Homework 12

550.433

1. (50 pts.) In class I simulated a solution to the Black-Scholes model (using the exact solution) and compared it to the solution that I got using the Euler-Maruyama approximation. Do the same thing for the Ornstein-Uhlenbrek process. Remember that stochastic differential equation was written as

$$dX(t) = \theta(\mu - X(t))dt + \sigma dW(t),$$

and the analytical solution is

$$X(t) = X(0)e^{-\theta t} + \mu(1 - e^{-\theta t}) + \int_0^t \sigma e^{\theta(s-t)} dW(s).$$

By the way, convenient values of the parameters are $\theta = 1$, $\mu = 20$, and $\sigma = 10$.

2. (30 pts.) Illustrate through a simulated `R` example the difference between the Itô integral and the Stratonovich integral. That is, find some value of $h(t)$ and simulated process $W(t)$ so that calculating the integral $\int_0^T h(t)dW(t)$ is considerably different for the two calculations.

3. (20 pts.) The `R` function that I've written entitled `blackScholesConvergence` calculates the average difference in the EM solution to the truth at the final time point. This average is calculated over `nmbSimulation` simulations of the function (note that `nmbSimulations` is an input to this function). The other inputs to the function include T (the total time), N (the number of time steps), λ, and μ. Run this code with a variety of values of T and N, and illustrate the approximation given in (5.4) of the assigned paper.

4. (Extra Credit: 20 pts.). Illustrate through simulation **Situation 2** discussed in the notes.