Problem Set 6

Ian McGroarty

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Problem 1

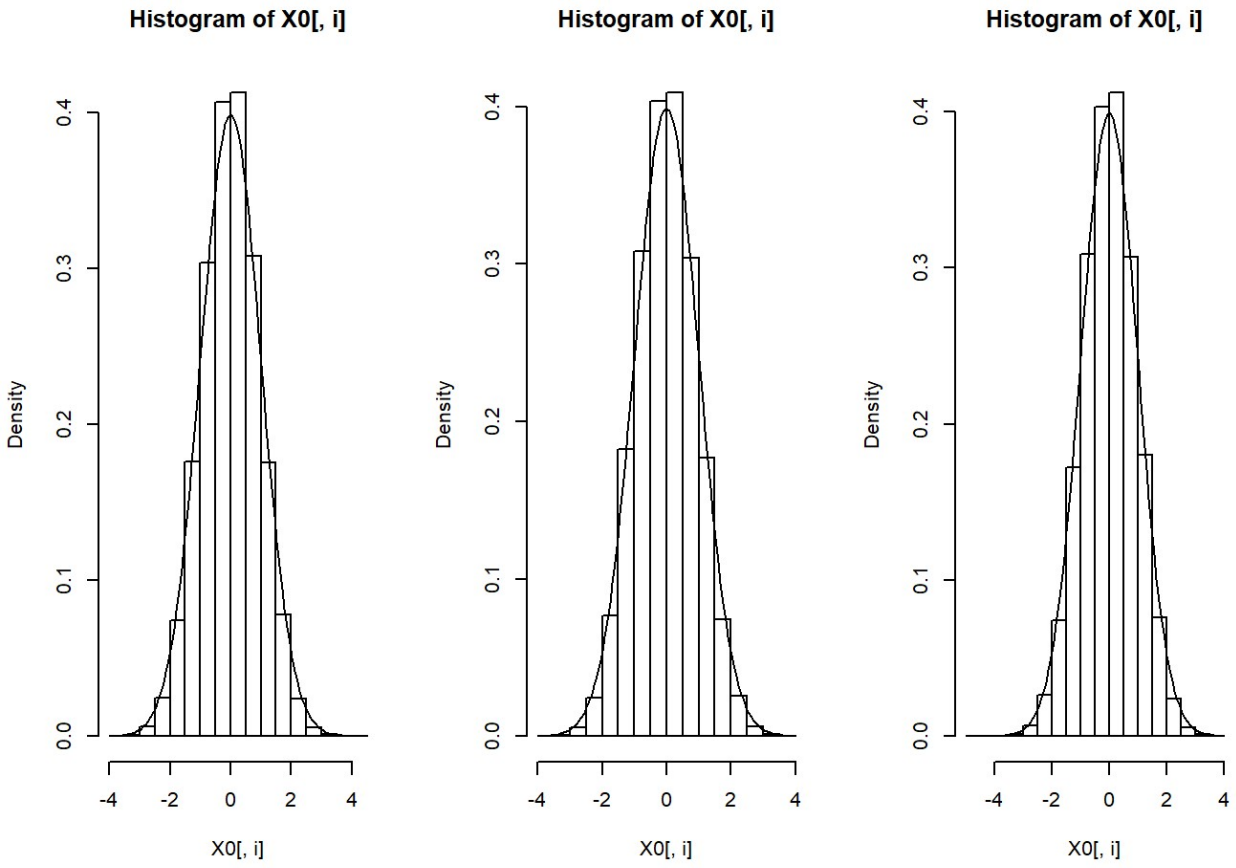
(a)

