Problem Sheet: Automata Graph Theory

These questions are largely taken from Sipser's book [1].

1. Draw a DFA that recognises all strings over $\{a, b\}$ that have at least three a's.

2. Draw a DFA that recognises strings over $\{a, b\}$ that have at least two b's.

3. Draw a DFA that recognises strings over $\{a, b\}$ that have exactly two a's.

4. Draw a DFA that recognises strings over $\{a, b\}$ that have an odd number of a's.

5. Draw a DFA that recognises strings over $\{a, b\}$ that have at least three a's and at least two b's.

6. Define all of the above DFA's.

7. Draw an NFA that recognises, over the alphabet $\{0, 1\}$, both strings that begin with a 1 and end with a 0 and strings that contain at least three 1's.

8. Draw an NFA that recognises, over the alphabet $\{0, 1\}$, both strings that contain the substring 0101 and strings that don't contain the substring 110.

9. Draw an NFA that recognises, over the alphabet $\{0, 1\}$, the concatenations of strings of length at most five and strings where every odd position is a 1.

10. Draw an NFA that recognises, over the alphabet $\{0, 1\}$, the Kleene star of the language containing strings with at least two 0's and at most one 1.

11. Define all of the above deterministic finite automata.

12. Use the pumping lemma to show that the following language over the alphabet $\{0, 1\}$ is not regular.
$$L = \{0^n 1^n \mid n \in \mathbb{N}\}$$

13. Use the pumping lemma to show that the following language over the alphabet $\{0, 1\}$ is not regular.

$$L = \{w \mid w \text{ has an equal number of 0's and 1's}\}$$

References

[1] Michael Sipser. *Introduction to the Theory of Computation*. International Thomson Publishing, 3rd edition, 1996.