Given $x_j \in X$ and $x_i^* \in X^*$, let $x_1^* x_n* $ be n independent trials, each resulting in either success or failure, with success defined as $x_i^* = x_j$. The probablity of k success is a binomial random variable with the probability of success for any trial $p = \frac{1}{n}$. Therefore, by Theorem 3.2.1 (Larsen & Marx (2018), p. 104):

$$\mathrm{P(k\ successes)} = \binom{n}{k}(\frac{1}{n})^k(\frac{n-1}{n})^{n-k}$$

We can see this in R by calculating the theoretical probability and creating a bootstrap sample for

$$\hat{\theta}_i^* = \sum_{i=1}^{n} I_{(x_j^*=x_i)}$$

Where $I(\phi)$ is a Binomial random variable with success probability $$

```
## Define nchoosek (Larsen & Marx (2018), p. 84).
  nchoosek <- function(n,k){
    factorial(n)/((factorial(k))*(factorial(n-k)))
  }

## Define full function (Larsen & Marx (2018), p. 104)
 binom.nk <- function(n,k){
   nchoosek(n,k) * ((1/n)^k) * ((n-1)/n)^(n-k)
 }


## Get a base X of size n
 vctrzdData <- c(1,2,3.5,4,7.3,8.6,12.4,13.8)

## Set parameters
 n <- length(vctrzdData)
 k <- 2
 B <- 1000

## Boot Strap
  count <- c()
    for (i in 1:B){
      ## Get B samples from X
        btstrpSmpl = sample(vctrzdData,length(vctrzdData), replace = TRUE)
      ## Count how many times x_i shows up in the sample
        count[i] <- sum(btstrpSmpl == vctrzdData[3])
    }
## Probability
  (sum(count == k))/B
```

```
## [1] 0.211
```

```
## Compare to theoretical probability
  binom.nk(length(vctrzdData),k)
```