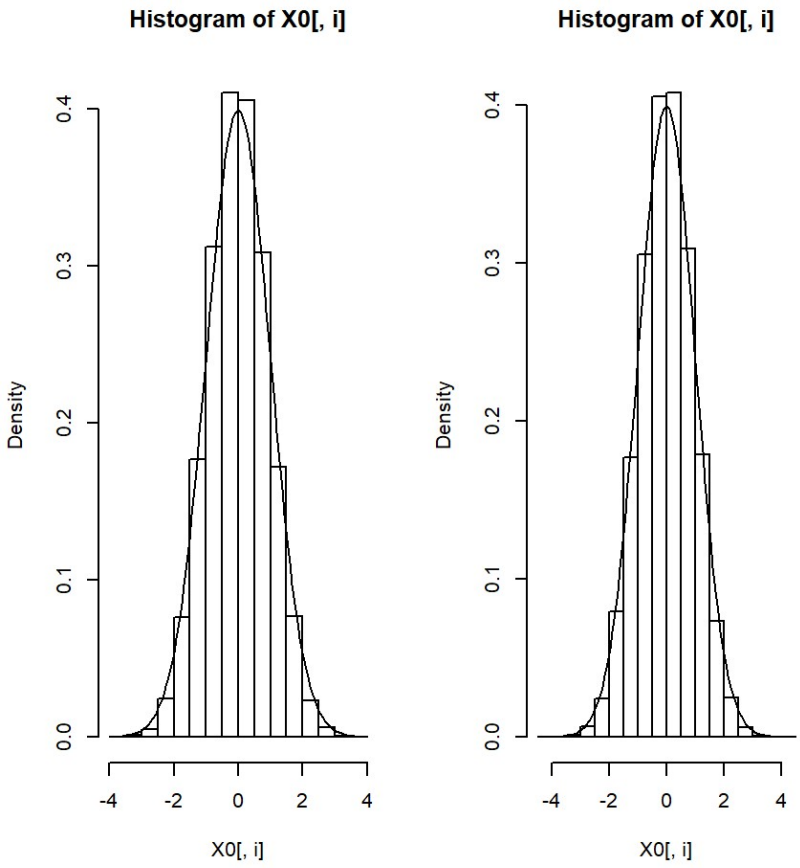
Write a Gibbs sampler using the conditional distributions provided in Example 7.4. Run your R code for p=5 and $\rho = 0.25$. Verify that the marginals are all $N(, 01)$ NOTE: Okay the 500 iterations just says system.time = 0 so I'm bumping it up to 50000 so I can compare.

```
# Set up the parameters as in the question
  Nsim <- 50000
  p <- 5
  r <- 0.25

# Get the starting distributions

  ## X0 should have p samples from the normal distribution
    X0 <- rnorm(p)
  ## The first x should have the save values as X0
    XI <- X0

# Run the sampling
  system.time( for (t in 2:Nsim){
     for (i in 1:p) {
      # Get X bar from the pervious iteration
        avgx <- sum(XI[i-1])/(p-1)
      # Gather the current X with the formuala from 7.4
        XI[i] <- rnorm(1,(p-1)*r*avgx/(1+(p-2)*r),sqrt((1+(p-2)*r-(p-1)*r^2)/(1+(p-2)*
r)))
     }
     X0 <- rbind(X0,XI)
   })
```

```
##    user  system elapsed
##   53.62    2.52   56.15
```

```
# Plots
  par(mfrow=c(1,3))
  for (i in 1:p){
    hist(X0[,i],freq=FALSE)
    curve(dnorm(x),add=TRUE)
  }
```





(b)

