

Chapter 13: Binomial Trees

1. The current price of a non-dividend-paying stock is \$30. Over the next six months it is expected to rise to \$36 or fall to \$26. Assume the risk-free rate is zero. An investor sells call options with a strike price of \$32. Which of the following hedges the position?
- A. Buy 0.6 shares for each call option sold
 - B. Buy 0.4 shares for each call option sold
 - C. Short 0.6 shares for each call option sold
 - D. Short 0.6 shares for each call option sold

Answer: B

The value of the option will be either \$4 or zero. If Δ is the position in the stock we require

$$36\Delta - 4 = 26\Delta$$

so that $\Delta = 0.4$. it follows that 0.4 shares should be purchased for each option sold.

2. The current price of a non-dividend-paying stock is \$30. Over the next six months it is expected to rise to \$36 or fall to \$26. Assume the risk-free rate is zero. What is the risk-neutral probability of that the stock price will be \$36?
- A. 0.6
 - B. 0.5
 - C. 0.4
 - D. 0.3

Answer: C

The formula for the risk-neutral probability of an up movement is

$$p = \frac{e^{rT} - d}{u - d}$$

In this case $u = 36/30$ or 1.2 and $d = 26/30 = 0.8667$. Also $r = 0$ and $T = 0.5$. The formula gives

$$p = (1 - 0.8667) / (1.2 - 0.8667) = 0.4.$$

3. The current price of a non-dividend-paying stock is \$30. Over the next six months it is expected to rise to \$36 or fall to \$26. Assume the risk-free rate is zero. An investor sells call options with a strike price of \$32. What is the value of each call option?
- A. \$1.6
 - B. \$2.0
 - C. \$2.4
 - D. \$3.0

Answer: A

The formula for the risk-neutral probability of an up movement is

$$p = \frac{e^{rT} - d}{u - d}$$

In this case $u = 36/30$ or 1.2 and $d = 26/30 = 0.8667$. Also $r = 0$ and $T = 0.5$. The formula gives

$$p=(1-0.8667)/(1.2-0.8667)=0.4.$$

The payoff from the call option is \$4 if there is an up movement and \$0 if there is a down movement. The value of the option is therefore $0.4 \times 4 + 0.6 \times 0 = \1.6 . (We do not do any discounting because the interest rate is zero.)

4. The current price of a non-dividend-paying stock is \$40. Over the next year it is expected to rise to \$42 or fall to \$37. An investor buys put options with a strike price of \$41. Which of the following is necessary to hedge the position?
- A. Buy 0.2 shares for each option purchased
 - B. Sell 0.2 shares for each option purchased
 - C. Buy 0.8 shares for each option purchased
 - D. Sell 0.8 shares for each option purchased

Answer: C

The payoff from the put option is zero if there is an up movement and 4 if there is a down movement. Suppose that the investor buys one put option and buys Δ shares. If there is an up movement the value of the portfolio is $\Delta \times 42$. If there is a down movement it is worth $\Delta \times 37 + 4$. These are equal when $37\Delta + 4 = 42\Delta$ or $\Delta = 0.8$. The investor should therefore buy 0.8 shares for each option purchased.

5. The current price of a non-dividend-paying stock is \$40. Over the next year it is expected to rise to \$42 or fall to \$37. An investor buys put options with a strike price of \$41. What is the value of each option? The risk-free interest rate is 2% per annum with continuous compounding.
- A. \$3.93
 - B. \$2.93
 - C. \$1.93
 - D. \$0.93

Answer: D

The formula for the risk-neutral probability of an up movement is

$$p = \frac{e^{rT} - d}{u - d}$$

In this case $r=0.02$, $T=1$, $u=42/40=1.05$ and $d=37/40=0.925$ so that $p=0.76$ and the value of the option is $(0.76 \times 0 + 0.24 \times 4)e^{-0.02 \times 1} = 0.93$

6. Which of the following describes how American options can be valued using a binomial tree?
- A. Check whether early exercise is optimal at all nodes where the option is in-the-money
 - B. Check whether early exercise is optimal at the final nodes
 - C. Check whether early exercise is optimal at the penultimate nodes and the final nodes
 - D. None of the above

Answer: A

For an American option we must check whether exercising is better than not exercising at each node where the option is in the money. (It is clearly not worth exercising when the option is out of the money)

7. In a binomial tree created to value an option on a stock, the expected return on stock is
- A. Zero
 - B. The return required by the market
 - C. The risk-free rate
 - D. It is impossible to know without more information

Answer: C

The expected return on the stock on the tree is the risk-free rate. This is an application of risk-neutral valuation.

