Problem Set 11 *

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*Problem list

Problem 9.2.6.

$$
\begin{aligned}
H_0 &: \mu_x = \mu_y \\
H_1 &: \mu_x < \mu_y \\
t &= \frac{\bar{x} - \bar{y}}{S_p\sqrt{\frac{1}{n} + \frac{1}{m}}} && \text{Theorem 9.2.2 (pg 452)} \\
t &= \frac{6\bar{5}.2 - 7\bar{5}.5}{13.9\sqrt{\frac{1}{9} + \frac{1}{12}}} = -1.6804
\end{aligned}
$$

By Theorem 9.2.2.b (pg 452) H_0 should be rejected if $t \leq -t_{\alpha,n+m-2}$. In this case, $t_{0.05,19} = -1.729$. Since $-1.6804 > -1.729$ we can not reject the null hypothesis. There does not appear to be a significant difference in the lifespans of alcoholic authors and non alcoholic authors.

Problem 9.3.4.

$$
\begin{aligned}
H_0 &: \sigma_x^2 = \sigma_y^2 \\
H_1 &: \mu_x < \mu_y
\end{aligned}
$$

By Theorem 9.3.1.c (pg 464) we can reject the null hypothesis if:

$$
\begin{aligned}
s_y^2/s_x^2 &\leq f_{\alpha/2,m-1,n-1} \ or \ \geq f_{1-\alpha/2,m-1,n-1} \\
3.18/5.67 &\leq f_{0.025,9,9} \ or \ \geq f_{0.975,9,9} \\
0.561 &\leq 0.248 \ or \ \geq 4.03
\end{aligned}
$$

Since neither of these conditions are met, we can not reject the null hypothesis. It does not appear that a strong magnetic field

Problem 9.4.4.

$$H_0 : p_S = p_{NS}$$
$$H_1 : p_S \neq p_{NS}$$

$$p_e = \frac{x+y}{n+m} \qquad \text{Theorem 9.4.1 (pg 469)}$$
$$= \frac{53+705}{91+1117} = 0.6275$$
$$z = \frac{\frac{x}{n} - \frac{y}{m}}{\sqrt{\frac{p_e(1-p_e)}{n} - \frac{p_e(1-p_e)}{m}}} \qquad \text{Theorem 9.4.1 (pg 469)}$$
$$= \frac{\frac{53}{91} - \frac{705}{1117}}{\sqrt{\frac{0.63(1-0.63)}{91} - \frac{0.63(1-0.63)}{1117}}} \quad = -0.9246$$

By Theorem 9.4.1 (pg 469) we may reject H_0 if z is either $\leq -z_{\alpha/2}$ or $\geq z_{\alpha/2}$ Since z=-0.9246 satisfies neither of these conditions ($z_{0.005} = 2.575$) we can not reject the null hypothesis. Thus, it appears that there is not a significant difference in the probability of finding a UFO on the ground whether your in Spain or not. So basically this is guaranteed proof that aliens are real and they ARE AMONG US.

Problem 9.5.6. A $100(1-\alpha)\%$ confidence interval for the variance ratio , σ_X^2/σ_Y^2 is given by:

$$\begin{aligned}
&\frac{s_x^2}{s_Y^2}F_{\alpha/2,m-1,n-1}, \frac{s_X^2}{s_Y^2}F_{1-\alpha/2,m-1,n-1} && \text{Theorem 9.5.2}\\
&\frac{0.0002103}{0.0000955}F_{0.025,9,8}, \frac{0.0002103}{0.0000955}F_{0.975,9,8}\\
&\frac{0.0002103}{0.0000955}0.226, \frac{0.0002103}{0.0000955}4.43\\
&(0.498, 9.755)
\end{aligned}$$

No conclusions can be drawn about the *true* variances being different since the ratio $\sigma_X^2/\sigma_Y^2 = 1$ is contained in the confidence interval. Therefore, the case study was within reason to assume that the variances are equal.

