Homework 13

625.433

1. (20 pts.) In the example I've provided, I didn't calculate a smoothed estimate for any values of X. Simulate $n = 100$ and calculate a smoothed estimate of X_{50}.

2. (20 pts.) Consider the two state space models:

$$\begin{aligned} \text{Model 1} \qquad \mathbf{X}_{t+1,1} &= \mathbf{F}_1\mathbf{X}_{t,1} + \mathbf{V}_{t,1} \\ \mathbf{Y}_{t,1} &= \mathbf{G}_1\mathbf{X}_{t,1} + \mathbf{W}_{t,1}, \end{aligned}$$

and

$$\begin{aligned} \text{Model 2} \qquad \mathbf{X}_{t+1,2} &= \mathbf{F}_2\mathbf{X}_{t,2} + \mathbf{V}_{t,2} \\ \mathbf{Y}_{t,2} &= \mathbf{G}_2\mathbf{X}_{t,2} + \mathbf{W}_{t,2}, \end{aligned}$$

where $\mathbf{V}_{t,1}, \mathbf{W}_{t,1}, \mathbf{V}_{t,2}$, and $\mathbf{W}_{t,2}$ are all uncorrelated. Derive a state-space represenation for $\left(\mathbf{Y}'_{t,1}, \mathbf{Y}'_{t,2}\right)'$.

3. (20 pts.) Some of the work I do involves estimating the position of an unmanned underwater vehicle at a particular time. The Kalman Filter can be used to do this. We do this by letting $\mathbf{x}_t$ be the state at time t where

$$\mathbf{x}_t = \begin{pmatrix} u_t \\ y_t \\ z_t \\ \dot{u}_t \\ \dot{y}_t \\ \dot{z}_t \end{pmatrix},$$

u_t is the East position of the vehicle at time t, y_t is the North position at time t, z_t is the down position at time t, $\dot{u}_t$ is the East velocity at time t, and so on. The state equation for the motion of the vehicle is

$$\mathbf{x}_{t+1} = \mathbf{F}\mathbf{x}_t + \mathbf{V}_{t+1},$$

where

$$\mathbf{F} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

and $\mathbf{V}_t \sim N\left(\mathbf{0}, .02 \times \mathbf{I}_6\right)$. The position and velocity are observed with error, and this can be described by the measurement equation, which is

$$\mathbf{Y}_{t+1} = \mathbf{X}_{t+1} + \mathbf{W}_{t+1},$$

where $\mathbf{W}_{t+1} \sim N\left(\mathbf{0}, .01 \times \mathbf{I}_6\right)$. The measurement for the first 100 steps are given in the data set `underwaterVehicle.rds`. With this data, answer the following quesetions:

   (a) Explain the state equation. In particular, explain the matrix $\mathbf{F}$ and why the entries in $\mathbf{F}$ make sense in the context of this problem.

   (b) Given the dat you have, estimate the position of the vehicle at time 53.

4. (40 pts.) This is homework you're going to do on the internet. Do some research online and find an article or some website which describes an application of the Kalman Filter. Write two (detailed) paragraphs describing what you found. Let the first paragraph describe the context of the application, and the second describe how the Kalman Filter is used in the application and why it's necessary.