

$$\begin{array}{c|c} vt \\ \hline \\ ct' \end{array} \qquad \begin{array}{c|c} ct \\ \hline \end{array}$$

$$\begin{aligned} & \mathbf{\hat{\chi}} & (ct)^2 = (ct')^2 + (vt)^2 \\ & \Rightarrow c^2t^2 - v^2t^2 = c^2(t')^2 \\ & \Rightarrow t^2(c^2 - v^2) = c^2(t')^2 \\ & \Rightarrow (t')^2 = t^2(1 - \frac{v^2}{c^2}) \\ & \Rightarrow t' = t\sqrt{1 - \frac{v^2}{c^2}} \end{aligned}$$