Problem 10.2.6.

$$\begin{aligned}
P_{X_1,\ldots,X_i}(k_1,\cdots,k_t) &= \frac{n!}{k_1!\cdots k_t!}p_1^{k_1}\cdots p_t^{k_t} \qquad \text{Theorem 10.2.1 (pg 485)}\\
&= \frac{5!}{(2!)(2!)(1!)(0!)}(0.713)^2(0.270)^2(0.01)^1(0.007)^0\\
&= 0.01112
\end{aligned}$$

Problem 10.3.6.

$$\begin{aligned}
&H_0 : p_1 = \frac{1291.1}{5139} = 0.2514, && p_2 = 0.749\\
&H_1 : p_1 \neq 0.2514, && p_2 \neq 0.749\\
&d = \Sigma_{i=1}^t \frac{(k_i - np_i)^2}{np_{i_0}} && \geq \chi^2_{1-\alpha,t-1} && \text{Theorem 10.3.1 (pg 489)}\\
&= \frac{(1383 - 5139(0.2451))^2}{5139(0.2514)} + \frac{(3756 - 5139(0.749))^2}{5139(0.749)} && \geq \chi^2_{0.95,1}\\
&= 11.97 + 2.25 = 14.04 && \geq 3.841
\end{aligned}$$

Since d satisfies the rejection criteria in Theorem 10.3.1 we can reject the null hypothesis.

Problem 10.4.10. To claim that a Poisson pdf can model these data is to say that:

$$H_0 : \text{P(i turnovers happen in a given game)} = e^{-\lambda}\lambda^i/i!, i = 0, 1, 2, ...$$

where λ is the expected number of turnovers in a given game:

$$\lambda_e = \frac{800}{440} = 1.82.$$

The estimated frequencies are calculated by: $(440e^{-1.82}(1.82)^i/)i!$ The fourth column of table one lists the entire set of $n\hat{p}'_{i_0}s$. (Note that for the 6+ row, the probability is forced such that the total probability is 1.) Now even though it is close, to comply with the $n\hat{p}_{i_0} \geq 5$ requirement dictated by theorem 10.4.1 (pg 499), we must combine the last 2 row into a "5+" category. This is demonstrated in table 2. Now using Theorem 10.4.1.b, the test statistic is defined as 3.52 (calculations shown in the 4th column of table 2):

$$d_1 = \Sigma_{i=1}^t \frac{(k_i - np_i)^2}{np_{i_0}}$$

With 6 classes and one estimated parameter, the number of degrees of freedom associated with d_1 is 4(=6-1-1). In orer to test H_0 at the $\alpha = 0.05$ level of significance, we should reject H_0 is

$$d_1 \geq \chi^2_{0.95,4}$$

Since $3.52 < 9.488$ we can not reject the null hypothesis. Thus, it appears that the distribution of turnovers does in fact follow a Poisson distribution.

Problem 10.5.6. Well I wasn't about to do this by hand! I couldn't find an R package that does it so I used excel! Figure 1 has the R_iC_j matrix and Figure 2 shows the d_2 calculation. With r = 3 and c =3 the number of degrees of freedom associated with the test statistic is 4. By Theorem 10.5.1 (pg 510)H_0 should be rejected if $d_2 \geq \chi^2_{0.95,4} = 9.488$. Hence, we can not reject the null hypothesis. It appears that the blood pressure of the father and child are dependent.

Problem Randomization Test. I got a p-value of 0.0013. This feels like not enough of an answer. So here is my R code :).

1 Tables

i	k	ik	p	np
0	75	0	0.16	71.29
1	125	125	0.29	129.75
2	126	252	0.27	118.07
3	60	180	0.16	71.63
4	34	136	0.07	32.59
5	13	65	0.03	11.86
6	7	42	0.01	4.80
	440	800	1	440

Table 1:

i	k	np	d
0	75	71.29	0.19
1	125	129.75	0.17
2	126	118.07	0.53
3	60	71.63	1.89
4	34	32.59	0.06
5	20	16.66	0.67
			3.52

Table 2:

	lower	middle	upper
lower	11.12	11.48	10.40
middle	10.45	10.78	9.77
upper	9.43	9.74	8.83

Figure 1: R_iC_j table

	lower	middle	upper	Total
lower	0.75	0.02	0.55	1.32
middle	0.03	0.00	0.06	0.09
upper	1.25	0.01	1.14	2.40
Total	2.03	0.03	1.76	3.81

Figure 2: d_1's table