Hadamard Gate

ian.mcloughlin@atu.ie

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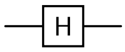
Gate



Single systems – ibm quantum learning, 2023. URL `https://learning.quantum-computing.ibm.com/course/basics-of-quantum-information/single-systems#unitary-operations`

Matrix

$$\mathsf{H} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$|\phi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$|\phi_H\rangle \, \mathsf{H} \, |\phi\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \alpha + \beta \\ \alpha - \beta \end{bmatrix}$$

Probability

$$\langle \phi_H | = \frac{1}{\sqrt{2}} \begin{bmatrix} (\alpha+\beta)^* & (\alpha-\beta)^* \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \alpha^* + \beta^* & \alpha^* - \beta^* \end{bmatrix}$$

$$\begin{aligned} \langle \phi_H | \phi_H \rangle &= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) \begin{bmatrix} \alpha^* + \beta^* & \alpha^* - \beta^* \end{bmatrix} \begin{bmatrix} \alpha + \beta \\ \alpha - \beta \end{bmatrix} \\ &= \frac{1}{2} \left[(\alpha^* + \beta^*)(\alpha + \beta) + (\alpha^* - \beta^*)(\alpha - \beta) \right] \\ &= \frac{1}{2} \left[\alpha^*\alpha + \beta^*\alpha + \alpha^*\beta + \beta^*\beta + \alpha^*\alpha - \beta^*\alpha - \alpha^*\beta + \beta^*\beta \right] \\ &= \frac{1}{2} \left[2\alpha^*\alpha + 2\beta^*\beta \right] \\ &= \alpha^*\alpha + \beta^*\beta \end{aligned}$$

$$\langle \phi | \phi \rangle = \begin{bmatrix} \alpha^* & \beta^* \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha^*\alpha + \beta^*\beta = 1$$