

# Heap Sort

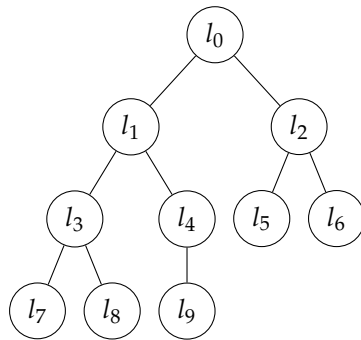
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## Sorting

Search for  $l = (l_0, l_1, l_2, \dots, l_{n-1})$  where  $l_0 \leq l_1 \leq \dots \leq l_{n-1}$ .

On a tree



## Heaps

Almost complete binary tree with heap property.

Max heap: each parent bigger than children.

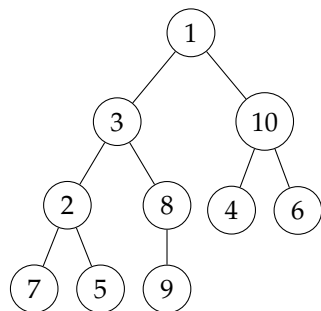
Min heap: each parent smaller than children.

## To min or max heap

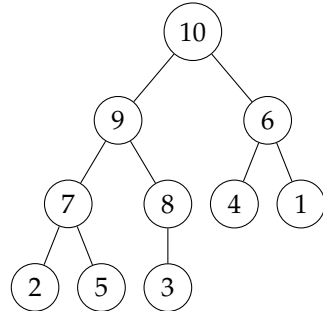
1. Start with last node, moving backwards.
2. Compare node to children, swap if needed.
3. Swap parent down tree until we have a heap.

## Example heap

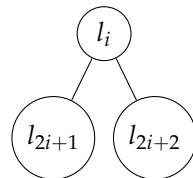
Sort ascending  $\rightarrow$  use max heap.



Last five nodes have no children. Sixth-last has one child, is smaller so swap. Now have a heap from sixth-last. Same for seventh-last: swap 2 for 7. Third node is a heap. Second node swaps 9 for 3, and filters down swapping 3 for 8. Finally, the root is swapped with 10 and then 6.



*As an array*



(	$l_0$ ,	$l_1$ ,	$l_2$ ,	$l_3$ ,	$l_4$ ,	$l_5$ ,	$l_6$ ,	$l_7$ ,	$l_8$ ,	$l_9$ )
(	1,	3,	10,	2,	8,	4,	6,	7,	5,	9)
(	1,	3,	10,	2,	9,	4,	6,	7,	5,	8)
(	1,	3,	10,	7,	9,	4,	6,	2,	5,	8)
(	1,	3,	10,	7,	9,	4,	6,	2,	5,	8)
(	1,	9,	10,	7,	3,	4,	6,	2,	5,	8)
(	1,	9,	10,	7,	8,	4,	6,	2,	5,	3)
(	10,	9,	1,	7,	8,	4,	6,	2,	5,	3)
(	10,	9,	6,	7,	8,	4,	1,	2,	5,	3)

### Heap Sort

1. Convert complete binary tree to heap.
2. Swap root for last child, ignore last child.
3. Repeat  $n - 1$  times.

(	$l_0$ ,	$l_1$ ,	$l_2$ ,	$l_3$ ,	$l_4$ ,	$l_5$ ,	$l_6$ ,	$l_7$ ,	$l_8$ ,	$l_9$ )
(	10,	9,	6,	7,	8,	4,	1,	2,	5,	3)
(	3,	9,	6,	7,	8,	4,	1,	2,	5,	10)
(	9,	8,	6,	7,	3,	4,	1,	2,	5,	10)
(	5,	8,	6,	7,	3,	4,	1,	2,	9,	10)
(	8,	7,	6,	5,	3,	4,	1,	2,	9,	10)
(	2,	7,	6,	5,	3,	4,	1,	8,	9,	10)

### *Comparisons*

*To heap:*  $O(n \log n)$

*Replace root:*  $O(\log n)$  but  $O(n)$  times.

*Check heap:*  $O(n)$