Problem Set 9 *

Ian McGroarty

Course Number: 625.641

July 30, 2019

*Problems:2,4,6,8,18

Problem 2 (Proportional Carrying Charges). Suppose that a forward contract on an asset is written at time zero and there are M periods Carrying charge is $qS(k)$.

Period	Quantity	Carrying Cost
0	x_0	$-S(0)$
1	$x_0 - qx_0 = x_0(1-q)$	$-(S(0) + qS(0) = S(1-q)$
2	$x_0(1-q)^2$	$S(1-q)^2$
$\vdots$	$\vdots$	$\vdots$
M	$x_0(1-q)^M$	$S(0)$ - Repay

$$\begin{aligned}
Profit \ = \pi = F \cdot Q - \frac{S}{d(0,M)} & \quad \text{Profit function} \\
F \cdot Q = \frac{S}{d(0,M)} & \quad \text{Set profit to zero} \\
F \cdot x_0(1-q)^M = \frac{S}{d(0,M)} & \quad \text{Substitute quantity at period M} \\
F = \frac{S}{(1-q)^M \cdot d(0,M)} & \quad \text{Set } x_0 = 1 \text{ and solve}
\end{aligned}$$

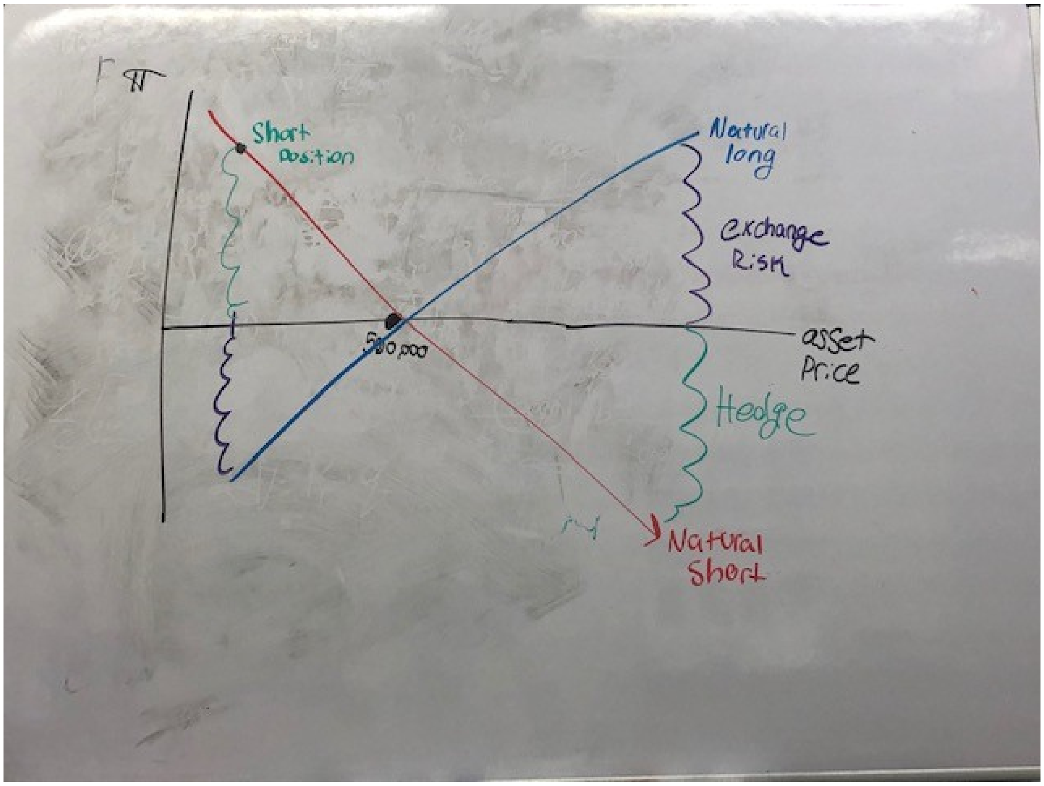
Problem 4 (Continuous time Carrying Charge). Suppose continuous compounding framework is used with a fixed interest rate r.

