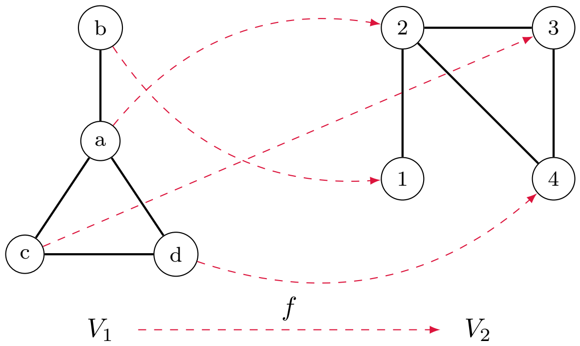
Graph

Simple graph: $G = (V, E)$; V a set; E a set of two-subsets of V.

Isomorphism

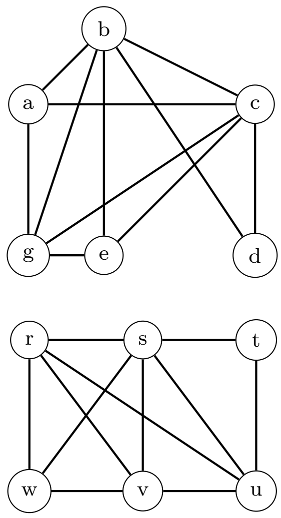
Graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$. Bijection $f : V_1 \to V_2$ such that $f(E_1) = E_2$ where $f(E_1) = \{\{f(v_1), f(v_2)\} | \{v_1, v_2\} \in E_1\}$.

Example



$$
\begin{aligned}
f(E_1) &= \{\{f(a), f(b)\}, \{f(a), f(c)\}, \\
&\quad \{f(a), f(d)\}, \{f(c), f(d)\}\} \\
&= \{\{1,2\}, \{1,3\}, \{1,4\}, \{3,4\}\} = E_2
\end{aligned}
$$

Non-isomorphism



No of maps

$f(a) \to 6$ choices; $f(b) \to 5$ choices; $f(c) \to 4$ choices; etc. So, $n!$ maps between the vertex sets of two graphs with n vertices.

Some invariants

- Degrees.
- Paths.
- Connection.

Adjacency matrix

Fix a listing of V. $[a_{ij}]$ where a_{ij} is 1 if $\{v_i, v_j\} \in E$ else 0.

Example

$$
\begin{bmatrix}
0 & 1 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 0 & 1 & 0
\end{bmatrix}
\qquad
\begin{bmatrix}
0 & 1 & 0 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 \\
1 & 1 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 & 0 & 1 \\
1 & 1 & 0 & 0 & 1 & 0
\end{bmatrix}
$$

Encoding