20 #(1 minute with 20 shots per minute)
> shots_high = 15 #(5 minutes with 3 shots per minute)
>
> # Probability that a plane gets hit while flying low
> p_low = 0.9*0.8*0.05
>
> # Probability that a plane gets hit while flying high
> p_high = 0.75*0.95*0.70
>
> # Probability that at least one plane gets hit while flying low
> x = seq(1,shots_low)
> sum(dbinom(x,shots_low,p_low))
[1] 0.5196689
>
> # Probability that at least one plane gets hit while flying high
> y = seq(1,shots_high)
> sum(dbinom(y,shots_high,p_high))
[1] 0.9999683
```

Flying low is optimal

(b) Each individual bomber has a 70% chance to destroy the target. Use the results of part (a) to determine the chances of success (target destroyed) for this mission.

```
> # Chance of success if low
> expected_low = sum(1-dbinom(planes,shots_low,p_low))
> planes_low = seq(1,15) #created based on line above expecting 15.4 planes to make it through
> sum(dbinom(planes_low,16,0.7))
[1] 0.9966767
>
> # Chance of success if high
> expected_high = sum(1-dbinom(planes,shots_high,p_high))
> planes_high = seq(1,14) #created based on line above expecting 15.4 planes to make it through
> sum(dbinom(planes_high,16,0.7))
[1] 0.9738884
```

(c) Determine the minimum number of bombers necessary to guarantee a 95% chance of mission success.

*** see R code ***

(d) Perform a sensitivity analysis with respect to the probability p = 0.7 that an individual bomber can destroy the target. Consider the number of bombers that must be sent to guarantee a 95% chance of mission success.

*** see R code ***

(e) Bad weather reduces both Pdetect and p, the probability that a bomber can destroy the target. If all of these probabilities are reduced in the same proportion, which side gains an advantage in bad weather?

The bombers (see R code)