Hadamard Gate

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Last updated: 28 September 2023

Gate

Η

Matrix

$$\mathsf{H} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$|\phi
angle = egin{bmatrix} lpha \ eta \end{bmatrix}$$

$$\ket{\phi_H}\mathsf{H}\ket{\phi} = rac{1}{\sqrt{2}}egin{bmatrix} 1 & 1 \ 1 & -1 \end{bmatrix}egin{bmatrix} lpha \ eta \end{bmatrix} = rac{1}{\sqrt{2}}egin{bmatrix} lpha + eta \ lpha - eta \end{bmatrix}$$

Probability

$$\langle \phi_H | = rac{1}{\sqrt{2}} \left[(lpha + eta)^* \quad (lpha - eta)^*
ight] = rac{1}{\sqrt{2}} \left[lpha^* + eta^* \quad lpha^* - eta^*
ight]$$

$$\langle \phi_H | \phi_H \rangle = \left(\frac{1}{\sqrt{2}}\right) \left(\frac{1}{\sqrt{2}}\right) \left[\alpha^* + \beta^* \quad \alpha^* - \beta^*\right] \left[\alpha + \beta \atop \alpha - \beta\right]$$

$$= \frac{1}{2} \left[(\alpha^* + \beta^*)(\alpha + \beta) + (\alpha^* - \beta^*)(\alpha - \beta)\right]$$

$$= \frac{1}{2} \left[\alpha^* \alpha + \beta^* \alpha + \alpha^* \beta + \beta^* \beta + \alpha^* \alpha - \beta^* \alpha - \alpha^* \beta + \beta^* \beta\right]$$

$$= \frac{1}{2} \left[2\alpha^* \alpha + 2\beta^* \beta\right]$$

$$= \alpha^* \alpha + \beta^* \beta$$

$$\langle \phi | \phi
angle = egin{bmatrix} lpha^* & eta^* \end{bmatrix} egin{bmatrix} lpha \ eta \end{bmatrix} = lpha^*lpha + eta^*eta = 1$$