So the interest rate will be the discount factor for continuous time (page 75) $d(0, M) = e^{-rM}$. We see that the relation between discrete time $(1 + r)^M$ and continuous time e^{rM} that quantity will be e^{qm} Plugging these values into the result from problem 2 we get:

$$F = \frac{S}{e^{qM} \cdot e^{-rM}} = \frac{S}{e^{(q-r)M}} = Se^{(r-q)M}$$

Problem 6 (Foreign Currency Alternative. he US electronics company received and order to sell equipment to a German customer in 90 days. Price specified as 500,000 D-Mark (wow this book is old) . To understand how to hedge the risk we must first understand the risk itself. Since the US firm will want to convert the payment into dollars, the true price that the US is receiving is 500,000 E where is is the exchange rate. If the value of the D-Mark goes down relative to the dollar, then the US will loose value on the transaction. If the value of the D-Mark goes up relative to the dollar then the US will make money on the transaction. Thus, the US is the 'natural long' in this scenario (see the graph below), with the asset D-Mark. By shorting the D-Mark the US hedges against risk. If the D-Mark depreciates relative to the dollar, the short position will cover the losses from the transaction. Conversely, if the D-Mark appreciates relative to the dollar, the short position will lose money and thereby offset the gains of the transaction.

As the problem states, there is another way to offset the exchange rate risk. By borrowing $500,000/(1 + r_G)$ D-Mark and converting the money to USD and then into treasury bills. The firm effectively removes their dependency on the D-Mark since they paid themselves. If the D-Mark depreciates then the value of the loan also depreciates and vice versa. Note that they don't need to borrow the full 500,000 D-Mark since this reflects the present value of a bond with a face value of 500,000.


π
Short position
Natural long
exchange Risk
500,000
asset Price
Hedge
Natural Short

Problem 8 (Simple Formula). Derive the formula (10.6) b converting a cash flow of a bond to that of the fixed portion of the swap.

Solution Note the the discount rate $d(0, M) = \frac{1}{(1+\lambda/m)^n}$.

$$
\begin{aligned}
P &= \frac{F}{(1+\lambda/m)^n} + \Sigma\frac{C/m}{(1+\lambda/m)^n} && \text{(3.1) page 53} \\
P &= F(d(0,M)) + \Sigma(C/m)(d(0,M)) && \text{substitute} \\
P - F(d(0,M)) &= \Sigma(C/m)(d(0,M)) \\
P - F(d(0,M)) &= C \cdot \Sigma(d(0,M)) && \text{Since C is constant} \\
\frac{1}{C}(P - F(d(0,M)) &= \Sigma(d(0,M)) \\
\frac{X}{C}(P - F(d(0,M)) &= \Sigma(d(0,M))X && \text{Multiply by x?}
\end{aligned}
$$

Then we have $P = B(M, C)$ and $F = 100$ as stated on page 275.

Problem 18 (Symmetric Probability). uppose the wealth that is to be received at a time T in the future has the following form. Where X is a random variable with $E(x) = 0$:

(a) Show that the optimal choice is $h = 0$.

$$
\begin{aligned}
W &= a + hx + cx^2 \\
\frac{d}{dx}W &= \frac{d}{dx}a + hx + cx^2 && \text{take the derivative w respect to x} \\
0 &= h + 2cx && \text{Set to 0} \\
h &= 2cx
\end{aligned}
$$

It is clear that as $x \to 0 \implies h \to 0$.

(b) Apply this result to the corn farm problem

$$
\begin{aligned}
R &= 10C - \frac{C^2}{1000} + \frac{\bar{C} - C}{1000}h && \text{page 29} \\
\frac{d}{dx}R &= \frac{d}{dx}10C - \frac{C^2}{1000} + \frac{\bar{C} - C}{1000}h && \text{take the derivative w respect to C} \\
0 &= 10 - \frac{2C}{1000} + \frac{h}{1000} && \text{Set to 0}
\end{aligned}
$$

$$10000 - 2C = h$$

$$10000 - 2(3000) = h = 4000 \qquad \bar{C} = 3000$$