Unitary Error Basis

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Pauli Matrices

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

Inner Product

A, B: $d \times d$ matrices.

$\langle A, B \rangle = \mathrm{tr}(A^* B)/d$

$\mathrm{tr}(A) = \sum_i a_{ii}$

Conjugate Transpose

$A^* = A^{\mathsf{H}} = A^{\dagger} = \bar{A}^{\mathsf{T}}$

$$\begin{bmatrix} i & 2+i \\ 1 & 3-2i \end{bmatrix}^* = \begin{bmatrix} -i & 1 \\ 2-i & 3+2i \end{bmatrix}$$

Trace

$\mathrm{tr}(A) = \sum_i a_{ii}$

$\mathrm{tr}(A \otimes B) = \mathrm{tr}(A)\mathrm{tr}(B)$

$\mathrm{tr}(cA) = c\mathrm{tr}(A)$

$\mathrm{tr}(A) = \sum_i \lambda_i$

Orthogonality

$$A = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^* = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$A^* B = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$$

$\mathrm{tr}(A^* B) = 0 + 0 = 0$

$d = 2$

$\mathrm{tr}(A^* B)/2 = 0/2 = 0$

Normality

$A = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad A^* = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

$A^*A = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} -i^2 & 0 \\ 0 & -i^2 \end{bmatrix}$

$\mathsf{tr}(A^*A) = -i^2 + -i^2 = 2$

$d = 2$

$\mathsf{tr}(A^*B)/2 = 2/2 = 1$