Deutsch's Algorithm

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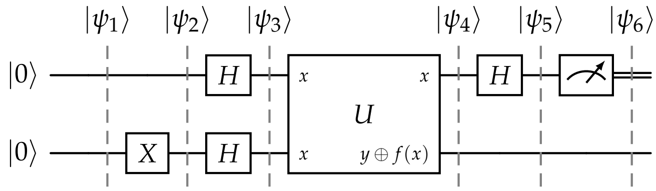
Quantum query algorithms – ibm quantum learning, 2023. URL `https://learning.quantum-computing.ibm.com/course/fundamentals-of-quantum-algorithms/quantum-query-algorithms#deutschs-algorithm`



$$|\psi_1\rangle = |0\rangle \otimes |0\rangle = |00\rangle \to \begin{bmatrix}1\\0\end{bmatrix} \otimes \begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\begin{bmatrix}1\\0\end{bmatrix}\\ \\ 0\begin{bmatrix}1\\0\end{bmatrix}\end{bmatrix} = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

$$|\psi_2\rangle = (I \otimes X)\,|\psi_1\rangle = \begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix} \begin{bmatrix}1\\0\\0\\0\end{bmatrix} = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}$$

$$I \otimes X = \begin{bmatrix}1\begin{bmatrix}0&1\\1&0\end{bmatrix} & 0\begin{bmatrix}0&1\\1&0\end{bmatrix}\\ \\ 0\begin{bmatrix}0&1\\1&0\end{bmatrix} & 1\begin{bmatrix}0&1\\1&0\end{bmatrix}\end{bmatrix} = \begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$$

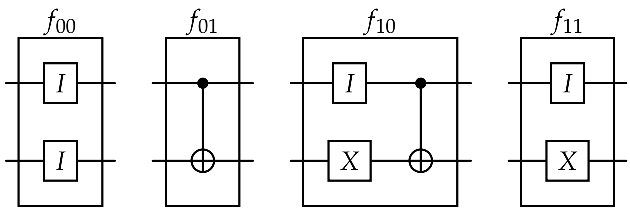
$$|\psi_3\rangle = (H \otimes H)\,|\psi_2\rangle = \frac{1}{2}\begin{bmatrix}1&1&1&1\\1&-1&1&1\\1&1&-1&-1\\1&1&-1&1\end{bmatrix} \begin{bmatrix}0\\1\\0\\0\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1\\-1\\1\\1\end{bmatrix}$$

$$H \otimes H = \frac{1}{\sqrt{2}\sqrt{2}} \begin{bmatrix}1\begin{bmatrix}1&1\\1&-1\end{bmatrix} & 1\begin{bmatrix}1&1\\1&-1\end{bmatrix}\\ \\ 1\begin{bmatrix}1&1\\1&-1\end{bmatrix} & -1\begin{bmatrix}1&1\\1&-1\end{bmatrix}\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1&1&1&1\\1&-1&1&1\\1&1&-1&-1\\1&1&-1&1\end{bmatrix}$$

U_f

x	y	$f_{00}(x)$	$y \oplus f_{00}(x)$	$f_{01}(x)$	$y \oplus f_{01}(x)$	$f_{10}(x)$	$y \oplus f_{10}(x)$	$f_{11}(x)$	$y \oplus f_{11}(x)$
0	0	0	0	0	0	1	1	1	1
0	1	0	1	0	1	1	0	1	0
1	0	0	0	1	1	0	0	1	1
1	1	0	1	1	0	0	1	1	0

	f_{00}	f_{01}	f_{10}	f_{11}
$\|00\rangle$	$\|00\rangle$	$\|00\rangle$	$\|01\rangle$	$\|01\rangle$
$\|01\rangle$	$\|01\rangle$	$\|01\rangle$	$\|00\rangle$	$\|00\rangle$
$\|10\rangle$	$\|10\rangle$	$\|11\rangle$	$\|10\rangle$	$\|11\rangle$
$\|11\rangle$	$\|11\rangle$	$\|10\rangle$	$\|11\rangle$	$\|10\rangle$



$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$$

f_{00}

$$|\psi_4\rangle = (H \otimes I)\,|\psi_3\rangle = \frac{1}{2\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

$$H \otimes I = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & 1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ 1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & -1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$