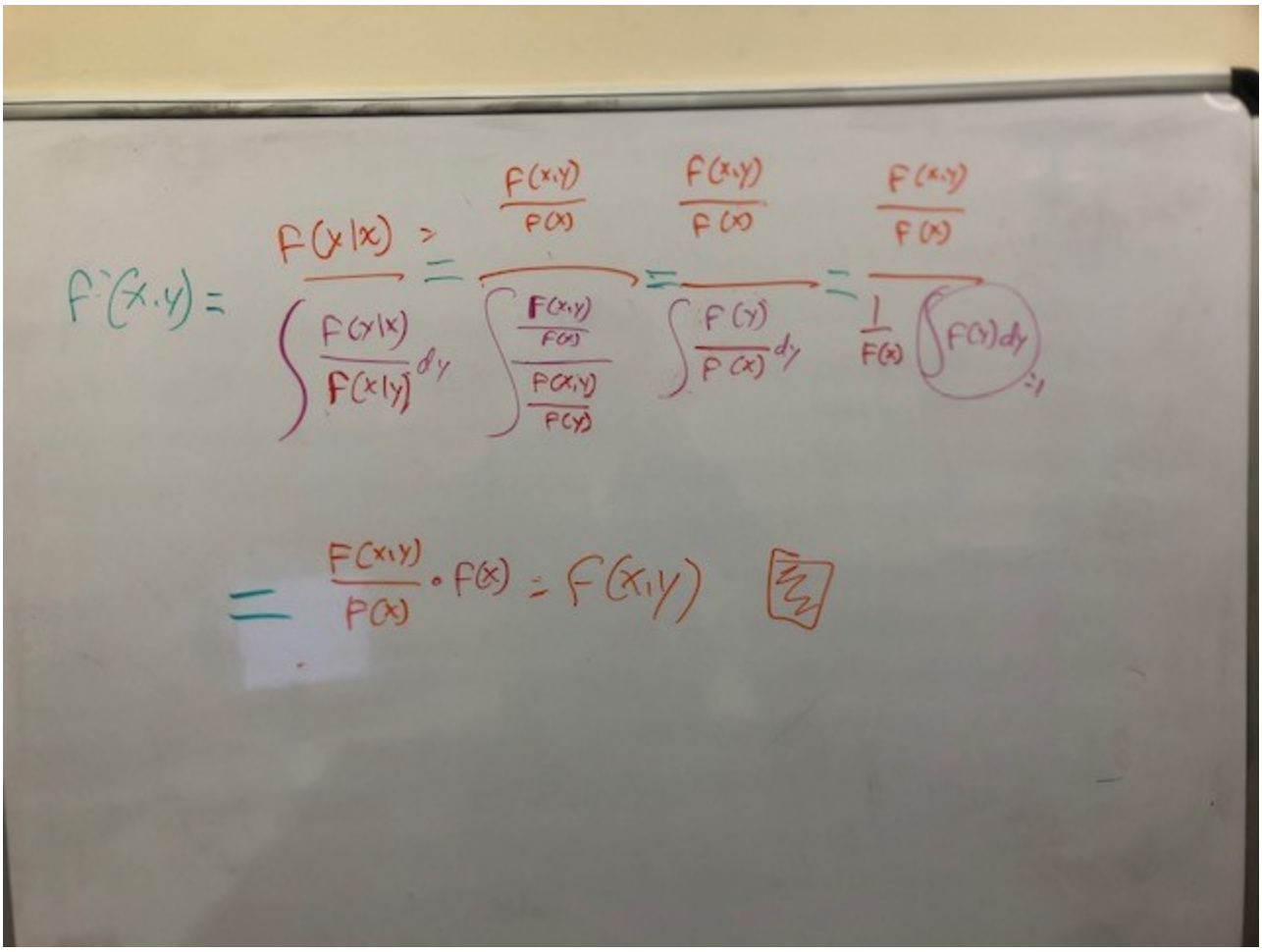
Compare your algorithm using 500 Iterations with rnorm described in 2.2.1 in terms of execution time. Okay the 500 iterations just says system.time = 0 so I'm bumping it up to 50000. It is very clearly way faster.

```
## I basically just used  the example from
  # https://www.rdocumentation.org/packages/lgarch/versions/0.6-2/topics/rmnorm
system.time(
x <- rmnorm(n=Nsim,varcov = diag(c(1,1,1,1,1)))
)
```

```
##    user  system elapsed
##    0.22    0.05    0.04
```

Problem 2

My latex is such that I'm not sure I could make this proof easy to read (too many fractions of fractions). So I've taken a picture of my proof on my white board. I hope that will suffice.


f(x,y) = f(y|x) = F(x,y)/F(x) = F(x,y)/F(x) = F(x,y)/F(x)
∫ F(y|x)/F(x|y) dy
∫ F(x,y)/F(x) / F(x,y)/F(y)
∫ F(y)/F(x) dy
1/F(x) ∫ F(y) dy = 1
= F(x,y)/F(x) · F(x) = f(x,y)

text