

Tasks: Emerging Technologies

ian.mcloughlin@atu.ie

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1. The Collatz conjecture¹ is a famous unsolved problem in mathematics. The problem is to prove that if you start with any positive integer x and repeatedly apply the function $f(x)$ below, you always get stuck in the repeating sequence 1, 4, 2, 1, 4, 2, ...

$$f(x) = \begin{cases} x \div 2 & \text{if } x \text{ is even} \\ 3x + 1 & \text{otherwise} \end{cases}$$

For example, starting with the value 10, which is an even number, we divide it by 2 to get 5. Then 5 is an odd number so, we multiply by 3 and add 1 to get 16. Then we repeatedly divide by 2 to get 8, 4, 2, 1. Once we are at 1, we go back to 4 and get stuck in the repeating sequence 4, 2, 1 as we suspected.

Your task is to verify, using Python, that the conjecture is true for the first 10,000 positive integers.

2. Square roots are difficult to calculate. In Python, you typically use the power operator (a double asterisk) or a package such as `math`. In this task,² you should write a function `sqrt(x)` to approximate the square root of a floating point number x without using the power operator or a package.

Rather, you should use the Newton's method.³ Start with an initial guess for the square root called z_0 . You then repeatedly improve it using the following formula, until the difference between some previous guess z_i and the next z_{i+1} is less than some threshold, say 0.01.

$$z_{i+1} = z_i - \frac{z_i \times z_i - x}{2z_i}$$

3. Consider all possible functions taking four bits as input and outputting a single bit. How many such possible functions⁴ are there?

Write Python code to select one such function at random out of all the possibilities. Suppose the only way you can figure out what the function is, is by calling it with different inputs and checking the outputs. How many times do you need to call the function to be certain which function it is?

¹ *The Simple Math Problem We Still Can't Solve* | Quanta Magazine. Sept. 22, 2020. URL: <https://www.quantamagazine.org/why-mathematicians-still-cant-solve-the-collatz-conjecture-20200922/> (visited on 08/18/2023).

² *A Tour of Go*. Aug. 18, 2023. URL: <https://go.dev/tour/flowcontrol/8> (visited on 08/18/2023).

³ *Square Roots via Newton's Method*. Feb. 4, 2015. URL: <https://math.mit.edu/~stevenj/18.335/newton-sqrt.pdf> (visited on 08/18/2023).

⁴ You can ignore any internal workings of the function, just focus on how many ways there are of having four bits of input and one bit of output.