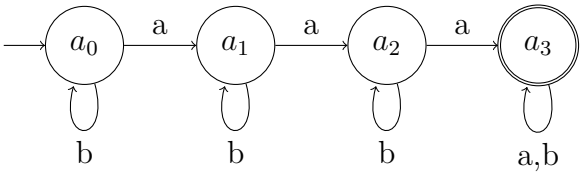
These questions are largely taken from Sipser's book [1].

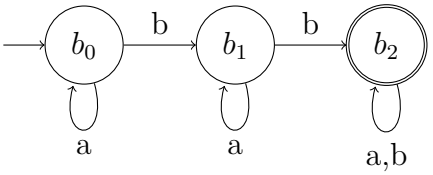
1. Draw a DFA that recognises all strings over $\{a, b\}$ that have at least three a's.

Solution:



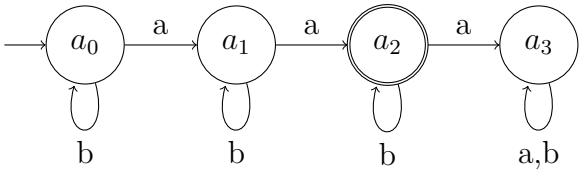
2. Draw a DFA that recognises all strings over $\{a, b\}$ that have at least two b's.

Solution:



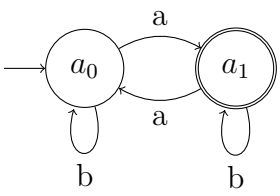
3. Draw a DFA that recognises all strings over $\{a, b\}$ that have exactly two a's.

Solution:



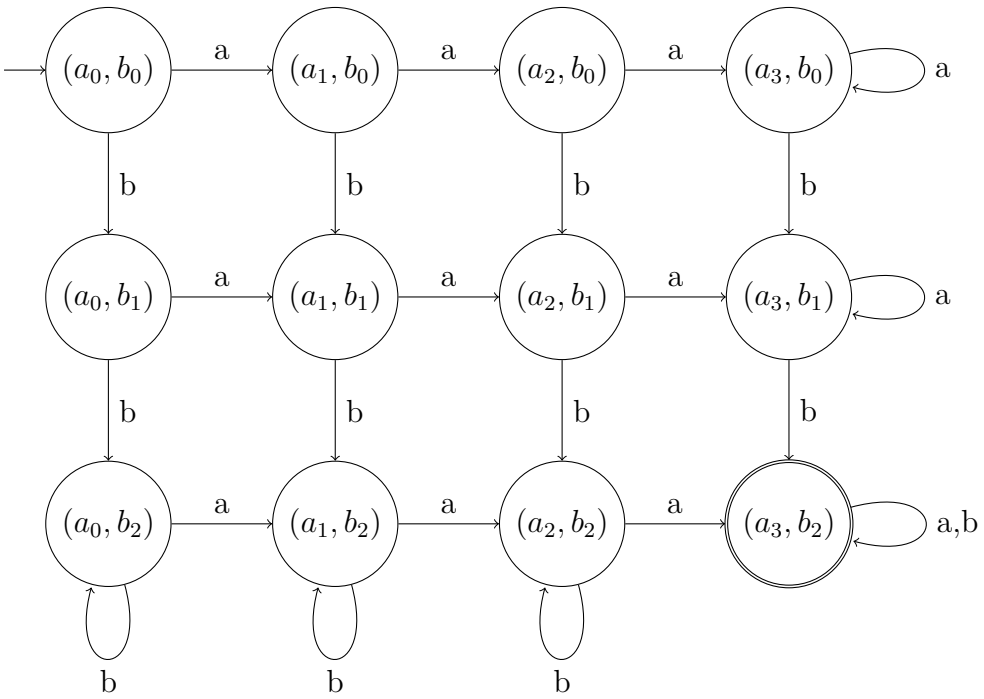
4. Draw a DFA that recognises all strings over $\{a, b\}$ that have an odd number of a's.

Solution:



5. Draw a DFA that recognises all strings over $\{a, b\}$ that have at least three a's and at least two b's.

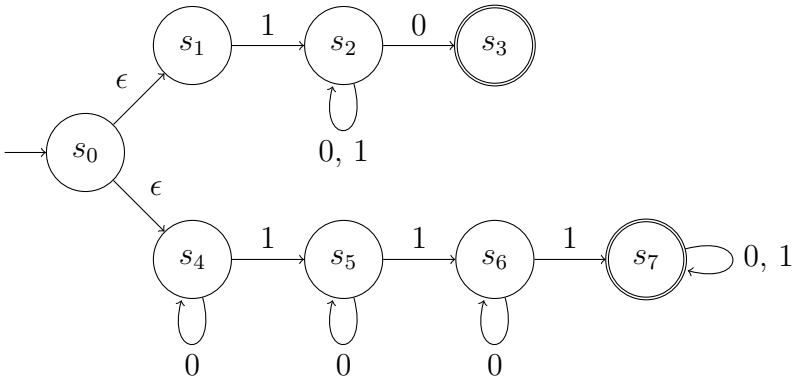
Solution:



