A Fundamental Domain of a Nontrivial Circle Bundle over a Surface

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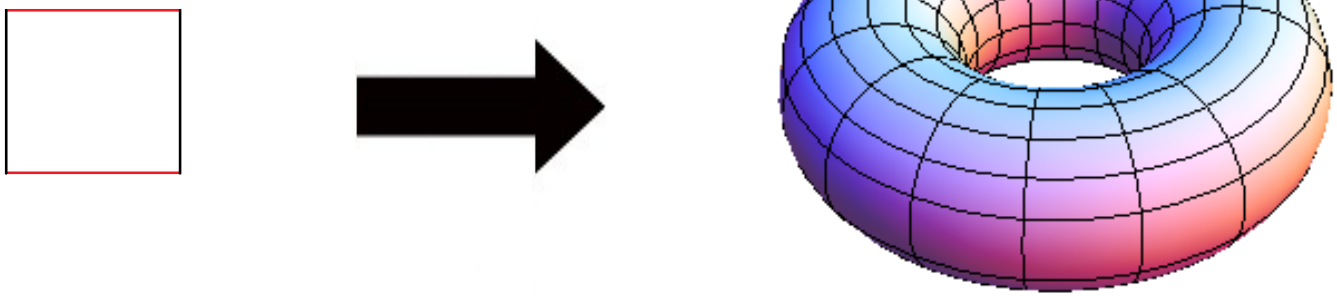
May 12, 2014

Fundamental Domains

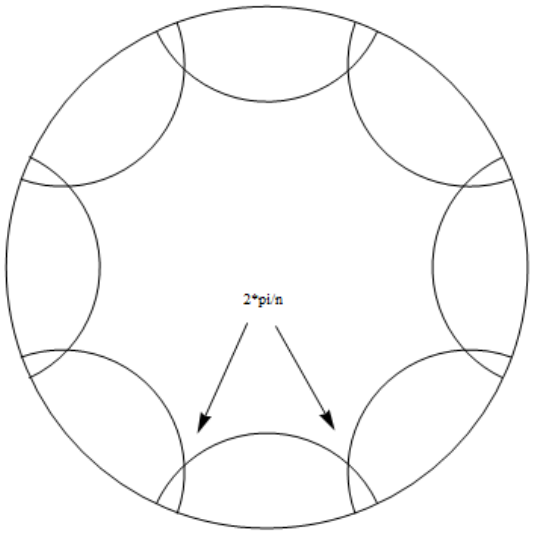
Definition (Fundamental Domain)

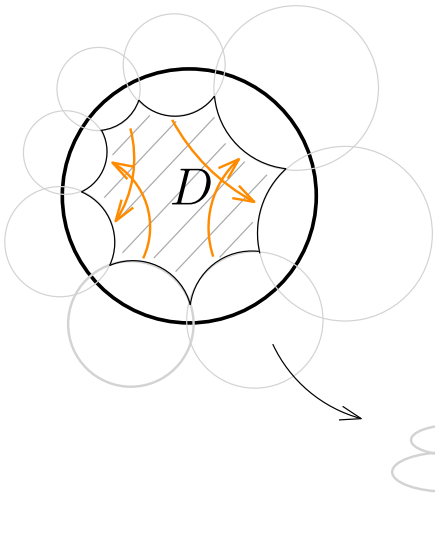
A "polyhedron" where side identifications yield the manifold.