8. In a binomial tree created to value an option on a stock, what is the expected return on the option?
- A. Zero
 - B. The return required by the market
 - C. The risk-free rate
 - D. It is impossible to know without more information

Answer: C

The expected return on the option on the tree is the risk-free rate. This is an application of risk-neutral valuation. The expected return on all assets in a risk-neutral world is the risk-free rate.

9. A stock is expected to return 10% when the risk-free rate is 4%. What is the correct discount rate to use for the expected payoff on an option in the real world?
- A. 4%
 - B. 10%
 - C. More than 10%
 - D. It could be more or less than 10%

Answer: D

The correct answer is D. There is no easy way of determining the correct discount rate for an option's expected payoff in the real world. For a call option the correct discount rate in the real world is often quite high and for a put option it is often quite low (even negative). The example in the text illustrates this.

10. Which of the following is true for a call option on a stock worth \$50
- A. As a stock's expected return increases the price of the option increases
 - B. As a stock's expected return increases the price of the option decreases
 - C. As a stock's expected return increases the price of the option might increase or decrease
 - D. As a stock's expected return increases the price of the option on the stock stays the

same

Answer: D

The option price when expressed in terms of the underlying stock price is independent of the return on the stock. To put this another way, everything relevant about the expected return is incorporated in the stock price.

11. Which of the following are NOT true

- A. Risk-neutral valuation and no-arbitrage arguments give the same option prices
- B. Risk-neutral valuation involves assuming that the expected return is the risk-free rate and then discounting expected payoffs at the risk-free rate
- C. A hedge set up to value an option does not need to be changed
- D. All of the above

Answer: C

The hedge set up to value an option needs to be changed as time passes. A and B are true.

12. The current price of a non-dividend paying stock is \$30. Use a two-step tree to value a European call option on the stock with a strike price of \$32 that expires in 6 months. Each step is 3 months, the risk free rate is 8% per annum with continuous compounding. What is the option price when $u = 1.1$ and $d = 0.9$.

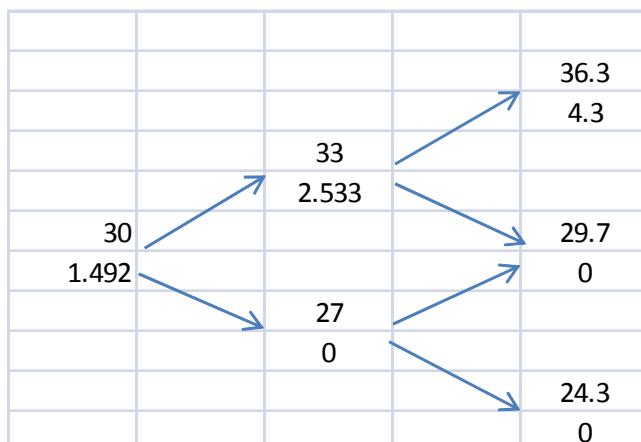
- A. \$1.29
- B. \$1.49
- C. \$1.69
- D. \$1.89

Answer: B

The probability of an up movement is

$$p = \frac{e^{rT} - d}{u - d} = \frac{e^{0.08 \times 0.25} - 0.9}{1.1 - 0.9} = 0.6010$$

The tree is



13. The current price of a non-dividend paying stock is \$30. Use a two-step tree to value a European put option on the stock with a strike price of \$32 that expires in 6 months with $u = 1.1$ and $d = 0.9$. Each step is 3 months, the risk free rate is 8%.

