Collatz Turing Machine

Language

$A = \{0, 1\}$

$T = A \ \cup \ \{,\} \ \cup \ \{\sqcup, X\}$

$L = \{1, 10, 11, 100, 101, 110, 111, 1000, 1001, \ldots\}$

$f(s) = 1s, s$

Duplicate and append 1

State	Description
q_0	Append a comma.
q_1	Move left to start.
q_2	Check if 0 or 1, mark X.
q_3	Move right to end, append 0.
q_4	Move right to end, append 1.
q_5	Move left to X, overwrite 0, back to q_2.
q_6	Move left to X, overwrite 1, back to q_2.
q_7	Move to start, prepend 1.
q_8	Back to start, end.

State	Input	Write	Move	Next
q_0	$\sqcup$	,	L	q_1
q_0	0	0	R	q_0
q_0	1	1	R	q_0
q_0	,	,	R	q_f
q_0	X	X	R	q_f
q_1	$\sqcup$	$\sqcup$	R	q_2
q_1	0	0	L	q_1
q_1	1	1	L	q_1
q_1	,	,	R	q_f
q_1	X	X	R	q_f
q_2	$\sqcup$	$\sqcup$	R	q_f
q_2	0	X	R	q_3
q_2	1	X	R	q_4
q_2	,	,	R	q_7
q_2	X	X	R	q_f
q_3	$\sqcup$	0	L	q_5
q_3	0	0	R	q_3
q_3	1	1	R	q_3
q_3	,	,	R	q_3
q_3	X	X	R	q_f
q_4	$\sqcup$	1	L	q_6
q_4	0	0	R	q_4
q_4	1	1	R	q_4
q_4	,	,	R	q_4
q_4	X	X	R	q_f
q_5	$\sqcup$	$\sqcup$	L	q_f
q_5	0	0	L	q_5
q_5	1	1	L	q_5
q_5	,	,	L	q_5
q_5	X	0	R	q_2
q_6	$\sqcup$	$\sqcup$	L	q_f
q_6	0	0	L	q_6
q_6	1	1	L	q_6
q_6	,	,	L	q_6
q_6	X	1	R	q_2
q_7	$\sqcup$	1	L	q_8
q_7	0	0	L	q_7
q_7	1	1	L	q_7
q_7	,	,	L	q_7
q_7	X	X	L	q_7
q_8	$\sqcup$	$\sqcup$	R	q_a
q_8	0	0	R	q_f
q_8	1	1	R	q_f
q_8	,	,	R	q_f
q_8	X	X	R	q_f

Add

State	Description
q_0	Right to comma.
q_1	Decrease by 1, go to final states if no 1.
q_2	Left to start.
q_3	Increase by 1, use X as comma if overflow.
q_4	Delete to comma/X, replace X with 1.
q_5	Left to start.

State	Input	Write	Move	Next
q_0	⊔	⊔	L	q_f
q_0	0	0	R	q_0
q_0	1	1	R	q_0
q_0	,	,	R	q_1
q_0	X	X	R	q_1
q_1	⊔	⊔	L	q_4
q_1	0	1	R	q_1
q_1	1	0	L	q_2
q_1	,	,	L	q_f
q_1	X	X	L	q_f
q_2	⊔	⊔	R	q_3
q_2	0	0	L	q_2
q_2	1	1	L	q_2
q_2	,	,	L	q_2
q_2	X	X	L	q_2
q_3	⊔	⊔	R	q_f
q_3	0	1	R	q_0
q_3	1	0	R	q_2
q_3	,	X	L	q_0
q_3	X	X	L	q_f
q_4	⊔	⊔	L	q_f
q_4	0	⊔	L	q_4
q_4	1	1	L	q_f
q_4	,	⊔	L	q_5
q_4	X	1	L	q_5
q_5	⊔	⊔	R	q_a
q_5	0	0	L	q_5
q_5	1	1	L	q_5
q_5	,	,	L	q_f
q_5	X	X	L	q_f

Divide by two

State	Description
q_0	Delete 0, move right.

State	Input	Write	Move	Next
q_0	⊔	⊔	L	q_f
q_0	0	⊔	R	q_0
q_0	1	1	R	q_f
q_0	,	,	R	q_f
q_0	X	X	R	q_f

Remove leading zeros

Fails on string meaning zero.

State	Description
q_0	Move right to end.
q_1	Delete zeros.
q_2	Left to end.

State	Input	Write	Move	Next
q_0	⊔	⊔	L	q_1
q_0	0	0	R	q_0
q_0	1	1	R	q_0
q_0	,	,	R	q_f
q_0	X	X	R	q_f
q_1	⊔	⊔	R	q_f
q_1	0	⊔	L	q_0
q_1	1	1	L	q_2
q_1	,	,	R	q_f
q_1	X	X	R	q_f
q_2	⊔	⊔	R	q_a
q_2	0	0	R	q_2
q_2	1	1	R	q_2
q_2	,	,	R	q_f
q_2	X	X	R	q_f

Check if 1

State	Description
q_0	Check if first bit 1.
q_1	Check if second bit blank.

State	Input	Write	Move	Next
q_0	⊔	⊔	R	q_f
q_0	0	0	R	q_f
q_0	1	1	R	q_1
q_0	,	,	R	q_f
q_0	X	X	R	q_f
q_1	⊔	⊔	L	q_a
q_1	0	0	L	q_f
q_1	1	1	L	q_f
q_1	,	,	R	q_f
q_1	X	X	R	q_f

Check if even

State	Description
q_0	Check if first bit 0.

State	Input	Write	Move	Next
q_0	⊔	⊔	R	q_f
q_0	0	0	R	q_a
q_0	1	1	R	q_f
q_0	,	,	R	q_f
q_0	X	X	R	q_f