Problem Set 3 *

Ian McGroarty

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Problem 3.4.4. Remission time is $f_Y(y) = \frac{1}{9}y^2$, $0 \leq y \leq 3$. What is the probability the patients malaria lasts longer than one year?

Solution The probability that a malaria patients remisssion lasts longer than one year is 0.963, or 96.3%.

$$
\begin{aligned}
F_Y(y) &= \int f_Y(y)dy && \text{Def 3.4.3 pg 135} \\
&= \int \frac{1}{9}y^2 dy \\
&= \frac{1}{27}y^3 && \text{This is the cdf.} \\
P(Y > s) &= 1 - F_Y(s) && \text{Theorem 3.4.2(a) pg 135} \\
P(Y > 1) &= 1 - \frac{1}{27}(1^3) \\
&= \frac{26}{27} \\
P(Y > 1) &= 0.963
\end{aligned}
$$

*Problem list - 3.4.4, 3.5.14, 3.5.32, 3.6.2, 3.6.10

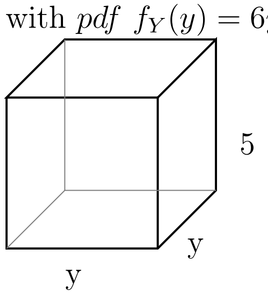
Problem 3.5.14. 15 observations are chosen at random from pdf $f_Y(y) = 3y^2,\ 0 \le y \le 1$. Let X denote the number that lie in the interval $(\frac{1}{2}, 1)$. Find $E(X)$.

Solution First, we must determine the probability that any given obseravtion is in the interval $(\frac{1}{2}, 1)$. To do this we evaluate the area under the pdf on the given interval.

$$
\begin{aligned}
P(r < Y \le s) &= F_Y(s) - F_Y(r) && \text{Theorem 3.4.2 pg 135} \\
&= \Big|_{\frac{1}{2}}^{1} F_Y && \text{Interested in the interval 1/2 to 1} \\
&= \int_{\frac{1}{2}}^{1} f_Y(y)dy && \text{Def 3.4.3 pg 135} \\
&= \int_{\frac{1}{2}}^{1} 3y^2 dy \\
&= y^3\Big|_{\frac{1}{2}}^{1} \\
&= 1^3 - (\frac{1}{2})^3 \\
&= \frac{7}{8}
\end{aligned}
$$

Since the events of X are mutually exclusive, axiom 3 (pg 26) apples. $E(X) = 15(7/8) = 105/8$.

Problem 3.5.32. Box with height 5in. and base YxY inches. Where Y is a random variable with *pdf* $f_Y(y) = 6y(1-y),\ 0 < y < 1$. Find the expected volume of the box.



Solution The box has an expected area of 1.5 inches.

The area of the box is A=5(Y^2) Let area be defined as g(Y), a continuous funtion. Then:

$$
\begin{aligned}
E[g(Y)] &= \int g(y) \cdot f_Y(y)dy && \text{Theorem 3.5.3 pg 148} \\
&= \int_0^1 (5y^2) \cdot 6y(1-y)dy && \text{Interested in interval 0 to 1} \\
&= 30 \int_0^1 y^3 - y^4 dy \\
&= 30(\frac{y^4}{4} - \frac{y^5}{5}\Big|_0^1) \\
&= \frac{30}{4} - 305 \\
&= \frac{30}{20} = \;\; \text{1.5 inches.}
\end{aligned}
$$

Problem 3.6.2. Find the variance of Y if:

$$
f_Y(y) = \begin{cases} \frac{3}{4}, & 0 \leq y \leq 1 \\ \frac{1}{4}, & 2 \leq y \leq 3 \\ 0, & \text{elsewhere} \end{cases}
$$

Solution $Var(Y) = \frac{5}{8}$ or 0.833.

$$
\begin{aligned}
E(Y) = \mu &= \int_0^1 y(\frac{3}{4})dy + \int_2^3 y(\frac{1}{4})dy && \text{First need to find Expected value of y} \\
&= (\frac{3}{8})y^2\Big|_0^1 + (\frac{1}{8})y^2\Big|_2^3 \\
&= (\frac{3}{8}) + (\frac{9}{8} - \frac{4}{8}) = 1 \\
E(Y^2) = \mu &= \int_0^1 y^2(\frac{3}{4})dy + \int_2^3 y^2(\frac{1}{4})dy && \text{Now need to find Expected value of } y^2 \\
&= (\frac{3}{12})y^3\Big|_0^1 + (\frac{1}{12})y^3\Big|_2^3 \\
&= (\frac{3}{12}) + (\frac{27}{12} - \frac{8}{12}) = \frac{22}{12} \\
Var(Y) &= \sigma^2 = E(Y^2) - \mu^2 && \text{Theorem 3.6.1 pg 155} \\
&= \frac{22}{12} - \frac{12}{12} = \frac{10}{12}
\end{aligned}
$$

Problem 3.6.10. Let Y be a random variabe whose pdf is given by $f_Y(y) = 5y^4,\ 0 \leq y \leq 1$. Find Var(Y).

Solution $Var(Y) = \frac{5}{32}$ or 0.1562

$$
\begin{aligned}
E(Y) = \mu &= \int_0^1 y \cdot 5y^4 dy && \text{First need to find Expected value of y} \\
&= \int_0^1 5y^5 dy \\
&= (\frac{5}{6})y^6 \Big|_0^1 = \frac{5}{6} \\
E(Y^2) = \mu_2 &= \int_0^1 y^2 \cdot 5y^4 dy && \text{Now to find expected value of } y^2 \\
&= \int_0^1 5y^6 dy \\
&= (\frac{5}{7})y^7 \Big|_0^1 = \frac{5}{7} \\
Var(Y) = \sigma^2 &= E(Y^2) - E(Y)^2 = \mu_2 - \mu^2 && \text{Theorem 3.6.1 pg 155} \\
&= \frac{5}{7} - (\frac{5}{6})^2 = \frac{5}{252}
\end{aligned}
$$