Reversible Computing

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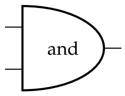
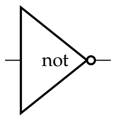
Logical Reversibility



x_1	x_2	$\wedge$
0	0	0
0	1	0
1	0	0
1	1	1



x	$\bar{x}$
0	1
1	0

Landauer's Principle

$\Delta E \geq kT \ln 2$

$k = 1.380649 \times 10^{-23} \mathsf{JK}^{-1}$

$\Delta E \geq (1.380649 \times 10^{-23})(293)(0.693147) \approx 2.8 \times 10^{-21} \mathsf{J}$ at $20^\circ \mathsf{C}$[1]

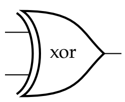
R. Landauer. Irreversibility and heat generation in the computing process. *IBM Journal of Research and Development*, 5(3):183–191, 1961. DOI: 10.1147/rd.53.0183

Charles H. Bennett. The thermodynamics of computation—a review. *International Journal of Theoretical Physics*, 21 (12):905–940, Dec 1982. ISSN 1572-9575. DOI: 10.1007/BF02084158. URL https://doi.org/10.1007/BF02084158

Charles H. Bennett. Notes on landauer's principle, reversible computation and maxwell's demon, 2003

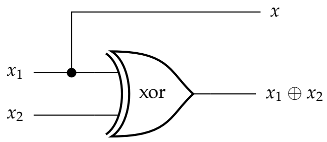
[1] Somewhere in th

XOR



x_1	x_2	$\oplus$
0	0	0
0	1	1
1	0	1
1	1	0

CNOT



x_1	x_2	x_1	$x_1 \oplus x_2$
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

Toffoli

$y = x_3 \oplus (x_1 \wedge x_2)$

x_1	x_2	x_3	x_1	x_2	y
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Fredkin

$y_1 = x_1 \oplus (x_0 \wedge (x_1 \oplus x_2))$

$y_2 = x_2 \oplus (x_0 \wedge (x_1 \oplus x_2))$

x_0	x_1	x_2	x_0	y_1	y_2
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	0	1
1	1	1	1	1	1

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$