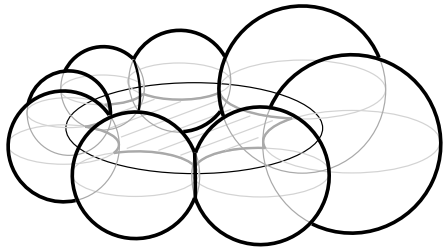


The Trivial Example

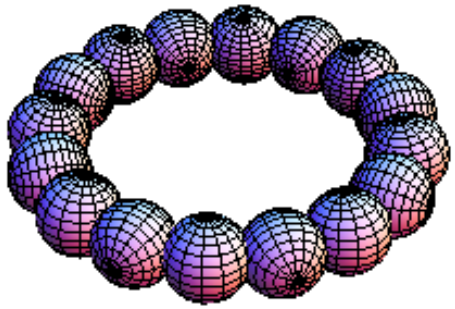
$$S^3 = \mathbb{R}^3 \cup \{\infty\}$$



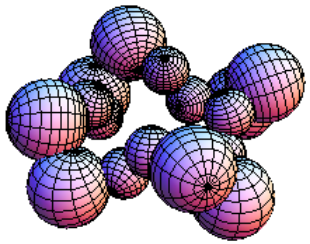


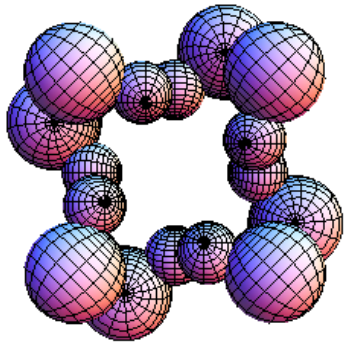


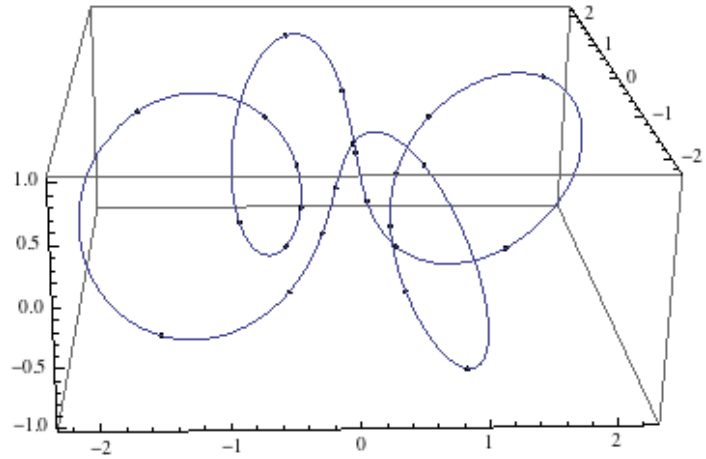
For Comparison - Euler Number Equals 0



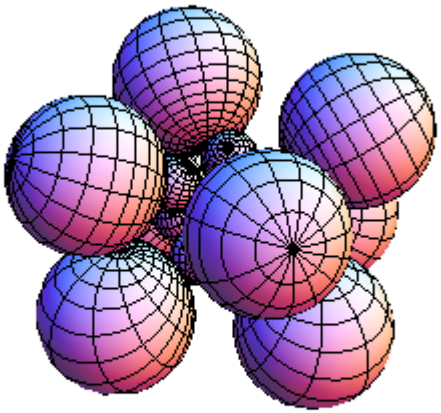
The Nontrivial Example - Euler Number Equals 1

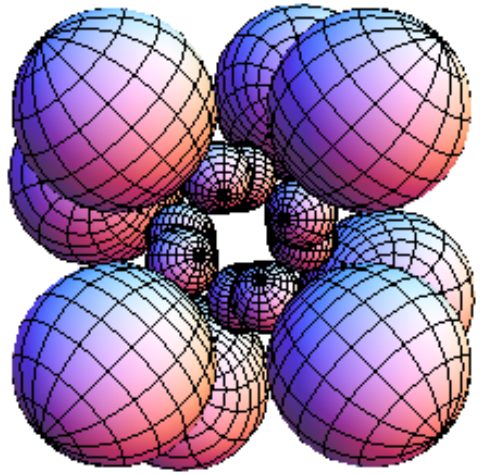






The Nontrivial Example - Euler Number Equals 2





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- *StereographicProjection* : $S^2 - (0, 0, 1) \to \mathbb{R}^2$

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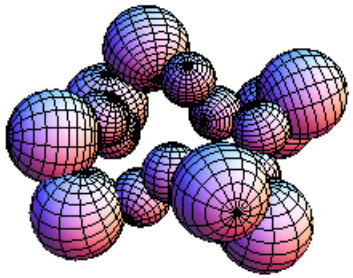
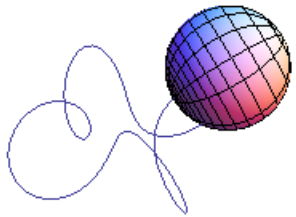
$Curve(t) = (e^{it} z_0, e^{ipt} w_0)$
Rotates sphere around the curve (not continuously) stepwise.





Future Projects

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- Get exact angles
  $\alpha = \frac{2\pi}{n}$
  where n $=$ number of spheres in the domain

Acknowledgements

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- Gromov, Lawson, Thurston - 1988

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- Son Lam Ho and Dr. Michelle Lee