Heap Sort

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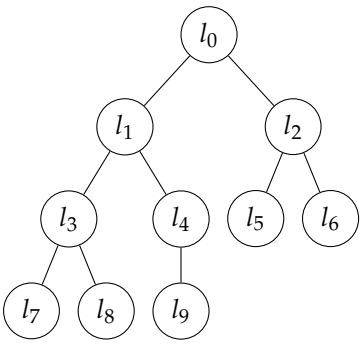
Sorting

Search for $l = (l_0, l_1, l_2, \ldots, l_{n-1})$ where $l_0 \leq l_1 \leq \cdots \leq l_{n-1}$.

On a tree



Heaps

Almost complete binary tree with heap property.

Max heap: each parent bigger than children.

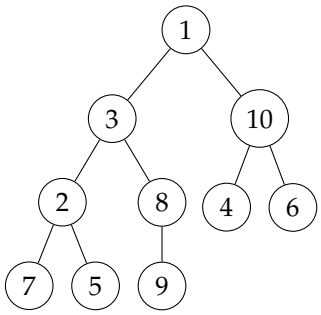
Min heap: each parent smaller than children.

To min or max heap

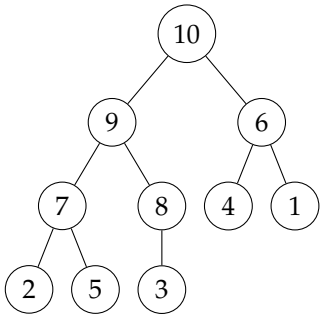
1. Start with last node, moving backwards.
2. Compare node to children, swap if needed.
3. Swap parent down tree until we have a heap.

Example heap

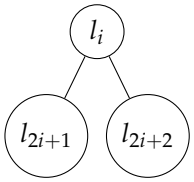
Sort ascending $\rightarrow$ use max heap.



Last five nodes have no children. Sixth-last has one child, is smaller so swap. Now have a heap from sixth-last. Same for seventh-last: swap 2 for 7. Third node is a heap. Second node swaps 9 for 3, and filters down swapping 3 for 8. Finally, the root is swapped with 10 and then 6.



As an array



(	l_0,	l_1,	l_2,	l_3,	l_4,	l_5,	l_6,	l_7,	l_8,	l_9)
(	1,	3,	10,	2,	8,	4,	6,	7,	5,	9)
(	1,	3,	10,	2,	9,	4,	6,	7,	5,	8)
(	1,	3,	10,	7,	9,	4,	6,	2,	5,	8)
(	1,	3,	10,	7,	9,	4,	6,	2,	5,	8)
(	1,	9,	10,	7,	3,	4,	6,	2,	5,	8)
(	1,	9,	10,	7,	8,	4,	6,	2,	5,	3)
(	10,	9,	1,	7,	8,	4,	6,	2,	5,	3)
(	10,	9,	6,	7,	8,	4,	1,	2,	5,	3)

Heap Sort

1. Convert complete binary tree to heap.
2. Swap root for last child, ignore last child.
3. Repeat $n - 1$ times.

	l_0,	l_1,	l_2,	l_3,	l_4,	l_5,	l_6,	l_7,	l_8,	l_9)
(	10,	9,	6,	7,	8,	4,	1,	2,	5,	3)
(	3,	9,	6,	7,	8,	4,	1,	2,	5,	10)
(	9,	8,	6,	7,	3,	4,	1,	2,	5,	10)
(	5,	8,	6,	7,	3,	4,	1,	2,	9,	10)
(	8,	7,	6,	5,	3,	4,	1,	2,	9,	10)
(	2,	7,	6,	5,	3,	4,	1,	8,	9,	10)

Comparisons

To heap: $O(n \log n)$

Replace root: $O(\log n)$ but $O(n)$ times.

Check heap: $O(n)$