Homework 5

625.433

1. (25 pts.) Apply the inverse-transform method to generate random variables from a *Laplace distribution* which has pdf

$$f(x) = \frac{\lambda}{2} \exp\{-\lambda|x-\theta|\} \quad -\infty < x < \infty.$$

(let $\lambda = 2$ and $\theta = 2$)

2. (25 pts.) Let the random variable X have the pdf

$$f(x) = \begin{cases} \frac{1}{4} & 0 < x < 1 \\ x - \frac{3}{4} & 1 \le x \le 2 \end{cases}$$

Generate a random variable from $f(x)$ using

(a) the inverse-transform method

(b) the accept-reject method, using the proposal density $g(x) = \frac{1}{2} \quad 0 \le x \le 2$.

3. (25 pts.) Let the random variable X have pdf

$$f(x) = \begin{cases} \frac{1}{2}x & 0 < x < 1 \\ \frac{1}{2} & 1 \le x \le \frac{5}{2} \end{cases}$$

Generate a random variable from $f(x)$ using

(a) The inverse-transform method

(b) The accept-reject method, using the proposal density

$$g(x) = \frac{8}{25}x \quad 0 \le x \le \frac{5}{2}.$$

4. (25 pts.) Apply the inverse-transform method to generate a random variable from the discrete uniform distribution with pdf

$$f(x) = \begin{cases} \frac{1}{n+1} & x = 0, 1, \ldots, n \\ 0 & \text{otherwise} \end{cases}$$

5. (**Bonus** of 10 pts.) The adaptive accept-reject algorithm required that the density being sampled from is log-concave. Let's assume you want to do an adaptive accept-reject algorithm for a density that is not log-concave. Is there any way you could imagine you would adjust/correct for what was discussed in class? Be as specific as possible, and draw a picture if you have to.