Problem Set 8 *

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*Problems:5.2,5.4,5.6,5.7,5.12,5.16,5.17,5.20,5.27

Problem 11.7. $S = 19, K = 20, c = 1, r = 0.04.$

$$\begin{aligned} c + Ke^{-rT} &= p + S_0 && \text{Put-Call parity} \\ 1 + 20e^{0.04*(3/12)} &= p + 19 \\ p &= 1.8 \end{aligned}$$

Problem 11.14. K=30, c=2, $S_0 = 29$ D=0.5, r=0.1, p = ?.

$$\begin{aligned} c + Ke^{-rT} + D &= p + S_0 && \text{Put Call Parity} \\ 2 + 30e^{0.1*(6/12)} + 0.5e^{-0.1*(2/12)} + 0.5e^{-0.1*(5/12)} - 29 &= p \\ p &= 2.51 \end{aligned}$$

Problem 11.15. So the price of the put would be overvalued so we can buy the call and short the stock and the put: This would give a positive upfront gain of $(3 - 2 + 29) = 30$. When invested at the risk free rate this grows to $30e^{.1*3/12} = 30.76$. No matter what happens the investor will buy a stock for \$30 at time T and have a profit of \$0.76.

Problem 11.18. Based on the hint in the book, I'll consider:

Portfolio A:One European Call and K in cash

Portfolio B:One American Put and one share of the stock

The value of portfolio A at time T is: $max(S_T - K, 0) + Ke^{rt} \rightarrow max(S_T, K) - K + Ke^{rt}$. The value of portfolio B at time T is a little more compliacted because of the posibility of early exercise. To see this first consider that for a similar portfolio with a european put instead, the value is $max(K - S_T, 0) + S_T$ But since there is the posibility of early exercise you have to forward the profit from selling the put: $max([K - S_T]e^{r(T-t)}, 0) + S_T \rightarrow max(K, S_T)e^{r(T-t)}$ It is pretty clear from these two valuation to see that portfolio A is worth more than portfolio B. So.

$$
\begin{aligned}
c + K &\geq P + S_0 \\
c - P &\geq S_0 - K \\
C - P &\geq S_0 - K \qquad \text{Since c=C pg 243}
\end{aligned}
$$

To continue we can use the put call parity

$$
\begin{aligned}
c + Ke^{-rT} &= p + S_0 \qquad \text{Eqn. 11.6 pg 239} \\
c - p &= S_0 - Ke^{-rT} \\
C - p &= S_0 - Ke^{-rT} \quad \text{see above} \\
C - P &\leq S_0 - Ke^{-rT} \quad p \leq P \text{ pg 246} \\
S_0 - K \leq C - P &\leq S_0 - Ke^{-rT}
\end{aligned}
$$

Problem 11.19. Based on the hint from the book, I consider:

Portfolio A:One European Call and (K+D) in cash

Value A: $max(S_T - K, 0) + (K + D)e^{rT} \to max(S_T, K) - K + (K + D)e^{rT}$

Portfolio B:One American Put and one share of the stock

Value B:$max(K - S_T, 0)e^{r(T-t)} + S_T + De^{rT}$

This is similar to 11.18 so I won't go into it as much. But similarly we can see the $A \geq B$ so

$$c + K + D \geq P + S_0$$
$$C - P \geq S_0 - K - D$$

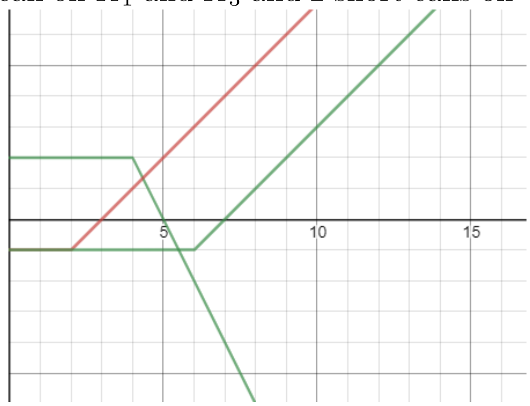
From the put call parity for options with dividends:

$$c + D + Ke^{-rT} = p + S_0$$
$$C - P + D \leq S_0 - Ke^{-rT}$$

Since dividends decrease C and increase P the result holds that

$$C - P \leq S_0 - Ke^{-rT}$$

Problem 11.25. With $K_1 < K_2 < K_3$ and $K_3 - K_2 = K_2 - K_1$. Using the hint from the book we consider 1 long call on K_1 and K_3 and 2 short calls on K_2. An example graph is shown below.



From the figure we can see easy that $c_2 = 0.5(c_1 + c_3)$. So let us consider this. First note that $c_1 = max(S - K_1, 0)$, and $c_2 = 2 \cdot max(K_2 - S, 0)$, and finally, $c_3 = max(S - K_3, 0)$. One quick way to see this is to put the three options in one portfolio and see that:

Case 1:$S \leq K_1 \implies$ All worth 0

Case 2:$K_1 < S \leq K_2 \implies c_1 > 0 \& c_2 = 0$

Case 3:$K_1 < K_2 \leq S < K_3 \implies (S - K_1) - 2(S - K_2) = 2K_2 - S - K \geq 0$

Case 4:$K_3 < S \implies (S - K_1) - 2(S - K_2) + (S - K_3) = 2K_2 - K_1 = K_3 \geq 0$

Since: $c_1 - 2c_2 + c_3 \geq 0 \implies 0.5(c_1 + c_3) \geq c_2$