Chapter 14: Wiener Processes and Ito's Lemma

1. A variable x starts at 10 and follows the generalized Wiener process

   $$dx = a\,dt + b\,dz$$

   where time is measured in years. If a = 2 and b =3 what is the expected value after 3 years?

   A. 12

   B. 14

   C. 16

   D. 18

   Answer: C

The drift is 2 per year and so the expected increase over three years is 2×3 = 6 and the expected value at the end of 3 years is 10+6 = 16.

2. A variable x starts at 10 and follows the generalized Wiener process

   $$dx = a\,dt + b\,dz$$

   where time is measured in years. If a = 3 and b =4 what is the standard deviation of the value in 4 years?

   A. 4

   B. 8

   C. 12

   D. 16

   Answer: B

The variance per year is 4^2 or 16. The variance over four years is 16×4 = 64. The standard deviation is $\sqrt{64} = 8$.

3. A variable x starts at 10 and follows the generalized Wiener process

   $$dx = a\,dt + b\,dz$$

   If a = 3 and b =4 what is the standard deviation of the value in three months?

   A. 1

   B. 2

   C. 3

   D. 4

   Answer: B

The variance per year is 4^2 or 16. The variance over three months is 16×0.25 = 4. The standard deviation is $\sqrt{4} = 2$.

4. The variance of a Wiener process in time t is

   A. t

B. t squared
C. the square root of t
D. t to the power of 4

Answer: A

The variance of a Wiener process is 1 per unit time or t in time t.

5. The process followed by a variable X is

$$dX = mX\,dt + sX\,dz$$

What is the coefficient of dz in the process for the square of X.

A. sX
B. sX^2
C. $2sX^2$
D. msX

Answer: C

From Ito's lemma, the coefficient of dz is $sX\,\partial f/\partial X$ where $f = X^2$. Because $\partial f/\partial X = 2X$, the coefficient of dz is $2sX^2$.

6. The process followed by a variable X is

$$dX = mX\,dt + sX\,dz$$

What is the coefficient of dt in the process for the square of X.

A. $2mX^2+s^2X^2$
B. $2mX^2$
C. $mX^2+2s^2X^2$
D. $mX^2+s^2X^2$

Answer: A

From Ito's lemma, the coefficient of dt is

$$mX\frac{\partial f}{\partial X}+\frac{1}{2}s^2X^2\frac{\partial^2 f}{\partial X^2}$$

where $f = X^2$. Because $\partial f/\partial X = 2X$ and $\partial^2 f/\partial X^2 = 2$ the coefficient of dt is $2mX^2+s^2X^2$

7. Which of the following is true when the stock price follows geometric Brownian motion

A. The future stock price has a normal distribution
B. The future stock price has a lognormal distribution
C. The future stock price has geometric distribution

D. The future stock price has a truncated normal distribution

Answer: B

Ito's lemma show that the log of the stock price follows a generalized Wiener process. This means that the log of the stock price is normally distributed so that the stock price is lognormally distributed.

8. If a stock price follows a Markov process which of the following could be true
   A. Whenever the stock price has gone up for four successive days it has a 70% chance of going up on the fifth day.
   B. Whenever the stock price has gone up for four successive days there is almost certain to be a correction on the fifth day.
   C. The way the stock price moves on a day is unaffected by how it moved on the previous four days.
   D. Bad years for stock price returns are usually followed by good years.

   Answer: C

A Markov process is a particular type of stochastic process where only the current value of a variable is relevant for predicting the future. Stock prices are usually assumed to follow Markov processes. This corresponds to a weak form market efficiency assumption.

9. A variable x starts at zero and follows the generalized Wiener process
   $$dx = a\,dt + b\,dz$$
   where time is measured in years. During the first two years a=3 and b=4. During the following three years a=6 and b=3. What is the expected value of the variable at the end of 5 years
   A. 16
   B. 20
   C. 24
   D. 30

   Answer: C

During the first two years, the drift per year is 3 and so the total drift is 3×2 or 6. During the next three years, the drift per year is 6 and the total drift is 6×3 = 18. The total drift over the five years is 6+18 =24. Given that the variable starts at zero, its expected value at the end of the five years is therefore 24.

