Problem Set 2 *

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*Problems 2,3,7,9,11

Problem 2. The Bridge at Cay Road is actually part of the road between Augen and Burger. This means that if project 2 and 4 are to be carried out, either projects 6 or 7 must also be carried out.

Solution If 2 or 4 then 6 or 7 so if $x_2 + x_4 = 1 \implies x_6 + x_7 = 1$ equivalently, $x_2 + x_4 = 1 \implies x_5 = 0$ By adding the constraint: $x_2 + x_4 + x_5 = 1$ we can solve our zero-one programing problem. This is done in R using the lp package and gets the result: that we should choose projects (4,6,10). I show the matrix optimization below.

$$\text{maximize} \begin{bmatrix} 4 & 5 & 3 & 4.3 & 1 & 1.5 & 2.5 & .3 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_{10} \end{bmatrix}$$

$$\text{Subject to: } \begin{bmatrix} 2 & 3 & 1.5 & 2.2 & 0.5 & 1.5 & 2.5 & 0.1 & 0.6 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_{10} \end{bmatrix} \leq \begin{bmatrix} 5 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Problem 3. A company has identified a number of promising projects. Managers can allocate up to \$250,000 in each of the first 2 years. If less than \$250,000 is used in the first year, the balance can be invested at 10% and used to augment next year's budget.

Solution First I need to make some notes. We want to make the constraints such that $\Sigma c_{ij}x_i \leq y_i$. This is because we want to make sure that we spend less than our budget. To make this easier to see, I multiply the costs by negative 1 thus giving us cost $\leq$ budget. Now to account for how much budget will be left over we need add another variable x_8 (not a binary vector). For the first year, we add x_8 to the cash flow and say that the cash flow must equal \$250,000. For the second year we subtract $0.1x_6$ and say that the cash flow must be less than or equal to \$250,000. Subtracting out x_6 gives us the extra room in our budget. To maintain the matrix dimensions, we also have to add a zero to the net present value calculation.

Using R's lpSolve package - We want to maximize:

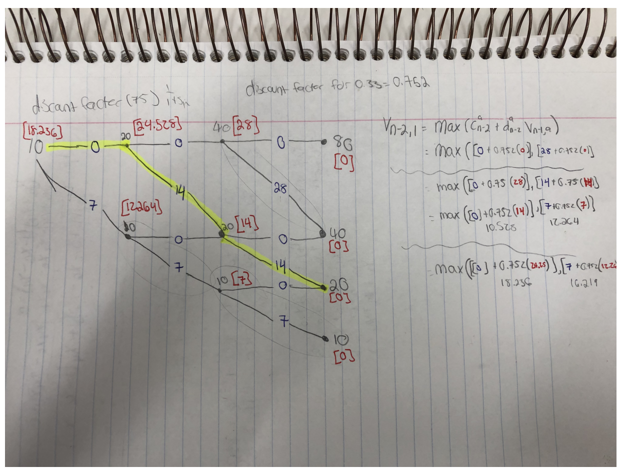
$$\begin{bmatrix} 150 & 200 & 100 & 100 & 120 & 150 & 240 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_8 \end{bmatrix}$$

Subject to:

$$\begin{bmatrix} 90 & 80 & 50 & 20 & 40 & 80 & 80 & 1 \\ 58 & 80 & 100 & 64 & 50 & 20 & 100 & -0.1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_8 \end{bmatrix} \leq \begin{bmatrix} \$\ 250{,}000 \\ \$\ 250{,}000 \end{bmatrix}$$

Putting this into R lpSolve package, showed that we should invest in projects (4,5,6,7). It also shows us that we invest \$30,000 in the first year.

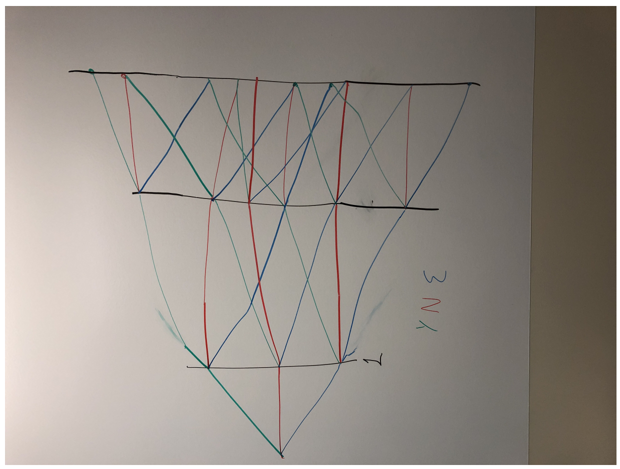
Problem 7. Find a solution to the fishing problem when interest rate is 33%. Are the decisions different than when the interest rate is 25%. At what critical values of the discount factor does the solution change. **solution** The photo below shows the node values for interest rate = 33% and the highlighted path is the same as when interest rate = 25%. The discount factor when the interst rate is 33% is given by the definition of discount factor (pg 75) $d_k = \frac{1}{(1+s_k)^k} = 1/1.33 = 0.752$



Problem 9. Three choices to pump oil: (a) not pump oil, (b) pump normally with operating cost 50,000 and you will pump out %20 of what the reserves were at at the beginning, (c) enhanced pumping with 120,000 op cost and %36 of your reserves. Price of oil is $10/barrel and the interest rate is 10% . 3 year period. Honestly, this art project took 45 minutes so I'm out of time and the rest of this seems like an easy time consuming exercise and I'm tired so I'll take the point. If I fail the class because of it so be it.

(a) Show how to set up a trinomial lattice to represent the possible states of the oil reserves.

Well that was a pointless art project. The photo is below.



b What is the maximum present value of your profits, and what is the corresponding pumping strategy.

Problem 11. how that for $g < r$

$$\begin{aligned}
\Sigma_{k=1} \frac{(1+g)^{k-1}}{(1+r)^k} &= \frac{1}{r-g} \\
&= S = \frac{1}{(1+r)} + \frac{S(1+g)}{1+r} && \text{hint} \\
S * (1 - \frac{(1+g)}{(1+r)}) &= \frac{1}{(1+r)} && \text{solve for S} \\
S * (\frac{r-g}{1+r}) &= \frac{1}{(1+r)} \\
S &= \frac{1}{r-g}
\end{aligned}$$