6. Define all of the above DFA's.

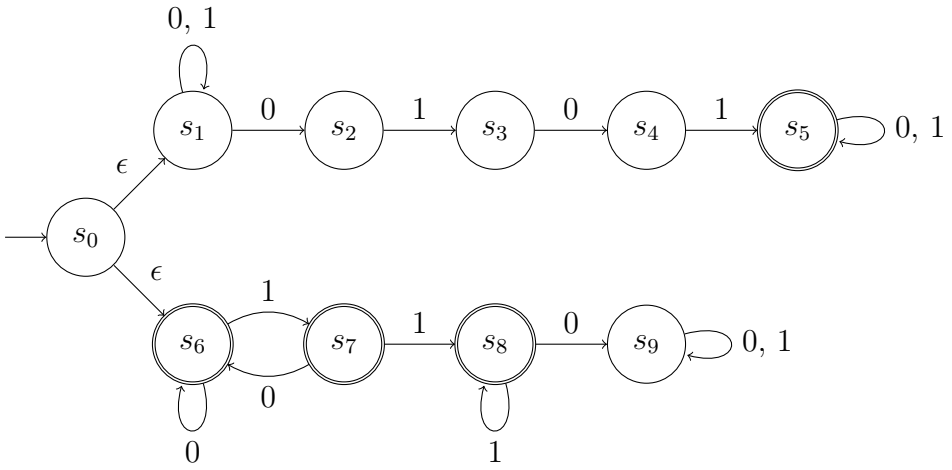
7. Draw an NFA that recognises any string over $\{0, 1\}$ that begins with a 1 and ends with a 0 or contains at least three 1's.

Solution:



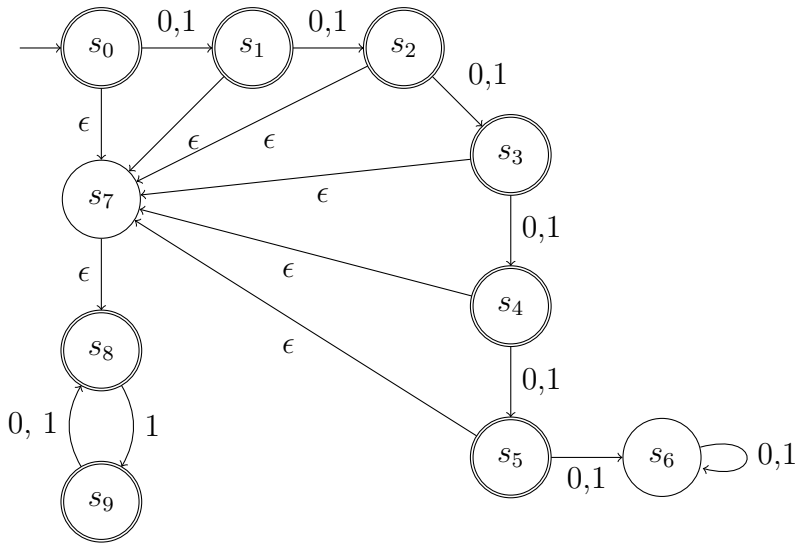
8. Draw an NFA that recognises strings over $\{0, 1\}$ that contains the substring 0101 or doesn't contain the substring 110.

Solution:



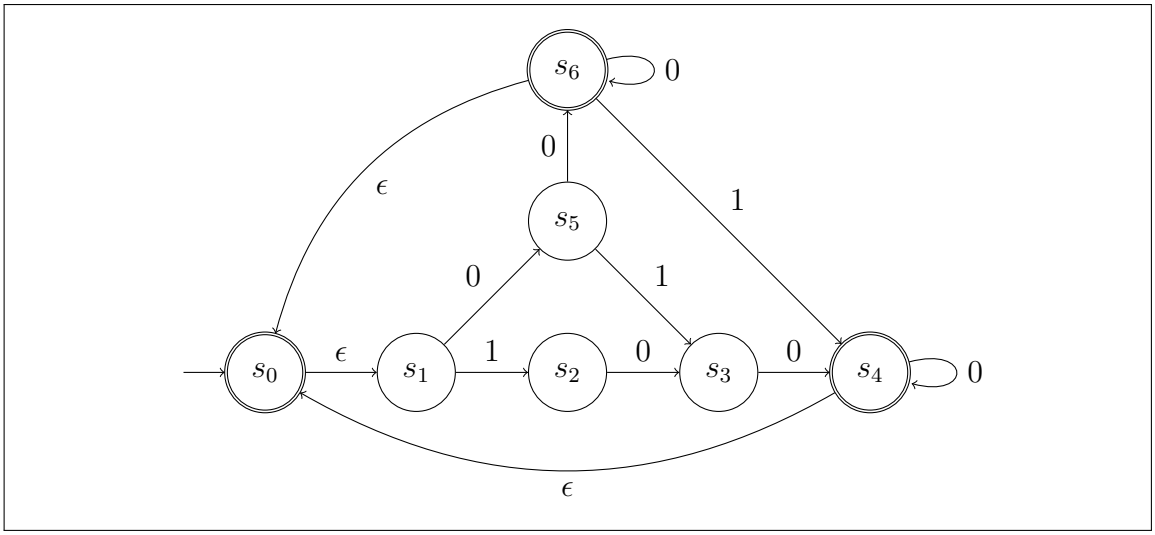
9. Draw an NFA that recognises strings over $\{0, 1\}$ that are the concatenation of strings of length at most five and strings where every odd position is a 1.

Solution:



10. Draw an NFA that recognises strings in the Kleene star of the language over over $\{0, 1\}$ containing only strings with at least two 0's and at most one 1.

Solution:



11. Define all of the above deterministic finite automata.

12. Use the pumping lemma to show that the following language over the alphabet $\{0, 1\}$ is not regular.

$$L = \{0^n 1^n \mid n \in \mathbb{N}\}$$

13. Use the pumping lemma to show that the following language over the alphabet $\{0, 1\}$ is not regular.

$$L = \{w \mid w\text{has an equal number of 0's and 1's}\}$$

References

[1] Michael Sipser. *Introduction to the Theory of Computation*. International Thomson Publishing, 3rd edition, 1996.