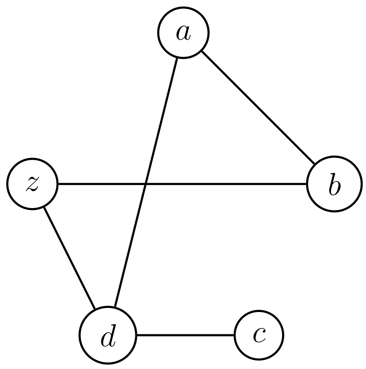
The following problems relate to the fundamentals of Graph Theory. They are mainly taken from Norman Biggs' Discrete Mathematics [1].

1. Consider the following graph [1].



   (a) Determine the vertex set.

   (b) Determine the edge set.

   (c) Determine the adjacency list.

   (d) For each of the vertices, determine the degree.

2. Professor McBrain and his wife April give a party at which there are four other married couples. Some pairs of people shake hands when they meet, but naturally no couple shake hands with each other. At the end of the party the Professor asks everyone else how many people they have shaken hands with, and he receives nine different answers [1].

   (a) Draw a graph representing the handshakes exchanged at the party.

   (b) How many people shook hands with April?

3. Three houses A, B, C each has to be connected to the gas, water and electricity supplies: G, W, E. Write down the adjacency list for the graph which represents this problem, and construct a pictorial representation of it. Can you find a picture in which the lines representing the edges do not cross? [1]

4. The pathways in a formal garden are to be laid out in the form of a wheel graph W_n whose vertex set is $V = \{0, 1, 2, \ldots, n\}$ and whose edges are:

$$\{0,1\}, \{0,2\}, \ldots, \{0,n\},$$
$$\{1,2\}, \{2,3\}, \ldots, \{n-1,n\}, \{n,1\}$$

   Describe a route around the pathways which starts and ends at vertex 0 and visits every vertex once only [1].

5. For each positive integer n we define the complete graph K_n to be the graph with n vertices in which each pair of vertices is adjacent [1].

   (a) How many edges has K_n?

   (b) For which values of n can you find a pictorial representation of K_n with the property that the lines representing the edges do not cross?

6. A 3-cycle in a graph is a set of three mutually adjacent vertices. Construct a graph with five vertices and six edges which contains no 3-cycles [1].

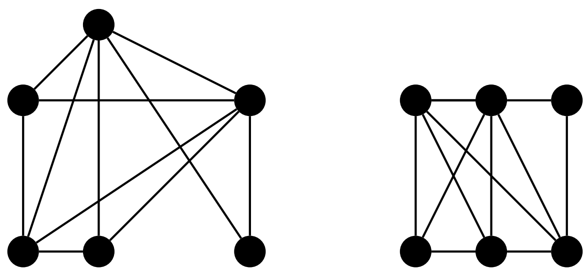
7. Consider the following two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ where:

$$\begin{aligned} V_1 &= \{a, b, c, d\} \\ E_1 &= \{\{a, b\}, \{b, c\}, \{b, d\}, \{c, d\}\} \\ V_2 &= \{1, 2, 3, 4\} \\ E_2 &= \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{3, 4\}\} \end{aligned}$$

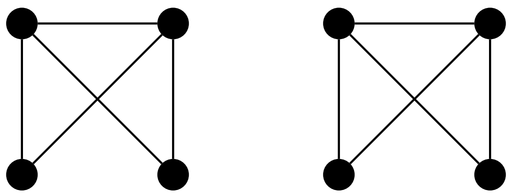
   (a) Draw a picture of each of the graphs.

   (b) Determine a bijection between the vertex sets that preserves the edges.

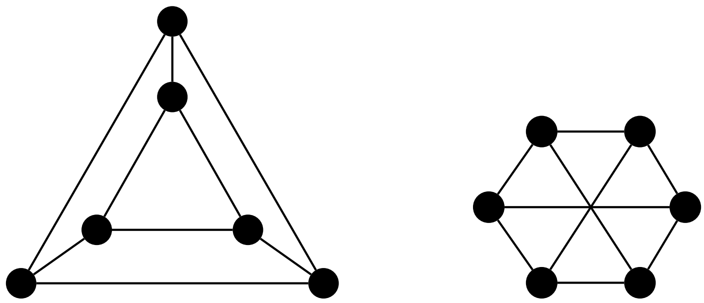
8. Determine if the following two graphs are isomorphic.



9. Determine three different isomorphisms between the following two graphs:



10. Prove that these graphs are not isomorphic [1]:



11. Find an isomorphism between the graphs defined by the following adjacency lists [1].

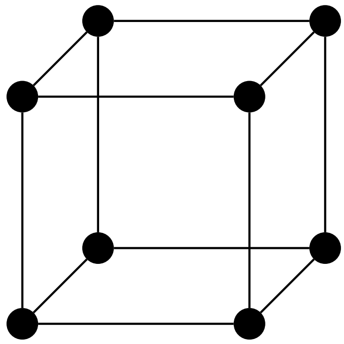
a	b	c	d	e	f	g	h	i	j
b	a	b	c	d	a	b	c	d	e
e	c	d	e	a	h	i	j	f	g
f	g	h	i	j	i	j	f	g	h

0	1	2	3	4	5	6	7	8	9
1	2	3	4	5	0	1	0	2	6
5	0	1	2	3	4	4	3	5	7
7	6	8	7	6	8	9	9	9	8

12. Show that the graph given by the following adjacency list [1]:

000	001	010	011	100	101	110	111
001	000	011	010	000	100	111	110
010	011	000	001	110	111	100	101
100	101	110	111	101	001	010	011

is isomorphic to the following graph:



References

[1] N. Biggs. *Discrete Mathematics.* Oxford science publications. OUP Oxford, 2002.