(0)Title:
Ian: say title.
“Our project this semester was to graph a fundamental domain for a nontrivial circle bundle over a surface”

(1)Fundamental Domain:
Ian: <First read definition on pdf>
“The fundamental domain gives a geometry to a topological surface”
“We require 4*n sides for a genus n surface. For example, a one holed torus requires 4 sides, as shown here.”
“Although we should note, that this won’t necessarily always give us a ‘nice’ geometry, meaning one that we are accustomed to dealing with.”

(2)The Trivial Example:
Hasan: “this (image on left) surface gives us a ‘nice’ geometry for a genus 2 surface”

“If you take the surface D and cross it with S^1, then identify the spheres, you will get the fundamental domain.”

**“what we see here (image on right) is the complement of the fundamental domain inside S^3, which is several intersecting S^2”

**“if we take the complement of D and cross it with S^1 we get the image on the right. Then the fundamental domain is the complement of that surface inside S^3, i.e. take S^3 and remove all the S^2 and that will be our fundamental domain”

“then if we identify the spheres in our fundamental domain we will get our topological surface.”

(3)Euler Number 0:
Ian : Here, we have drawn the fundamental domain of {sigma cross S1} in Mathematica. Note that we are looking at the complement in S3 of this union of spheres that are shown. We can identify this surface as a fundamental domain and yield a genus 4 surface, because there are 16 spheres.

(4)Nontrivial Example 1:
Hasan: “Here are the pictures that we made. This is using Euler number equal to 1, so our surface will be more complex than just sigma cross S^1”
“The complement of this surface is the fundamental domain of a circle bundle over a genus 4 surface.”

(5)Nontrivial Example 2:

Ian: “This is another example of a nontrivial fundamental domain.”
“Even though the spheres do not appear to be the same size, they actually are the same size in S3. This distortion comes from the stereographic projection from S3 onto R3.”

(6)A Lower Dimensional Space:
Hasan: “We want to figure out how to picture S^2 inside of S^3.”
“We did that by figuring out the process for the lower dimension problem, so we looked at S^1 inside S^2, because we can visualize this in our mind.”
“First, we chose an arbitrary circle in S^2. We rotated the center point to the z-axis to parameterize it.”
“After we parameterized the circle, we applied the inverse rotation. Now we have a parameterized arbitrary circle on S^2.”
“Finally, we used the point (0,0,1) to Stereographically project our circle on S^2 onto R^2.”

(7)Goal:
Ian: “Now that we figured out the process for S1 inside S2, we can apply the same process for S2 inside S3 using the point (0,0,0,1) for our stereographic projection. The process will give us the sphere in the image on the right.”
“Recall, the parameterized “circle” on S2 is actually a sphere.”
“To get what we want, which is the complement of the image on the left in S3, we have to rotate the sphere (not continuously) around the curve n times, which will allow us to obtain the genus n/4 surface.”

(8)Possible Future Projects:
Hasan: “We would like to find the equation so that we can stereographically project from any arbitrary point on S^3, not just (0,0,0,1). This will improve our visualization of the fundamental domain, because when we stereographically project from any point, we will have some amount of distortion. So the more points we stereographically project from, the better over all image we hope to get.”
“It’s also important to note that we “cheated” because we haven’t found the perfect radius for the sphere that will give us the needed angle for the fundamental domain. So this is something that requires further calculations.”

(9)Acknowledgements:
Ian: “The first examples of a nontrivial circle bundle were created by Gromov, Lawson and Thurston in 1988.”

“Feng Luo made further examples of nontrivial circle bundles in 1993. His research helped us form our algorithm to estimate the radius for our fundamental domain.”
“We would also like to thank our wonderful advisors for their enthusiasm and patience.”