Tasks: Fundamentals of Data Analysis

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1. The Fibonacci numbers are defined as follows. The first number, f_0 is 0 and the second, f_1 is 1. The other numbers are then defined as

$$f_i = f_{i-1} + f_{i-2}$$

   for i from 2 to infinity.

   In this task, write a function `fib(n)` to calculate the n^{th} Fibonacci number.

2. The Collatz conjecture[1] is a famous unsolved problem in mathematics. The problem is to prove that if you start with any positive integer x and repeatedly apply the function $f(x)$ below, you always get stuck in the repeating sequence $1, 2, 4, 1, 2, 4, \ldots$

$$f(x) = \begin{cases} x \div 2 & \text{if } x \text{ is even} \\ 3x + 1 & \text{otherwise} \end{cases}$$

   For example, starting with the value 10, which is an even number, we divide it by 2 to get 5. Then 5 is an odd number so, we multiply by 3 and add 1 to get 16. Then we repeatedly divide by 2 to get 8, 4, 2, 1. Once we are at 1, we go back to 4 and get stuck in the repeating sequence 4, 2, 1 as we suspected.

   Your task is to verify, using Python, that the conjecture is true for the first 10,000 positive integers.

[1] *The Simple Math Problem We Still Can't Solve | Quanta Magazine*. Sept. 22, 2020. URL: https://www.quantamagazine.org/why-mathematicians-still-cant-solve-the-collatz-conjecture-20200922/ (visited on 08/18/2023).