10. A variable x starts at zero and follows the generalized Wiener process
    $$dx = a\,dt + b\,dz$$
    where time is measured in years. During the first two years a=3 and b=4. During the following three years a=6 and b=3. What the standard deviation of the value of the variable at the end of 5 years
    A. 6.2

B. 6.7
C. 7.2
D. 7.7

Answer: D

The variance per year for the first two years is 4^2 or 16. The variance per year for the next three years is 3^2 or 9. The total variance of the change over five years is 2×16+3×9= 59. The standard deviation of the value of the variable at the end of the five years is therefore $\sqrt{59} = 7.7$

11. If a variable x follows the process $dx = b\ dz$ where dz is a Wiener process, which of the following is the process followed by y = exp(x).
    A. $dy = by\ dz$
    B. $dy = 0.5b^2y\ dt+by\ dz$
    C. $dy = (y+0.5b^2y)\ dt+by\ dz$
    D. $dy = 0.5b^2y\ dt+b\ dz$

    Answer: B

Ito's lemma shows that the process followed by y is $dy = 0.5b^2\exp(x)\ dt + b\exp(x)\ dz$. Substituting $y = \exp(x)$ we get the answer in B.

12. If the risk-free rate is r and price of a nondividend paying stock grows at rate m with volatility s, at what rate does a forward price of the stock grow for a forward contract maturing at a future time T.
    A. m
    B. $m-s^2/2$
    C. $m-r$
    D. $r-s^2/2$

    Answer: C

This is the application of Ito's lemma in Section 14.6.

13. When a stock price, S, follows geometric Brownian motion with mean return m and volatility s what is the process follows by X where X = ln S.
   A. $dX = m\ dt + s\ dz$
   B. $dX = (m-r)\ dt + s\ dz$
   C. $dX = (m - s^2)\ dt + s\ dz$
   D. $dX = (m - s^2/2)\ dt + s\ dz$

   Answer: D

This is the example in Section 14.7

14. Which of the following gives a random sample from a standard normal distribution in Excel?
   A. =NORMSINV()
   B. =NORMSINV(RAND())
   C. =RND(NORMSINV())
   D. =RAND()

   Answer: B

The correct instruction in Excel is =NORMSINV(RAND())

15. Which of the following defines an Ito process?
   A. A process where the drift is non-constant and can be stochastic
   B. A process where the coefficient of dz is non-constant and can be stochastic
   C. A process where either the drift or the coefficient of dz or both are non-constant and can be stochastic
   D. A process where proportional changes follow a generalized Wiener process

   Answer: C

In a generalized Wiener process the drift and coefficient of dz are both constant. In an Ito process they are not both constant .

16. A stock price is $20. It has an expected return of 12% and a volatility of 25%. What is the standard deviation of the change in the price in one day. (For this question assume that there are 365 days in the year.)
   A. $0.20
   B. $0.23
   C. $0.26
   D. $0.29

Answer: C

The standard deviation of the change in one day is $20 \times 0.25 \times \sqrt{1/365} = \$0.26$

17. A stock price is \$20. It has an expected return of 12% and a volatility of 25%. What is the stock price that has a 2.5% chance of being exceeded in one day? (For this question assume that there are 365 days in the year.)

A. \$20.41
B. \$20.51
C. \$20.61
D. \$20.71

Answer: B

From the previous question the standard deviation of the change in one day is \$0.26. There is a 2.5% chance that the stock price will increase by more than 1.96 standard deviations. The answer is therefore 20+1.96×0.26 = \$20.51. The expected return in one day is small and can be ignored.

18. Which of the following is NOT a property of a Wiener process?

A. The change during a short period of time dt has a variance dt
B. The changes in two different short periods of time are independent
C. The mean change in any time period is zero
D. The standard deviation over two consecutive time periods is the sum of the standard deviations over each of the periods

Answer D

Variances of Wiener processes are additive but standard deviations are not.

19. If e is a random sample from a standard normal distribution, which of the following is the change in a Wiener process in time dt .

A. e times the square root of dt
B. e times dt
C. dt times the square root of e
D. The square root of e times the square root of dt

Answer: A

The change is $e\sqrt{dt}$. This result is used when the process is simulated.

20. For what value of the correlation between two Wiener processes is the sum of the processes also a Wiener process?

A. 0.5

B. −0.5

C. 0

D. 1

Answer: B

The variance of each process is 1 per unit time. The variance of the sum is $1+1+2\rho$ where ρ is the correlation. This is 1 when $\rho=-0.5$.