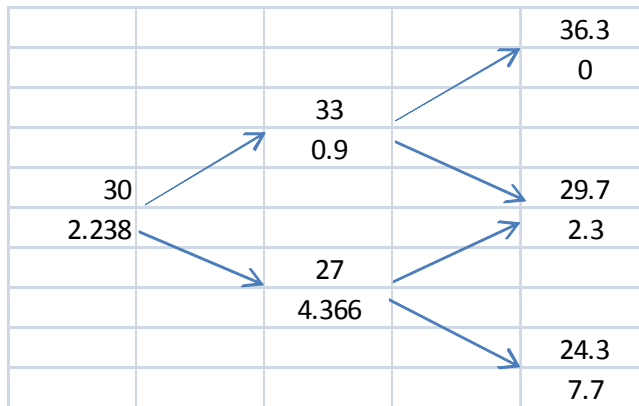
- A. \$2.24
- B. \$2.44
- C. \$2.64
- D. \$2.84

Answer: A

The probability of an up movement is

$$p = \frac{e^{rT} - d}{u - d} = \frac{e^{0.08 \times 0.25} - 0.9}{1.1 - 0.9} = 0.6010$$

The tree is



14. Which of the following is NOT true in a risk-neutral world?

- A. The expected return on a call option is independent of its strike price
- B. Investors expect higher returns to compensate for higher risk
- C. The expected return on a stock is the risk-free rate
- D. The discount rate used for the expected payoff on an option is the risk-free rate

Answer: B

In a risk-neutral world investors require an expected return equal to the risk-free rate and the discount rate that should be used for all expected payoffs is the risk-free rate.

15. If the volatility of a non-dividend paying stock is 20% per annum and a risk-free rate is 5% per annum, which of the following is closest to the Cox, Ross, Rubinstein parameter u for a tree with a three-month time step?

- A. 1.05
- B. 1.07
- C. 1.09
- D. 1.11

Answer: D

The formula for u is

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.2 \times \sqrt{0.25}} = 1.1052$$

16. If the volatility of a non-dividend-paying stock is 20% per annum and a risk-free rate is 5% per annum, which of the following is closest to the Cox, Ross, Rubinstein parameter p for a tree with a three-month time step?

- A. 0.50
- B. 0.54
- C. 0.58
- D. 0.62

Answer: B

The formula for p is

$$p = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.05 \times 0.25} - 0.9048}{1.1052 - 0.9048} = 0.538$$

17. The current price of a non-dividend paying stock is \$50. Use a two-step tree to value an American put option on the stock with a strike price of \$48 that expires in 12 months. Each step is 6 months, the risk free rate is 5% per annum, and the volatility is 20%. Which of the following is the option price?

- A. \$1.95
- B. \$2.00
- C. \$2.05
- D. \$2.10

Answer: B

In this case

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.2 \times \sqrt{0.5}} = 1.152 \quad d = 1/u = 0.868$$
$$p = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.05 \times 0.5} - 0.868}{1.152 - 0.868} = 0.5539$$

The tree is

				66.34482
				0
		57.5955		
		0		
50				50
1.999				0
		43.40617		
		4.593828		
				37.68192
				10.31808

18. Which of the following describes delta?

- A. The ratio of the option price to the stock price
- B. The ratio of the stock price to the option price
- C. The ratio of a change in the option price to the corresponding change in the stock price
- D. The ratio of a change in the stock price to the corresponding change in the option price

Answer: C

Delta is $\Delta f / \Delta S$ where ΔS is a small change in the stock price (with nothing else changing) and Δf is the corresponding change in the option price.

19. When moving from valuing an option on a non-dividend paying stock to an option on a currency which of the following is true?

- A. The risk-free rate is replaced by the excess of the domestic risk-free rate over the foreign risk-free rate in all calculations
- B. The formula for u changes
- C. The risk-free rate is replaced by the excess of the domestic risk-free rate over the foreign risk-free rate for discounting
- D. The risk-free rate is replaced by the excess of the domestic risk-free rate over the foreign risk-free rate when p is calculated

Answer: D

The formula for u does not change. The discount rate does not change. The formula for p becomes

$$p = \frac{e^{(r-r_f)\Delta t} - d}{u - d}$$

showing that D is correct.

20. A tree is constructed to value an option on an index which is currently worth 100 and has a volatility of 25%. The index provides a dividend yield of 2%. Another tree is constructed to value an option on a non-dividend-paying stock which is currently worth 100 and has a volatility of

25%. Which of the following are true?

- A. The parameters p and u are the same for both trees
- B. The parameter p is the same for both trees but u is not
- C. The parameter u is the same for both trees but p is not
- D. None of the above

Answer: C

The formula for u is the same in the two cases so that the values of the index on its tree are the same as the values of the stock on its tree. However, in the formula for p , r is replaced by $r-q$.