Problem Set 2 *

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Problem 2.3.10. An urn contains 24 chips. A is the event that the number is divisible by 2. B is the event that the number is divisible by 3. Find $P(A \cup B)$.

Solution: $P(A \cup B) = 0.667$

$$\begin{aligned} A &= \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24\} \\ B &= \{3, 6, 9, 12, 15, 18, 21, 24\} \\ A \cup B &= \{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24\} \\ P(A \cup B) &= 16/24 \\ &= 0.6667 \end{aligned}$$

Problem 2.4.4. Let A and B be two events such that $P((A \cup B)^C) = 0.6$ and $P(A \cap B) = 0.1$. Let E be the event that either A or B occurs but not both will occur. Find $P(E|A \cup B)$.

Solution: $P(E|A \cup B) = 0.75$

*Problem list - 2.3.10, 2.4.4, 2.4.36, 2.5.20, 2.6.12

Proof

$$\begin{aligned}
P(E|A\cup B) &= \frac{P(E\cap(A\cup B))}{P(A\cup B)} && \text{Def. Conditional Probability (pg 33)}\\
P(E\cap(A\cup B)) &= P(E)+P(A\cup B)-P(A\cup B) && \text{Theorem 2.3.6 (pg 27)}\\
&= P(E) && \text{This follows since} E\subseteq(A\cup B)\\
&= P(A\cup B)-P(A\cap B) && \text{Based on question.}\\
P(A\cup B) &= 1-P((A\cup B)^C) && \text{Theorem 2.3.1 (pg 27)}\\
P(E|A\cup B) &= \frac{(1-P((A\cup B)^C))-P(A\cap B)}{1-P((A\cup B)^C)}\\
&= \frac{(1-0.6-0.1)}{1-0.6}\\
&= 0.75
\end{aligned}$$

Problem 2.4.36. Probability of guilty verdict: 15% if the defense can discredit the police department and 80% if not. Attorneys has 70% chance of discrediting police department. What is the probability of a guilty verdict. Let G =1 if guilty verdict, 0 otherwise. Let C = 1 if the attorney convinces contamination, 0 otherwise: Find P(G).

$$P(G=1|C=0)=0.15$$

$$P(G=1|C=1)=0.80$$

$$P(G=0|C=0)=0.75$$

$$P(G=0|C=1)=0.20$$

Solution P(G)=0.345

Since $C_0\cap C_1=\emptyset$ and $C_0\cup C_1=S$ Where S is the total set of events. And $P(C_0)>0$ and

$P(C_1) > 0$. We can apply Theorem 2.4.1 (pg 41).

$$
\begin{aligned}
P(G) &= \Sigma_{i=0}^{i=1} P(G|C_i)P(C_i) \\
&= (0.3) * (0.8) + (0.7) * (0.15) \\
&= 0.345
\end{aligned}
$$

Problem 2.5.20. Players A,B, and C toss a fair coin in order. The first to throw a heads wins. What are their respective chances of winning.

SolutionLet $A_H, A_T, B_H, B_T, C_H, C_T$ denote the events that A,B,C throw H (heads) or T (tails) on individual tosses. the P(A throws the first head)$=P(A_H \cup (A_T \cap B_T \cap C_T) \cup \cdots)$ It follows similarly for B and C.

$$
\begin{aligned}
P(Awin) &= \frac{1}{2} + \frac{1}{2} * (\frac{1}{8}) + \frac{1}{2} * (\frac{1}{8})^2 + \cdots = \frac{1}{2} + (\frac{1}{1-(1/8)}) = \frac{4}{7} \\
P(Bwin) &= \frac{1}{4} + \frac{1}{4} * (\frac{1}{8}) + \frac{1}{4} * (\frac{1}{8})^2 + \cdots = \frac{1}{4} + (\frac{1}{1-(1/8)}) = \frac{2}{7} \\
P(Cwin) &= 1 - \frac{4}{7} - \frac{2}{7} = \frac{1}{7}
\end{aligned}
$$

Problem 2.6.12. What is the minimum number of (.,-) needed to represent any letter in the English alphabet?

Solution

It follows from the multiplication rule (pg 66) that with 2 symbols and n different characters. There are 2^k number of ways to arrange the (.,-). So the number of ways to arrange (.,-) in k slots is $\Sigma_{k=1}^{n} 2^k$. We need 26 combinations. So

$$
\Sigma_{k=1}^{n} 2^k < 26
$$

By counting, we can see that $\Sigma_{k=1}^{3} 2^k = 14$ and $\Sigma_{k=1}^{4} 2^k = 30$. Thus, we need a minimum of 4 (.,-) to represent every letter of the alphabet.

Problem Module 2 simulation. Consider a Baseball World Series (best of 7 game series) in which team A theoretically has a 0.55 chance of winning each game against team B. Simulate the probability that team A would win a World Series against team B by simulating 1000 World Series. You many use any software to conduct the simulation.

Solution Using the R code below, I ran this simulation a few times. I received the following outputs: (0.589,0.603,0.598). I also expanded the calculation to 100,000 and received (0.60607, 061046, 0.60552). This seems reasonable since performing the calculation using hypergeometric distribution, there is a probability of 0.608287 of A winning at least 4 games.

The following is R code used to conduct this simulation:

```
count = 0
for ( i in 1:1000) {
count <- count+ sum(sum(sample(c("A Win", "A lose"),7,replace=TRUE,prob=c(0.55 ,
0.45)) == "A Win")>=4)
}
count
count/1000
```