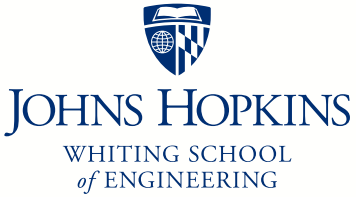
Johns Hopkins Engineering

Mathematical Finance

Module 1	Lecture 1



Basic Theory of Interest

Outline:

1. Simple Interest
2. Compound Interest
3. Present Value
4. Internal Rate of Return
5. Applications

Simple Interest

- An amount *A* invested at a simple annual rate *r*

will produce $A(1 + nr)$ after *n* years.

- In general, we have

$V(n) = A(1 + n*r)$, where r is the periodic rate, *n*

is the number of periods and $V(n)$ is the accumulated amount after *n* periods.

- Note that $V(n)$ is a linear function of *n*.

Compound Interest

- An amount *A* invested at a compound annual rate *r* will by a factor (1+r) each year. In fact the accumulated amount after *n* is given by:

$$V(n) = A(1+r)^n$$

- In general interest can be compounded semiannually , quarterly, daily, hourly, …etc. We have the following theorem.
- Theorem

  If the principal *A* is invested at a periodic rate r compounded *m* times in each period, then the amount *V* in the account after *n* periods is

$$V = A(1+r/m)^n$$

Compound Interest

- Effective Rate vs. Nominal Rate

The effective is the yearly rate that will produce the same result after 1 year without compounding.

Example

For an annual rate of 6% compounded daily,

$$\left(1+\frac{0.06}{365}\right)^{365} = 1.0618 = (1+0.0618)$$

the effective rate is 6.18% and the nominal rate is 6%.

In general for a nominal *r* compounded m times in a year the effective *r'* rate is

$$r' = \left(1+\frac{r}{m}\right)^{m} - 1$$

Continuous Compounding

- Recall

$$\lim_{m\to\infty}\left(1+\frac{r}{m}\right)^{m}=e^{r}$$

- Let the principal P be invested in an account with annual rate r compounded m times in a year. After t years the account will produce

$$A(t)=P\left(1+\frac{r}{m}\right)^{mt}$$

- Continuous compounding means that we let m goes to infinity, we obtain

$$A(t)=P\times\lim_{m\to\infty}\left(1+\frac{r}{m}\right)^{mt}=Pe^{rt}$$