Graph Isomorphism

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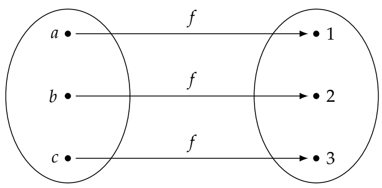
Basics

Set: $\{a, b, c\}$

Tuple: (a, a, a, b, c, d)

Map: $\{(a, d), (b, e), (c, d)\}$

Bijection



Map $f : X \longleftrightarrow Y$

Every y in Y is mapped to by exactly one x in X

Means map is reversible.

Counterexample: $f(x) = x^2$.

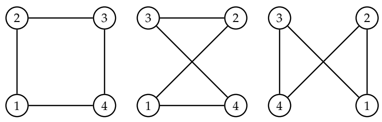
Graph

Graph: $G = (V, E)$

Vertex: element of $V = \{1, 2, 3, 4\}$

Edge: any 2-subset of V, e.g. $\{1, 2\}$

Example

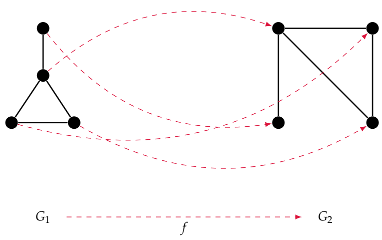


V: $\{1, 2, 3, 4\}$

E: $\{\{1, 2\}, \{1, 4\}, \{2, 3\}, \{3, 4\}\}$

Isomorphism

When two graphs have the exact same structure.



$G_1 = (V_1, E_1) \cong G_2 = (V_2, E_2)$

$f : V_1 \longleftrightarrow V_2$

$\{f(x), f(y)\} \in E_2 \Leftrightarrow \{x, y\} \in E_1$

Exercise

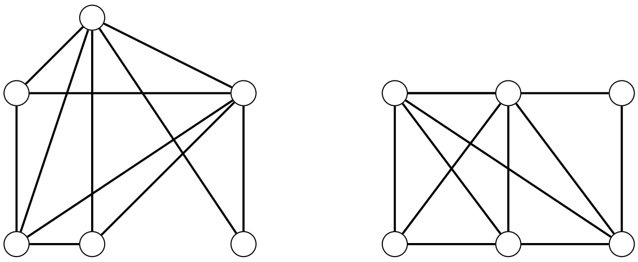
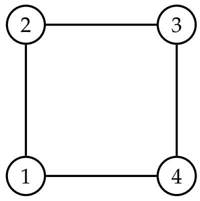


Figure 1: Are these isomorphic?

Adjacency Matrix

	1	2	3	4
1	0	1	0	1
2	1	0	1	0
3	0	1	0	1
4	1	0	1	0

	1	2	3	4
1	0	0	1	1
2	0	0	1	1
3	1	1	0	0
4	1	1	0	0



Decision Problem

Adjacency matrix 1: 0101 1010 0101 1010

Adjacency matrix 2: 0011 0011 1100 1100

String 1: 101 10 1

Ignore the spaces - they are just for explanation.

String 2: 011 11 0

Alphabet: $\{0, 1\}$

Language: $L \subseteq A^*$

Graph Isomorphism Problem: $L = \{st | s \cong t\}$

Example: $s_1 s_2 = 101101011110 \in L$