Quantum Latin Squares

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Quantum Latin Square

An $n \times n$ array of elements in $\mathbb{C}^n$ such that each row and each column is an orthonormal basis.

Benjamin Musto and Jamie Vicary. Quantum latin squares and unitary error bases, 2016

Example

$\|0\rangle$	$\|1\rangle$	$\|2\rangle$	$\|3\rangle$
$\frac{1}{\sqrt{2}}(\|1\rangle - \|2\rangle)$	$\frac{1}{\sqrt{5}}(i\|0\rangle + 2\|3\rangle)$	$\frac{1}{\sqrt{5}}(2\|0\rangle + i\|3\rangle)$	$\frac{1}{\sqrt{2}}(\|1\rangle + \|2\rangle)$
$\frac{1}{\sqrt{2}}(\|1\rangle + \|2\rangle)$	$\frac{1}{\sqrt{5}}(2\|0\rangle + i\|3\rangle)$	$\frac{1}{\sqrt{5}}(i\|0\rangle + 2\|3\rangle)$	$\frac{1}{\sqrt{2}}(\|1\rangle - \|2\rangle)$
$\|3\rangle$	$\|2\rangle$	$\|1\rangle$	$\|0\rangle$

$\|0\rangle$	$\|1\rangle$	$\|2\rangle$	$\|3\rangle$
$\|1\rangle$	$\|2\rangle$	$\|3\rangle$	$\|0\rangle$
$\|2\rangle$	$\|3\rangle$	$\|0\rangle$	$\|1\rangle$
$\|3\rangle$	$\|0\rangle$	$\|1\rangle$	$\|2\rangle$

Figure 1: Latin Square.

Normality

$$\left|\frac{1}{\sqrt{2}}(|1\rangle - |2\rangle)\right| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{-1}{\sqrt{2}}\right)^2} = 1$$

$$\left|\frac{1}{\sqrt{5}}(i\,|0\rangle + 2\,|3\rangle)\right| = \sqrt{\left(\frac{-i}{\sqrt{5}}\right)\left(\frac{i}{\sqrt{5}}\right) + \left(\frac{2}{\sqrt{5}}\right)^2} = \sqrt{\frac{5}{5}} = 1$$

$$\left|\frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)\right| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$$

$$\left|\frac{1}{\sqrt{5}}(2\,|0\rangle + i\,|3\rangle)\right| = \sqrt{\left(\frac{2}{\sqrt{5}}\right)^2 + \left(\frac{-i}{\sqrt{5}}\right)\left(\frac{i}{\sqrt{5}}\right)} = \sqrt{\frac{5}{5}} = 1$$

Orthogonality

$$\langle 0|1\rangle = (0)(1) + (1)(0) = 0$$

$$\left(\frac{1}{\sqrt{2}}(|1\rangle - |2\rangle)\right)^* \left(\frac{1}{\sqrt{5}}(i\,|0\rangle + 2\,|3\rangle)\right) = 0$$

$$\left(\frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)\right)^* \left(\frac{1}{\sqrt{2}}(|1\rangle - |2\rangle)\right) = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{-1}{\sqrt{2}}\right) = 0$$

$$\frac{1}{\sqrt{5}}(i\,|0\rangle + 2\,|3\rangle)^* \frac{1}{\sqrt{5}}(2\,|0\rangle + i\,|3\rangle) = \left(\frac{-i}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}\right) + \left(\frac{2}{\sqrt{5}}\right)\left(\frac{i}{\sqrt{5}}\right) = 0$$