Problem Set 12 *

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Course Number: 625.603

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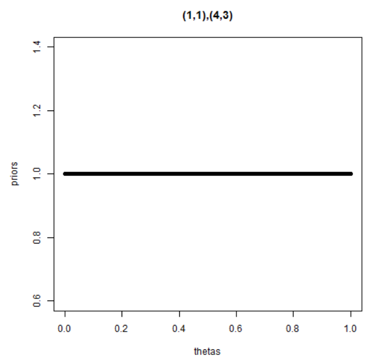
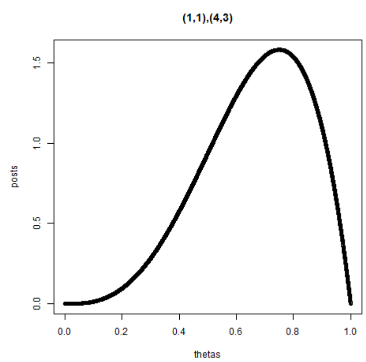
Problem (1,1),(4,3). .

Prior Distribution:



$$g(\theta) = \frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)}\theta^{r-1}(1-\theta)^{s-1}$$
$$g(\theta) = \frac{\Gamma(2)}{2\Gamma(1)}\theta^{0}(1-\theta)^{0}$$
$$g(\theta) = 1$$

Posterior Distribution:



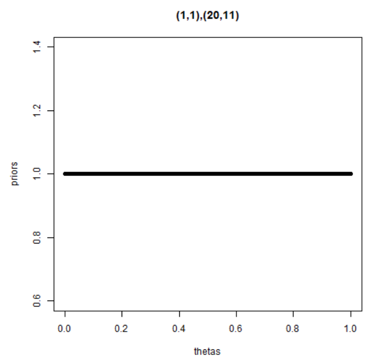
$$g(\theta|n,k) = \frac{\Gamma(n+r+s)}{\Gamma(r+k)\Gamma(n-k+s)} \cdot \theta^{r+k-1}(1-\theta)^{n+s-k-1}$$
$$g(\theta|n,k) = \frac{\Gamma(6)}{\Gamma(4)\Gamma(2)}\theta^{3}(1-\theta)^{1}$$
$$g(\theta|n,k) = \frac{6!}{4!8!}\theta^{3}(1-\theta)^{1}$$
$$g(\theta|n,k) = 15 \cdot \theta^{3}(1-\theta)^{1}$$

To find the probability of bias: $P(\theta > 0.5) = \int_{0.5}^{\infty} g(\theta)d\theta = 1/2$[1]

[1] Could not figure out how to do this in R so I used wolfram alpha. I still don't really get it though. Because it only lets me do 2 parameters not 4.

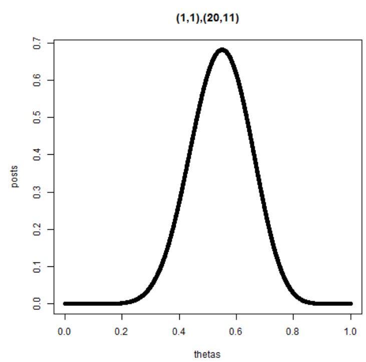
Problem (1,1),(20,11). .

Prior Distribution:



$$g(\theta) = \frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)}\theta^{r-1}(1-\theta)^{s-1}$$
$$g(\theta) = \frac{\Gamma(2)}{2\Gamma(1)}\theta^{0}(1-\theta)^{0}$$
$$g(\theta) = 1$$

Posterior Distribution:

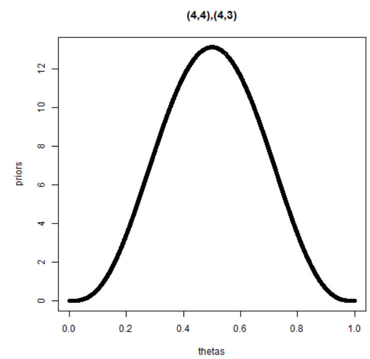


$$g(\theta|n,k) = \frac{\Gamma(n+r+s)}{\Gamma(r+k)\Gamma(n-k+s)} \cdot \theta^{r+k-1}(1-\theta)^{n+s-k-1}$$
$$g(\theta|n,k) = \frac{\Gamma(22)}{\Gamma(12)\Gamma(10)}\theta^{11}(1-\theta)^{9}$$
$$g(\theta|n,k) = \frac{22!}{12!32!}\theta^{11}(1-\theta)^{9}$$
$$g(\theta|n,k) = 646646 \cdot \theta^{11}(1-\theta)^{9}$$

