

Quantum Latin Squares

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Last updated: 30 November 2023

Quantum Latin Square

An $n \times n$ array of elements in \mathbb{C}^n such that each row and each column is an orthonormal basis.

Example

$ 0\rangle$	$ 1\rangle$	$ 2\rangle$	$ 3\rangle$
$\frac{1}{\sqrt{2}}(1\rangle - 2\rangle)$	$\frac{1}{\sqrt{5}}(i 0\rangle + 2 3\rangle)$	$\frac{1}{\sqrt{5}}(2 0\rangle + i 3\rangle)$	$\frac{1}{\sqrt{2}}(1\rangle + 2\rangle)$
$\frac{1}{\sqrt{2}}(1\rangle + 2\rangle)$	$\frac{1}{\sqrt{5}}(2 0\rangle + i 3\rangle)$	$\frac{1}{\sqrt{5}}(i 0\rangle + 2 3\rangle)$	$\frac{1}{\sqrt{2}}(1\rangle - 2\rangle)$
$ 3\rangle$	$ 2\rangle$	$ 1\rangle$	$ 0\rangle$

Benjamin Musto and Jamie Vicary.
Quantum latin squares and unitary
error bases, 2016

$ 0\rangle$	$ 1\rangle$	$ 2\rangle$	$ 3\rangle$
$ 1\rangle$	$ 2\rangle$	$ 3\rangle$	$ 0\rangle$
$ 2\rangle$	$ 3\rangle$	$ 0\rangle$	$ 1\rangle$
$ 3\rangle$	$ 0\rangle$	$ 1\rangle$	$ 2\rangle$

Figure 1: Latin Square.

Normality

$$\left| \frac{1}{\sqrt{2}}(|1\rangle - |2\rangle) \right| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{-1}{\sqrt{2}}\right)^2} = 1$$

$$\left| \frac{1}{\sqrt{5}}(i|0\rangle + 2|3\rangle) \right| = \sqrt{\left(\frac{-i}{\sqrt{5}}\right)\left(\frac{i}{\sqrt{5}}\right) + \left(\frac{2}{\sqrt{5}}\right)^2} = \sqrt{\frac{5}{5}} = 1$$

$$\left| \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle) \right| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$$

$$\left| \frac{1}{\sqrt{5}}(2|0\rangle + i|3\rangle) \right| = \sqrt{\left(\frac{2}{\sqrt{5}}\right)^2 + \left(\frac{-i}{\sqrt{5}}\right)\left(\frac{i}{\sqrt{5}}\right)} = \sqrt{\frac{5}{5}} = 1$$

Orthogonality

$$\langle 0|1\rangle = (0)(1) + (1)(0) = 0$$

$$\left(\frac{1}{\sqrt{2}}(|1\rangle - |2\rangle) \right)^* \left(\frac{1}{\sqrt{5}}(i|0\rangle + 2|3\rangle) \right) = 0$$

$$\left(\frac{1}{\sqrt{2}}(|1\rangle + |2\rangle) \right)^* \left(\frac{1}{\sqrt{2}}(|1\rangle - |2\rangle) \right) = \left(\frac{1}{\sqrt{2}} \right) \left(\frac{1}{\sqrt{2}} \right) + \left(\frac{1}{\sqrt{2}} \right) \left(\frac{-1}{\sqrt{2}} \right) = 0$$

$$\frac{1}{\sqrt{5}}(i|0\rangle + 2|3\rangle)^* \frac{1}{\sqrt{5}}(2|0\rangle + i|3\rangle) = \left(\frac{-i}{\sqrt{5}} \right) \left(\frac{2}{\sqrt{5}} \right) + \left(\frac{2}{\sqrt{5}} \right) \left(\frac{i}{\sqrt{5}} \right) = 0$$