



$$(ct)^2 = (ct')^2 + (vt)^2$$

$$\Rightarrow c^2 t^2 - v^2 t^2 = c^2 (t')^2$$

$$\Rightarrow t^2 (c^2 - v^2) = c^2 (t')^2$$

$$\Rightarrow (t')^2 = t^2 \left(1 - \frac{v^2}{c^2}\right)$$

$$\Rightarrow t' = t \sqrt{1 - \frac{v^2}{c^2}}$$