To find the probability of bias: $P(\theta > 0.5) = \int_{0.5}^{\infty} g(\theta)d\theta = 1/2$

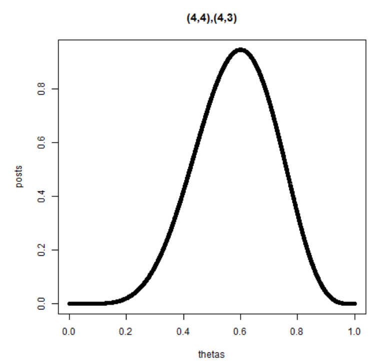
Problem (4,4),(4,3). .

Prior Distribution:



$$g(\theta) = \frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)}\theta^{r-1}(1-\theta)^{s-1}$$
$$g(\theta) = \frac{\Gamma(8)}{2\Gamma(4)}\theta^3(1-\theta)^3$$
$$g(\theta) = 840 \cdot \theta^3(1-\theta)^3$$

Posterior Distribution:

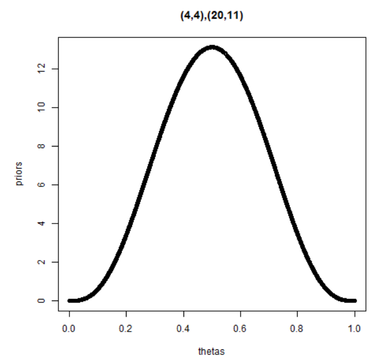


$$g(\theta|n,k) = \frac{\Gamma(n+r+s)}{\Gamma(r+k)\Gamma(n-k+s)} \cdot \theta^{r+k-1}(1-\theta)^{n+s-k-1}$$
$$g(\theta|n,k) = \frac{\Gamma(12)}{\Gamma(7)\Gamma(5)}\theta^6(1-\theta)^3$$
$$g(\theta|n,k) = \frac{12!}{7!11!}\theta^6(1-\theta)^3$$
$$g(\theta|n,k) = 792 \cdot \theta^6(1-\theta)^3$$

To find the probability of bias: $P(\theta > 0.5) = \int_{0.5}^{\infty} g(\theta)d\theta = 0.5785$

Problem (4,4),(20,11). .

Prior Distribution:

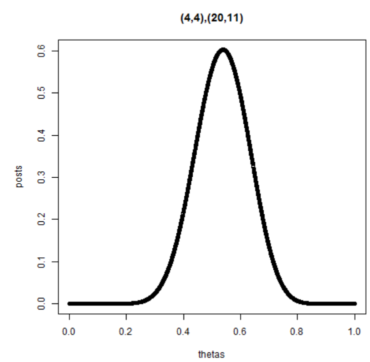


$$g(\theta) = \frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)}\theta^{r-1}(1-\theta)^{s-1}$$

$$g(\theta) = \frac{\Gamma(8)}{2\Gamma(4)}\theta^{3}(1-\theta)^{3}$$

$$g(\theta) = 840 \cdot \theta^{3}(1-\theta)^{3}$$

Posterior Distribution:



$$g(\theta|n,k) = \frac{\Gamma(n+r+s)}{\Gamma(r+k)\Gamma(n-k+s)} \cdot \theta^{r+k-1}(1-\theta)^{n+s-k-1}$$

$$g(\theta|n,k) = \frac{\Gamma(28)}{\Gamma(15)\Gamma(13)}\theta^{14}(1-\theta)^{12}$$

$$g(\theta|n,k) = \frac{28!}{15!13!}\theta^{14}(1-\theta)^{12}$$

$$g(\theta|n,k) = 37442160 \cdot \theta^{14}(1-\theta)^{12}$$

To find the probability of bias: $P(\theta > 0.5) = \int_{0.5}^{\infty} g(\theta)d\theta = 0.648$