Johns Hopkins Engineering

Mathematical Finance

Module 1	Lecture 3

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Internal Rate of Return

Theorem

Consider a cash flow stream $(x_0, x_1, \cdots, x_n)$ such that $x_0 < 0$ and $x_k \geq 0$ for k =1,2,…,n, with at least one term being strictly positive.

Then there exists a unique positive root to the equation

$$0 = x_0 + x_1 c + x_2 c^2 + \cdots + x_n c^n$$

Furthermore, if $\sum_{k=0}^{n} x_k > 0$ then the corresponding internal rate of return r=1/c-1 is positive.

Proof

The proof is based on an application of the intermediate value theorem to show the existence of a solution. The uniqueness is obtained due to the fact that the underlying function is strictly increasing.

Evaluation Criteria

How can we compare two cash flow streams?

The main methods used are: the Net Present Value and the IRR

- Net Present Value

Given two cash flow streams, the cash flow with the higher Net Present value represents the more desirable investment.

Note that in this case all cash outlays both negative an positive associated with the investment must be included.

Example

Consider the cash flow streams with $r = 0.05$

a. *(-2,1,1,3)* the NPV of this stream is NPV = 2.409

b. *(-2, 0, 2,3)* the NPV of this stream is NPV = 2.4056

Therefore, according to this criterion the investment a. is the better.

Evaluation Criteria

• Internal Rate of Return (IRR)

The IRR can also be used to rank cash flow streams. The stream with the higher IRR is therefore consider the better investment.

Example

Consider the cash streams used in the previous example a. *(-2,1,1,3)* and b. *(-2,0,2,3).*

1. For a. the IRR is obtained by solving the polynomial equation $-2+c+c^2+3c^3=0$
   The solution is *c= 0.666*, thus the IRR is *r = 1/c-1 = 0.5*
2. For b. the IRR is obtained by solving the equation
   $$-2+2c^2+3c^3=0$$
   the solution is c= 0.69875 and the IRR is r =1/ c-1=0.4311.

Finally, according to IRR the better investment is a.

Application

Net Flows

In cash flow analysis, it is crucial to use the net income minus expenses in order to have a clear view of the investment.

Example: Simplico gold mine

Assume that you want to lease a gold mine for a period of 5 years. Gold can be extracted at a rate of up to 20000 ounces per year and at a cost of $250 per ounce. This cost is the total cost of mining and refining exclusive of the cost of the lease.

Currently the market price of gold is $600 per ounce. The prevailing rate is $r = 6\%$ (and will remain constant for the next 5 years). What is the present value of the lease?

Applications

Solution:

The maximum profit we can get over one year of operation is

Profit = 20,000(600-250) =7,000,000 per year.

The present of the corresponding cash streams is

$$PV = \sum_{k=1}^{5} \frac{7{,}000{,}000}{(1.06)^k} = 29{,}486{,}520$$

Keep in mind that these profits occur at the end of each year, therefore at time t=0 the profit is zero.

Consequently the market value of this lease is 29,486,520.