



1	1	7	6	7	6	6
1	1	7	6	5	6	4
7	7	6	5	6	4	0
5	5	4	7	7	0	0
5	5	4	7	7	0	0

1	0	-1
2	0	-2
1	0	-1

Pad with 1

-17	-13	12	1	-7
-3	-3	1	9	8
4	-4	-9	22	23

	Pad	cdf	Scale		$(1-f) \hat{r}(r)$
$H_r(0) = 1$	$P_r(0) = \frac{1}{15}$	$F_r(0) = \frac{1}{15}$	$T(0) = \frac{7}{15}$	0	
$H_r(1) = 1$	$P_r(1) = \frac{1}{15}$	$\frac{2}{15}$	$T(1) = \frac{14}{15}$	0	
$H_r(2) = 0$	$P_r(2) = \frac{2}{15}$	$\frac{2}{15}$	$T(2) = \frac{14}{15}$	0	
$H_r(3) = 1$	$P_r(3) = \frac{1}{15}$	$\frac{3}{15}$	$T(3) = \frac{21}{15}$	1	
$H_r(4) = 2$	$P_r(4) = \frac{2}{15}$	$\frac{5}{15}$	$T(4) = \frac{35}{15}$	2	
$H_r(5) = 2$	$P_r(5) = \frac{2}{15}$	$\frac{7}{15}$	$T(5) = \frac{47}{15}$	3	
$H_r(6) = 4$	$P_r(6) = \frac{4}{15}$	$\frac{11}{15}$	$T(6) = \frac{77}{15}$	5	
$H_r(7) = 4$	$P_r(7) = \frac{4}{15}$	$\frac{15}{15}$	$T(7) = 7$	7	

$$C(y_n, \hat{y}_n) = \frac{1}{2} (y_n - \hat{y}_n)^2$$

Forward pass

$$a_1 = 1 \quad c_1 = 2 \quad \hat{y} = 2 \quad C = \frac{1}{2} (y - \hat{y})^2 = \frac{1}{2}$$

$$a_2 = 0$$

$$a_3 = 1 \quad c_2 = -4$$

$$a_4 = -4$$

Backward pass

$$\frac{\partial C}{\partial \hat{y}} = \frac{\partial}{\partial \hat{y}} \left( \frac{1}{2} (y - \hat{y})^2 \right) = -(y - \hat{y}) = 1 \quad \hat{y}' = 1$$

$$\frac{\partial C}{\partial c_1} = \frac{\partial \hat{y}}{\partial c_1} \frac{\partial C}{\partial \hat{y}} = \left( \frac{\partial \max(c_1, c_2)}{\partial c_1} = 1 \right) \cdot 1 = 1$$

$$\frac{\partial C}{\partial c_2} = \frac{\partial \hat{y}}{\partial c_2} \frac{\partial C}{\partial \hat{y}} = \left( \frac{\partial \max(c_1, c_2)}{\partial c_2} = 0 \right) \cdot 1 = 0$$

$$\frac{\partial C}{\partial b_1} = \frac{\partial c_1}{\partial b_1} \frac{\partial C}{\partial c_1} = \frac{\partial (a_1 + a_2 + b_1)}{\partial b_1} \cdot 1 = 1$$

$$\frac{\partial C}{\partial b_2} = \frac{\partial c_2}{\partial b_2} \frac{\partial C}{\partial c_2} = 0$$

$$\frac{\partial C}{\partial a_1} = \frac{\partial c_1}{\partial a_1} \frac{\partial C}{\partial c_1} = 1$$

$$\frac{\partial C}{\partial a_2} = 1$$

$$\frac{\partial \mathcal{L}}{\partial a_3} = 0 \quad \frac{\partial \mathcal{L}}{\partial a_4} = 0$$

$$\frac{\partial \mathcal{L}}{\partial w_1} = \frac{\partial a_1}{\partial w_1} \cdot \frac{\partial \mathcal{L}}{\partial a_1} = 1 \cdot x_1 = x_1 = -1$$

$$\frac{\partial \mathcal{L}}{\partial w_2} = \frac{\partial a_2}{\partial w_2} \cdot \frac{\partial \mathcal{L}}{\partial a_2} = 1 \cdot x_2 = x_2 = 0$$

$$\frac{\partial \mathcal{L}}{\partial w_3} = 0$$

$$\frac{\partial \mathcal{L}}{\partial w_4} = 0$$

$$C(y_n, \hat{y}_n) = \frac{1}{2} (y_n - \hat{y}_n)^2$$

Forward pass

$$\begin{aligned} a_1 &= 1 & c_1 &= 2 & \hat{y} &= 2 & C &= \frac{1}{2} (y - \hat{y})^2 = \frac{1}{2} \\ a_2 &= 0 \\ a_3 &= 1 & c_2 &= -4 \\ a_4 &= -4 \end{aligned}$$

Backward pass

$$\frac{\partial C}{\partial \hat{y}} = \frac{\partial}{\partial \hat{y}} \left( \frac{1}{2} (y - \hat{y})^2 \right) = -(y - \hat{y}) = 1 \quad \hat{y}' = 1$$

$$\frac{\partial C}{\partial c_1} = \frac{\partial \hat{y}}{\partial c_1} \frac{\partial C}{\partial \hat{y}} = \left( \frac{\partial \max(c_1, c_2)}{\partial c_1} = 1 \right) \cdot 1 = 1$$

$$\frac{\partial C}{\partial c_2} = \frac{\partial \hat{y}}{\partial c_2} \frac{\partial C}{\partial \hat{y}} = \left( \frac{\partial \max(c_1, c_2)}{\partial c_2} = 0 \right) \cdot 1 = 0$$

$$\frac{\partial C}{\partial b_1} = \frac{\partial c_1}{\partial b_1} \frac{\partial C}{\partial c_1} = \frac{\partial (a_1 + a_2 + b_1)}{\partial b_1} \cdot 1 = 1$$

$$\frac{\partial C}{\partial b_2} = \frac{\partial c_2}{\partial b_2} \frac{\partial C}{\partial c_2} = 0$$

$$\frac{\partial C}{\partial a_1} = \frac{\partial c_1}{\partial a_1} \cdot \frac{\partial C}{\partial c_1} = 1$$

$$\frac{\partial C}{\partial a_2} = 1$$

$$\frac{\partial C}{\partial a_3} = 0 \quad \frac{\partial C}{\partial a_4} = 0$$

$$\frac{\partial C}{\partial w_1} = \frac{\partial a_1}{\partial w_1} \cdot \frac{\partial C}{\partial a_1} = 1 \cdot x_1 = x_1 = -1$$

$$\frac{\partial C}{\partial w_2} = \frac{\partial a_2}{\partial w_2} \cdot \frac{\partial C}{\partial a_2} = 1 \cdot x_2 = x_2 = 0$$

$$\frac{\partial C}{\partial w_3} = 0$$

$$\frac{\partial C}{\partial w_4} = 0$$

## Gradient descent

$$w_1 : \quad \bar{w}_1 = w_1 + \alpha \cdot \left( -\frac{\partial \mathcal{L}}{\partial w_1} \right) = -0.9$$

$$w_3 : \quad \bar{w}_3 = w_3 + \alpha \left( -\frac{\partial \mathcal{L}}{\partial w_3} \right) = -1$$

$$b_1 : \quad \bar{b}_1 = b_1 + \alpha \left( -\frac{\partial \mathcal{L}}{\partial b_1} \right) = 0.9$$